



VALENTIN KONSTANTINOVICH IVANOV  
(1908–1992)

Valentin Konstantinovich was born on October 1, 1908 in Sankt-Petersburg (Russia) in a family of a rail-road official. He obtained the initial education in a grade school, but after the parents' passage to Sverdlovsk, he continued the apprenticeship in the secondary school by the name of N. A. Nekrasov.

The bright mathematical gift of Valentin Konstantinovich appeared very earlier. The following description of his abilities was given in his attestation after graduate from the secondary school in 1925: "... Ivanov V. K., 17 years old, a son of an official before the Revolution and now, has exceptional mathematical talent with fine general status, possesses deep analytical intellect, rare opportunity to synthesis, reach erudition, and can work systematically; he has strong intuition, deep percipience, and strict scientific manner of expression of his thoughts in various disciplines...".

After graduate from the secondary school, Valentin Konstantinovich had met a difficult dilemma with continuation of his education. The matter was in the fact that at this time there was not any mathematical faculty in high-school institutes in Sverdlovsk, but it was very difficult to enter some metropolitan university. Moreover, in that historical period there existed so-called the "social choice": the priority entering had the graduates from school for the working youngsters, then the children of workers, and, only after them, youngsters of other social origin, and, as a result, an enrolle from a family of an official was in a very disadvantage condition. So, by an advice of his patents, he entered the Ural Polytechnical institute and graduated from it on 1930.

After graduation from the Institute, Valentin Konstantinovich worked for a year in the State Institution for Projection of the Metallurgical Plants and after, till 1938, in the Ural State Institution for Projection of the Machine-Building Plants. His first scientific publications was joined with this period of his professional activity. Simultaneously (from 1933), he extramurally study in the Leningrad State University, which he excellently graduated from in 1939. In this year, his first mathematical work was published that was joined with analysis of convergence of the successive approximation and Seidel methods for systems of linear algebraic equations.

From Valentin Konstantinovich's recollections, this time he was got with algebra and participated the algebraic seminar headed by P. G. Kontorovich and S. N. Chernikov. As a result, he succeeded to solve a row of algebraic problems and to prepare his Candidate Dissertation titled "Some Questions of the Matrix Polynomials Theory", which he successfully defended in the Moscow State University in 1941.

In the same year, his article was published, in which an elegant solution for the N. G. Chebotarev's problem on construction of irreducible divisors of the polynomial  $x^m - 1$ . Namely, his hypothesis that all such divisors have only  $-1, 0$  and  $1$  as the coefficients was proved for the case when the exponent  $m$  contains not more than two prime divisors, but in the general case, the hypothesis was rejected by a correspondent counter-example.

In 1938, Valentin Konstantinovich passed to the Sverdlovsk Mining Institute. There he worked as an assistant and further as an associate professor on the Chair of High Mathematics till 1947 with an interruption in 1941–1942 for the military service. In 1947, several his works were published (in collaboration with P. V. Gel'd and A. S. Mikulinskii) on mathematical physics.

One of his work (in collaboration with Ya. S. Dubov) was remarkable; it had anticipated one very important result on the theory of algebras over a field. Under rather simple assumptions, this result establishes nilpotency of a nil-algebra. Similar result was obtained in 50s and was joined with the names of M. Nagata and G. Higman; in literature on the ring theory, it is often named as the Nagata–Higman theorem. But only in 80s, it was discovered that this result crucially (but in different terms) had been firstly established in the mentioned work and published in 1943, but this work was not noticed by algebraists for a long time.

From September of 1947 and till the end of his life, Valentin Konstantinovich worked in the Ural State University by the name of A. M. Gor'kii. With this university the most fruitful period of his scientific and pedagogical activity is joined. Here he worked as an associate professor and, further, as an professor; from 1951 and till 1980 (with short interruption for a doctor of sciences dissertation) he was the Chief of the Mathematical Analysis Chair.

