THE NATURAL AFFINORS ON THE $r$-JET PROLONGATIONS OF A VECTOR BUNDLE

Abstract. It is known that for integers $m \geq 2$, $n \geq 1$ and $r \geq 3$ there are only three $r$-jet prolongations of a vector bundle $E$ with $m$-dimensional bases and $n$-dimensional fibers. The first one is the usual $r$-jet prolongation $J^r E$, the second one is the vertical $r$-jet prolongation $J^r_v E$ and the third one is the $[r]$-jet prolongation $J^{[r]} E$. In this paper for integers $m \geq 2$, $n \geq 1$ and $r \geq 1$ we classify all natural affinors on $F^r E$, where $F^r E$ denotes $J^r E$ or $J^r_v E$ or $J^{[r]} E$. As corollaries we obtain similar results for $F^r E^*$, $(F^r E)^*$ and $(F^r E^*)^*$ instead of $F^r E$.

Introduction

One can prove (a paper in preparation) that for integers $r \geq 3$ and $m \geq 2$ there are only three $r$-jet prolongations of a vector bundle $E$ with $m$-dimensional basis. Namely, we have the usual $r$-jet prolongation $J^r E$ of $E$, the vertical $r$-jet prolongation $J^r_v E$ of $E$ and the $[r]$-jet prolongation $J^{[r]} E$ of $E$.

In [15] for integers $m \geq 2$, $n \geq 1$ and $r \geq 1$ we classified all natural linear operators $A$ lifting a linear vector field $X$ from a vector bundle $E$ with $m$-dimensional basis and $n$-dimensional fibers into a vector field $A(X)$ on $F^r E$, where $F^r E$ denotes $J^r E$ or $J^r_v E$ or $J^{[r]} E$. In the case $F^r E = J^r E$ we proved that $A(X)$ is a constant multiple of the flow operator $J^r X$. In the case $F^r E = J^r_v E$ we proved that $A(X)$ is a linear combination of the flow operator $J^r_v X$ and some explicitly constructed linear natural operator $U^{(1)}(X)$. In the case $F^r E = J^{[r]} E$ we proved that $A(X)$ is a linear combination of the flow operator $J^{[r]} X$ and some explicitly constructed linear natural operator $U^{(1)}(X)$.

An affinor $B$ on a manifold $M$ is a tensor field of type $(1,1)$ on $M$.

Key words and phrases: vector gauge bundle functors, natural operators, natural affinors, jets.

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A natural affinor $B$ on $F^rE$ is a system of invariant (with respect to vector bundle isomorphisms onto open vector subbundles) affinors $B : TF^rE \rightarrow TF^rE$ on $F^rE$ for any vector bundle $E$ with $m$-dimensional basis and $n$-dimensional fibers.

In the present paper for integers $m \geq 2$, $n \geq 1$ and $r \geq 1$ we classify all natural affinors $B$ on $F^rE$, where $F^rE$ denotes $J^rE$ or $J^r_E$ or $J^{[r]}E$. In the case $F^rE = J^rE$ we prove that $B$ is a constant multiple of the identity affinor $Id$ on $J^rE$. In the case $F^rE = J^r_E$ we proved that $B$ is a linear combination of the identity affinor $Id$ on $J^r_E$ and some explicitly constructed natural affinor $U$ on $J^r_E$. In the case $F^rE = J^{[r]}E$ we prove that $B$ is a linear combination of the identity affinor $Id$ on $J^{[r]}E$ and some explicitly constructed natural affinor $V$ on $J^{[r]}E$. As corollaries we obtain similar results for $F^rE^*$, $(F^rE)^*$ and $(F^rE^*)^*$ instead of $F^rE$.

Natural affinors can be used to study torsions of connections, see [5]. That is why they have been classified in many papers, [1]–[4], [6]–[14], [17]. e.t.c.

The category of vector bundles with $m$-dimensional bases and vector bundle maps with local diffeomorphisms as base maps will be denoted by $\mathcal{VB}_m$.

The category of vector bundles with $m$-dimensional bases and $n$-dimensional fibers and vector bundle isomorphisms onto open vector subbundles will be denoted by $\mathcal{VB}_{m,n}$.

The trivial vector bundle $\mathbb{R}^m \times \mathbb{R}^n$ over $\mathbb{R}^m$ with standard fiber $\mathbb{R}^n$ will be denoted by $\mathbb{R}^{m,n}$.