In 1948 (two years earlier of publication of the first volume of the L. Schwarz' monograph on the generalized functions), one noticeable V. K. Ivanov's work appeared where he had suggested a construction for quasi-functions and determined the Fourier transform for them. As it was opened later, these quasi-functions introduced by Ivanov coincide with the distributions of the slow increase, for which L. Schwarz had introduced the generalized Fourier transform. Further, in 1951–1952 V. K. Ivanov published interesting works, in which the classical (for the approximation theory) problem

is investigated on the uniform approximation of a function that is complex-valued and continuous on a compactum by quasi-polynomials (i.e., by a linear combination of a system of functions), and the subtle properties of the sets of points of the maximal deviation of polynomial for the function to be approximated were considered. In particular in these works, a convenient for applications criterion of the best approximating polynomial was obtained. This criterion differs from the known A. N. Kolmogorov's criterion.

During his work in the Sverdlovsk Mining Institute, Valentin Konstantinovich got into the problems that were interesting for geophysicists. These geophysical problems had interested him from the mathematical point of view. This stipulated his interest to solving the inverse problem of potential and to other problems of the exploratory geophysics. As a result in 1950s, a brilliant cycle of his works appeared that was devoted to these problems. In this cycle, it is possible to outline three directions of investigations.

1. Problems of uniqueness, equivalence, and stability of the inverse problems of the gravitational potential.

2. Methods for finding the harmonic moments of the anomalous masses on the basis of the gravitational observations.

3. Methods for solving the inverse problem of gravimetry and analytical continuation of the anomalous fields.

In works of the first direction, it is worthy to outline the article published in 1958, in which a generalization (in the two-dimensional case) of the classical P. S. Novikov's theorem on the uniqueness of the inverse problem of potential for the astro-bodies with known constant density. Also, it was shown that in the planar case there exist natural conditions on the boundary of the considered area, which provide not only the uniqueness but, also, the stability of solution for the inverse problem.

From 1953 up to 1955 V. K. Ivanov had a doctoral study in the Mathematical Institute by the name of V. A. Steklov of AN SSSR, where he defended the doctoral dissertation titled "Investigations on the inverse problem of the potential" (1955).

In work (1956), where some results of the doctoral dissertation were published, the proof is given for the uniqueness of solution for the inverse problem of Newtonian potential for a given body closed to the prescribed one; here, the formulation was investigated that is more general than one in earlier works by L. N. Sretenskii.

In the work devoted to the inverse problem of potential, an elegant proof of the following result was given: *for any distribution of the finite masses filling a finite area there exists the equi-potential distribution of the constant density.*

This result had given a powerful impulse for a large number of investigations implemented in further years by A. V. Tsirul'skii, V. N. Strakhov, S. V. Zakharov, V. G. Cherednichenko, and A. C. Morgulis.

In works joined with the second direction of investigations, it is necessary to outline the work published in 1950. This work essentially generalized and deepened researches of the previous explorers (by G. A. Gamburtsev, A. A. Zverev, A. P. Kazantsev, and others). Here, the compact and (that is the most important) general formulas were firstly given for defining the harmonic moments of the masses (with respect to the coordinates origin) of the outer gravitational field on the basis of the values of the outer

potential  $V$  and its normal derivative  $\partial V/\partial z$  that are given in the plane  $z = 0$ . These results by V. K. Ivanov were the initial point for investigations of other researchers and, first of all, of geophysicists.

Publications in 1956–1958 small in volume are very important for the theory of interpretation of the gravitational and magnetic anomalies. These publications are devoted to the inverse problem of the logarithmic potential, i.e., the third direction of the V. K. Ivanov's investigations. These works generated exclusively reach literature on this topic by other authors, mathematicians and geophysicists. This literature contains hundreds of titles where there are various generalizations of V. K. Ivanov's results and further development of his ideas (V. N. Strakhov, A. V. Tsurul'skii, Yu. A. Shaskin, L. E. Kazakova, A. A. Chudinova K. N. Gur'yanova, A. A. Kozmanova, and others).

In these works V. K. Ivanov firstly used the idea of characterization of a connected area  $D$  occupied by the masses by means of a conformal mapping of the unit circle onto the area  $D$ . For definition of this function in the case of the masses with the constant density, he had constructed the integral equation called *the equation of the inverse problem of the logarithmic potential*; he had investigated conditions for the finite solvability of such problems when the integral equation can be reduced to a system of nonlinear equations for determination of parameters of the conformal mapping. In addition, the formulas were obtained that are very important for the exploratory geophysics; the formulas describe the distance from the plane  $z = 0$  (or from the line  $y = 0$ ) to the set of peculiarities of the potential in the spatial and planar cases.

On the opinion of many authoritative scientists, these V. K. Ivanov's investigations in the topic of the inverse problems of the potential have the timeless value for geophysicists.

Working on the inverse problem of the potential, V. K. Ivanov had obtained a row of deep results in the theory of analytical and harmonic functions of many variables. In particular, he had proved the analogues of the Pólya's theorems on connection of the indicatrix of increase of the objective function with the support function of the convex envelope of the peculiarities for the function associated with the envelope. The approach was suggested to investigation of the increase of the entire functions of many complex variables, and their applications to summation of the multiple power series were considered. The same significant results were obtained by V. K. Ivanov in the analytical theory of numbers; these results are joined with the generalization of the Voronoy–Hardy identities and the Euler's multi-dimensional summational formulas.

As a rule, the inverse problems of the potential are reduced to solving the linear or nonlinear Fredholm equations of the first kind, which do not satisfy the classical Hadamard conditions of correctness. Apparently, this circumstance predetermined in early 60s the Valentin Konstantinovich's interest to investigation of the general theory of the ill-posed problems and methods for their solution.

By means of the notion of *the quasisolution* (1962) introduced by V. K. Ivanov as an element  $\bar{u}$ , which realizes the relation

$$\min \{ \|Au - f\| : u \in K \} = \|A\bar{u} - f\|$$

on a compact set  $K$ , it was succeeded to solve the problem of existence for the (quasi-) solution and to elaborate the first variational method (which is known now

as *Ivanov's method of quasisolutions* for constructing stable approximative solutions of the operator equation of the first kind in Hilbert and normed spaces. The results of these works were generalized both by V. K. Ivanov and other authors (I. N. Dombrovskaya and O. A. Liskovets) in various directions. In particular, the analogue for the method of the quasi-extensions for the topological spaces and operators had been constructed. It is necessary to note that the method of the quasisolutions together with the A. N. Tikhonov's regularization method is mostly usable method for solving the wide class of problems in the natural sciences.

The foundation of the another variational method belongs to V. K. Ivanov. This is *the method of the residual*. The idea of this method had been suggested by the American mathematician D. Phillips. In work 1966 for the equation with a nonlinear continuous operator, V. K. Ivanov proved the theorem on convergence of the method of the residual by using the idea of the compact imbedding. The case of the linear operator both continuous and closed was investigated in works of the V. K. Ivanov's apprentices (I. N. Dombrovskaya, V. V. Vasin, and V. P. Tanana). V. A. Morozov suggested *the principle of the residual* joined with this method as a regular rule for choice of the regularization parameter in the Tikhonov method.

The further direction in investigations fulfilled by V. K. Ivanov was concentrated on the problem of characterization of the sets of the uniform regularization and the joined problem for estimation of the errors of the variational methods.

The complete solution of thus problem was given in the work (1966) where the following remarkable result had been obtained.

*Let  $A$  be a linear completely continuous operator. A set  $M$  is a set of the uniform regularization if and only if it could be represented as the algebraic sum of a compactum and a finite-dimensional space.*

The knowledge of the continuity modulus for the inverse operator allows one to obtain the estimation of the error of the method used for solving the operator equation. In the work of 1966 (together with T. I. Korolyuk) for the important case when the problem operator commutes with the informational operator, the constructive formulas were obtained for the continuity modulus. This allowed one to create new effective techniques in the theory of estimation of the errors for the method of solving the ill-posed problems. This new method was widely used in works by V. K. Ivanov's apprentices and his followers.

The work of 1967 is of a special interest. Here, for the integral Fredholm equations, the necessary and sufficient conditions were formulated for the choice of the regularization parameter (in dependence on the error of the input data) for providing the convergence of the Tikhonov regularized solutions in the spaces  $L_2[a, b]$  and  $C[a, b]$ .