The coordinates on $\mathbb{R}^m$ will be denoted by $x^1, ..., x^m$. The fiber coordinates on $\mathbb{R}^{m,n}$ will be denoted by $y^1, ..., y^n$.

All manifolds are assumed to be finite dimensional and smooth. Maps are assumed to be smooth, i.e. of class $C^\infty$.

1. The $r$-jet prolongations of a vector bundle

The $r$-jet prolongation functor

Given a $\mathcal{VB}_m$-object $p : E \rightarrow M$ the $r$-jet prolongation $J^rE$ of $E$ is a vector bundle

$$J^rE = \{j^r_x \sigma \mid \sigma \text{ is a local section of } E, \ x \in M\}$$

over $M$. Every $\mathcal{VB}_m$-map $f : E_1 \rightarrow E_2$ covering $f : M_1 \rightarrow M_2$ induces a vector bundle map $J^r f : J^rE_1 \rightarrow J^rE_2$ covering $f$ such that

$$J^r f (j^r_x \sigma) = j^r_{f(x)} (f \circ \sigma \circ f^{-1}), \quad j^r_x \sigma \in J^rE_1 .$$

The functor $J^r : \mathcal{VB}_m \rightarrow \mathcal{VB}_m$ is a fiber product preserving vector gauge bundle functor.
The natural affinors

The vertical r-jet prolongation functor
Given a $\mathcal{VB}_m$-object $p : E \to M$ the vertical r-jet prolongation $J^r_v E$ of $E$ is a vector bundle

$$J^r_v E = \{ j^r_x \sigma \mid \sigma : M \to E_x, \ x \in M \}$$

over $M$. Every $\mathcal{VB}_m$-map $f : E_1 \to E_2$ covering $f : M_1 \to M_2$ induces a vector bundle map $J^r_v f : J^r_v E_1 \to J^r_v E_2$ covering $f$ such that

$$J^r_v f (j^r_x \sigma) = j^r_f (f \circ \sigma \circ f^{-1}) , \quad j^r_x \sigma \in J^r_v E_1.$$

The functor $J^r_v : \mathcal{VB}_m \to \mathcal{VB}_m$ is a fiber product preserving vector gauge bundle functor.

The $[r]$-jet prolongation functor
Let $p : E \to M$ be a $\mathcal{VB}_m$-object. For any $x \in M$ we have an unital associative algebra homomorphism $t_{[r]}^{[r]} : J^r_x (M, \mathbb{R}) \to gl(J^r_x (M, \mathbb{R}))$ given by

$$t_{[r]}^{[r]} (j^r_x \gamma)(j^r_x \eta) = j^r_x (\gamma \eta) - j^r_x (\eta(x) \gamma) + j^r_x (\eta(x) \gamma(x)) ,$$

$j^r_x \eta, j^r_x \gamma \in J^r_x (M, \mathbb{R}), \eta(x), \gamma(x) : M \to \mathbb{R}$ are constant maps. We have a vector bundle

$$J^{[r]} E = \bigcup_{x \in M} \text{Hom}_{t_{[r]}^{[r]}} (J^{[r]} C^\infty_{C^1}(E), J^r_x (M, \mathbb{R}))$$

over $M$. Here $\text{Hom}_{t_{[r]}^{[r]}} (J^{[r]} C^\infty_{C^1}(E), J^r_x (M, \mathbb{R}))$ is the vector space of all module homomorphisms over $t_{[r]}^{[r]} : J^r_x (M, \mathbb{R}) \to gl(J^r_x (M, \mathbb{R}))$ from the (free) $J^r_x (M, \mathbb{R})$-module $J^{[r]} C^\infty_{C^1}(E)$ of r-jets at $x$ of germs at $x$ of fiber linear maps $E \to \mathbb{R}$ into the $gl(J^r_x (M, \mathbb{R}))$-module $J^r_x (M, \mathbb{R})$. We call $J^{[r]} E$ the $[r]$-jet prolongation of $E$. Every $\mathcal{VB}_m$-map $f : E_1 \to E_2$ covering $f : M_1 \to M_2$ induces a vector bundle map $J^{[r]} f : J^{[r]} E_1 \to J^{[r]} E_2$ covering $f$ such that

$$J^{[r]} f (\Phi)(j^{[r]}_f (x) \xi) = J^{[r]} (f_\ast, id_\mathbb{R}) \circ \Phi(j^r_x (\xi \circ f))$$

for any $\Phi \in \text{Hom}_{t_{[r]}^{[r]}} (J^{[r]} C^\infty_{C^1}(E), J^r_x (M, \mathbb{R})), \ x \in M_1$ and any fiber linear map $\xi : E_2 \to \mathbb{R}$. The correspondence $J^{[r]} : \mathcal{VB}_m \to \mathcal{VB}_m$ is a fiber product preserving gauge bundle functor of order $r$, [16].