Also, V. K. Ivanov developed another approach to constructing the regularized family of the approximative solutions. This approach is based on application of the Fourier partial sums under corresponding choice of the number of members of the series when this number was agreed with the errors in the input data.

V. K. Ivanov's contribution into developing the stable methods for solving nonclassical problem of the mathematical physics is also interesting and significant. Investigations on the ill-posed problems were summarized in the monograph (together with V. V. Vasin and V. P. Tanana), which had been republished abroad [1].

Together with A. N. Tikhonov and M. M. Lavrent'ev, V. K. Ivanov is the acknowledged founder of the theory of the ill-posed problems, the theory that crucially changed the image of the contemporary natural sciences. For the cycle of works on the theory of the ill-posed problems, V. K. Ivanov and A. N. Tikhonov were awarded by the Lenin Award in 1966.

In 1970, V. K. Ivanov was elected as the correspondent-member of the USSR Academy of Sciences on the Mathematical Division.

Beginning from 1961 in connection with creation of the Sverdlovsk Division of the Mathematical Institution by the name of V. A. Steklov, Valentin Konstantinovich Ivanov becomes one of the first its collaborator. Here, for many years he headed the Mathematical Analysis Department, whose main topic was joined with generalized functions (called, also, *the distributions*). The theory of the generalized functions was another direction of investigations, with which V. K. Ivanov was joined through all his creative life.

The beginning of these investigations was made by the pioneer work (1948) mentioned above and, further (1950), which were devoted to the generalized Fourier transform defined on the quasi-functions. The quasi-functions were introduced as the ideal elements, which are the images of some family of operators on the space of integrable functions. The suggested construction of the space of quasi-functions that contains the integrable functions, slowly increasing functions, and their "generalized derivatives" allowed one to define the generalized Fourier transform for them. This definition is correct in the sense that the generalized Fourier transform coincides with the classical one on such elements where this transform can be defined. These works extended opportunities of the method of integral transformations in the theory of the linear equations of the mathematical physics.

The most part of the V. K. Ivanov's works on the theory of the generalized functions is devoted to the problem of multiplication of such functions. This problem appears in solving the nonlinear differential equations. The essence of this actual problem is in the fact that on the space of distributions it is impossible to introduce the multiplication with the associativity property:

$$\left(\frac{1}{x} \cdot x\right) \cdot \delta \neq \frac{1}{x} \cdot (x \cdot \delta).$$

This hampers application of the distributions to the nonlinear analysis. In the series of the V. K. Ivanov's works in 70s–80s, various approaches to definition of the multiplication of the distributions were suggested; in them, it is possible to outline two basic ones.

The first approach is in construction of the sub-algebras of the distributions sub-space that contains the generalized functions mostly important for applications. In works of this direction, the construction was suggested for various multiplications and corresponding (associative and commutative) sub-algebras, which contain such distributions as the delta-function and its derivatives and, also, the main (by the Hadamard) parts of the functions  $f(x) = 1/x^n$ , the Heaviside function, and others.

The second approach is in construction of some algebra, which is an extension of the space of distributions. Later, such an approach was successfully used by D. F. Colombo

in his constructed theory of multiplication. In works of this direction, the spaces of distributions were extended up to the algebras, which were constructed on the basis of application of various methods: the method of the asymptotic expansions, the method of approximation of the distributions by the infinitely differentiable functions, and the method of approximations by the combinations of the analytical functions in the upper and lower half-planes.

Some of these results on the multiplication of the generalized functions and their applications were described in the monograph [2] (together with V. V. Perminov).

In the last period of the Valentin Konstantinovich's life, the works on the generalized functions were devoted to solving problems for elimination of the divergence of various types that appear in investigations of problems, which are incorrect by Hadamar, in the quantum theory of the field and other scopes of the natural science.

In these works the construction of the quasi-values was suggested that allows one to separate some regular part in the diverging series, integrals, and  $S$ -matrices similarly to the main part by the Hadamard.