Remark 1. One can show that $J^r E$ and $J^{[r]} E$ can be constructed similarly as $J^{[r]} E$ using some other algebra homomorphisms $t_{x} : J^r_x (M, \mathbb{R}) \to gl(J^r_x (M, \mathbb{R}))$ instead of $t_{[r]}^{[r]}$. This justifies the name $[r]$-jet prolongation. If $r \geq 3$ and $m \geq 2$ then only $J^r E$, $J^{[r]} E$ and $J^{[r]} E$ admit such reconstruction (a paper in preparation).

From now on $F^r E$ denotes $J^r E$ or $J^{[r]} E$ or $J^{[r]} E$. 
2. The natural linear operators lifting linear vector fields to $F^rE$

In this section we will cite some results of [15].

Let $p : E \to M$ be a $\mathcal{VB}_{m,n}$-object. A projectable vector field $X$ on $E$ is called linear if $X : E \to TE$ is a vector bundle map from $p : E \to M$ into $Tp : TE \to TM$. Equivalently, the flow $Fl^X_l$ of $X$ is formed by $\mathcal{VB}_{m,n}$-maps. The space of linear vector fields on $E$ will be denoted by $\mathcal{X}_{lin}(E)$.

A natural linear operator $A : T_{lin}|\mathcal{VB}_{m,n} \to TF^r$ is an $\mathcal{VB}_{m,n}$-invariant family of $R$-linear operators $A : X_{lin}(E) \to X(F^rE)$ for any $\mathcal{VB}_{m,n}$-object $E$. The $\mathcal{VB}_{m,n}$-invariance means that for any $\mathcal{VB}_{m,n}$-map $f : E_1 \to E_2$ and any $f$-conjugate linear vector fields $X$ and $Y$ on $E_1$ and $E_2$ the vector fields $A(X)$ and $A(Y)$ are $\mathcal{VB}_{m,n}$-conjugate.

**Example 1.** (The flow operator) Let $X$ be a linear vector field on a $\mathcal{VB}_{m,n}$-object $p : E \to M$. The flow $Fl^X_l$ of $X$ is formed by $\mathcal{VB}_{m,n}$-maps on $E$. Applying functor $F^r$ we obtain a flow $F^r(Fl^X_l)$ on $F^rE$. The vector field $F^rX$ on $F^rE$ corresponding to the flow $F^r(Fl^X_l)$ is called the flow prolongation of $X$. The correspondence $F^r : T_{lin}|\mathcal{VB}_{m,n} \to TF^r$, $X \to F^rX$, is a natural linear operator.

**Example 2.** Given a linear vector field $X$ on a $\mathcal{VB}_{m,n}$-object $E$ covering a vector field $X$ on $M$ we define a vertical vector field $V^{<1>}(X)$ on $J^r_vE$ as follows. Let $y = j^r_v\sigma \in J^r_vE$, $\sigma : M \to E_x$, $x \in M$. We put

$$V^{<1>}(X)(y) = (y, j^r_v(X\sigma(x))) \in \{y\} \times (J^r_v)_xE = V^r_yJ^r_vE \subset T_yJ^r_vE,$$

where $X\sigma(x) : M \to E_x$ is the constant map. The correspondence $V^{<1>} : T_{lin}|\mathcal{VB}_{m,n} \sim TF^r$, $X \to F^rX$, is a natural linear operator.

**Example 3.** Given a linear vector field $X$ on a $\mathcal{VB}_{m,n}$-object $E$ covering a vector field $X$ on $M$ and a module homomorphism $\Phi : J^rC_\infty \cdot f.l(E) \to J^r_x(M, R)$ over $t^r_x : J^r_x(M, R) \to gl(J^r_x(M, R))$ (i.e. $\Phi \in J^r_x(E, x \in M)$) we have a linear map $\Phi_X : J^rC_\infty \cdot f.l(E) \to J^r_x(M, R)$ given by

$$\Phi_X(\sigma) = j^r_x(X\gamma(x)),$$

$\sigma \in J^rC_\infty \cdot f.l(E)$, $\gamma : M \to R$, $j^r_x\gamma = \Phi(\sigma)$, $X\gamma(x) : M \to R$ is the constant map. The linear map $\Phi_X : J^rC_\infty \cdot f.l(E) \to J^r_x(M, R)$ is module homomorphisms over $t^r_x : J^r_x(M, R) \to gl(J^r_x(M, R))$ as easily to verify. Consequently, we have vertical vector field $U^{(1)}(X)$ on $J^r_vE$ by

$$U^{(1)}(X)_{\Phi} = (\Phi, \Phi_X) \in \{\Phi\} \times J^r_xE = V_{\phi}J^r_vE,$$