In works of 1987–1992, the construction of the generalized functions was suggested, to which the non-bounded operators more general than the differential ones can be applied. This provided the opportunity for application of the constructed generalized functions to investigation of the wide class of the operator and differential-operator equations.

These results were described in monograph [3] (together with I. V. Mel'nikova and A. I. Filinkov).

It is worthy specially to note the unrepeatable style that is character for the works by Valentin Konstantinovich: the thoroughness in realization of intentions, purity and pithiness of describing the materials, simplicity and exactness of the formulations.

V. K. Ivanov deeply reacted not only onto the actual problems of mathematics, but, also, onto the philosophical aspects of relations between the theoretical and applied investigations in the science. By his opinion, the synthesis must be in the theory and applications. He declared many times that it is impossible to achieve the highest level of results by solving only applied problems without serious mathematical generalization; but in the turn, neither one pure mathematics itself can be productive. The practice inseminates the theory, gives the push for its developing, and lighted up the problems for investigations. He imaginary compared the pure mathematics with Antaeus, which loses his life-force being torn off the Earth.

In Valentin Konstantinovich, the gift of a scientist was successfully joined with the talent of a pedagogue. During his work in the Ural State University, he had read practically all mathematical courses, in which he always touched the most interesting aspects of the history of mathematical ideas and methods and their significance in other scopes and using in applications. His lectures differed by unusual emotionality, elegancy and entirety, they fascinated the listeners and generated deep impression.

In the Ural State University on the Chair of the Mathematical Analysis headed by Valentin Konstantinovich, the scientific seminar was working for many years. On the seminar, both the beginning young scientists and highly experienced ones made presentations, they were from the institutes of high-education and the scientific-research institutions of Sverdlovsk and many other cities of the USSR. All they always noted

the atmosphere of benevolence and interested discussion that was on this seminar, and the amazing erudition of Valentin Konstantinovich, which always made the indelible impression.

The scientific-pedagogical activity of V. K. Ivanov in the Ural State University favored for becoming the University into a large center of the mathematical education and the mathematical science in the Ural Region. The organizational activity of Valentin Konstantinovich was significant and fruitful for the science and education: he was the University pro-rector on the scientific work, the Chairman of the Directorate of the Ural Mathematical Society, the Chairman of the Board for dissertation defence, the member of the Editors Board in a row of scientific magazines, the member of the Inter-Departmental Board on Coordination of Scientific Investigations in the Presidium of the Ural Branch the the USSR Academy of Sciences, the member of the Scientific-Methodical Board of the USSR Ministry of the High Education on Mathematics, and others.

In spite of the amenity and amazing modesty, Valentin Konstantinovich always showed the fidelity and firmness when the matter touched his principles.

His life credo was: never to lose his dignity, never go to bargains with his conscience. He liked to quote the following German proverb:

*Gelden verloren — nichts verloren,  
Zeit verloren — wenig verloren,  
Freund verloren — viel verloren,  
Sich verloren — alles verloren.*

(To lose money means to lose nothing; to lose the time means to lose a little; to lose a friend means to lose many; to lose yourselves means to lose all).

Valentin Konstantinovich was a man of the wide interests and superordinate abilities: with the hot he studied various foreign languages and knew well German, English, French, and Italian; he had deep knowledge in history, painting, and literature.

The depth and richness of the scientific ideas, intelligence, exceptional generosity and benevolence had made him the Teacher for many ones.

*V. N. Strakhov, V. V. Vasin, V. V. Arestov  
K. N. Gur'yanova, I. V. Mel'nikova, L. N. Shevrin*

## References

1. V. K. Ivanov, V. V. Vasin, and V. P. Tanana, *Theory of Linear Ill-Posed Problems and its Applications*. VSP, Utrecht–Boston–Köln–Tokyo, 2002.
2. V. K. Ivanov and V. V. Perminov, *Nonlinear Convolution Operators: Ordinary Differential Equations*. Ural State University Publisher, Sverdlovsk, 1989 (in Russian).
3. V. K. Ivanov, I. V. Mel'nikova, and A. I. Filinkov. *Operator-Differential Equations and Ill-Posed Problems*. Nauka, Moscow, 1995 (in Russian).