$\Phi \in J^r_xE$, $x \in M$. The correspondence $U^{(1)} : T_{lin}|\mathcal{VB}_{m,n} \sim TJ^r$ is a natural linear operator.

In [15] we proved the following classification theorem.
The natural affinors

THEOREM 1. ([15]) Let $r \geq 1$, $m \geq 2$ and $n \geq 1$ be integers.

(a) Any natural linear operator $A : T_{\text{lin}}|\mathcal{V}B_{m,n} \rightarrow TJ^r$ is a constant multiple of the flow operator $J^r$.

(b) Any natural linear operator $A : T_{\text{lin}}|\mathcal{V}B_{m,n} \rightarrow TJ^r_v$ is a linear combination with real coefficients of the flow operator $J^r_v$ and $V^{<1>}$.

(c) Any natural linear operator $A : T_{\text{lin}}|\mathcal{V}B_{m,n} \rightarrow TJ^r[v]$ is a linear combination with real coefficients of the flow operator $J^r[v]$ and the operator $U^{(1)}$.

3. Examples of natural affinors on $F^rE$

A $\mathcal{V}B_{m,n}$-natural affinor $B$ on $F^rE$ is a family of $\mathcal{V}B_{m,n}$-invariant affinors $B : TF^rE \rightarrow TF^rE$ for any $\mathcal{V}B_{m,n}$-object $E$. The invariance means that $B \circ TF^rf = TF^rf \circ B$ for any $\mathcal{V}B_{m,n}$-map $f$.

EXAMPLE 4. (The identity affinor) For any $\mathcal{V}B_{m,n}$-object $E$ we have the identity map $\text{Id} : TF^rE \rightarrow TF^rE$. The family $\text{Id}$ is a $\mathcal{V}B_{m,n}$-natural affinor on $F^rE$.

EXAMPLE 5. Let $p : E \rightarrow M$ be a $\mathcal{V}B_{m,n}$-object. Define $U : TJ^r_vE \rightarrow VJ^r_vE$ by

$$U(v) = (y, j^r_x(Tp(v)\sigma)) \in \{y\} \times (J^r_v)_xE = V_yJ^r_vE,$$

$v \in T_yJ^r_vE$, $y = j^r_x\sigma \in (J^r_v)_xE$, $x \in M$. Here $Tp(v)\sigma : M \rightarrow E_x$ is the constant map, the differential of $\sigma : M \rightarrow E_x$ at $Tp(v)$. The family $U$ is a $\mathcal{V}B_{m,n}$-natural affinor on $J^r_vE$.

EXAMPLE 6. Let $p : E \rightarrow M$ be a $\mathcal{V}B_{m,n}$-object. Define $V : TJ^{[r]}E \rightarrow VJ^{[r]}E$ by

$$V(v) = (\Phi, \Phi_{Tp(v)}) \in \{\Phi\} \times J^{[r]}_xE = V\Phi J^{[r]}E,$$

$v \in T\Phi J^{[r]}E$, $\Phi \in J^{[r]}_xE$, $x \in M$. More precisely, $\Phi : J^r\mathcal{C}_c^{\infty,1}(E) \rightarrow J^r_x(M, \mathbb{R})$ is a module homomorphism over $t^{[r]}_x : J^r_x(M, \mathbb{R}) \rightarrow gl(J^r_x(M, \mathbb{R}))$. $\Phi_{Tp(v)} : J^r\mathcal{C}_c^{\infty,1}(E) \rightarrow J^r_x(M, \mathbb{R})$, $\Phi_{Tp(v)}(j^{r}_{x}\xi) = j^{r}_{x}(Tp(v)\gamma)$, $j^{r}_{x}\gamma = \Phi(j^{r}_{x}\xi)$, $j^{r}_{x}\xi \in J^r\mathcal{C}_c^{\infty,1}(E)$, is also a module homomorphism over $t^{[r]}_x$, i.e. $\Phi_{Tp(v)} \in J^{[r]}_xE$, see Example 3. The family $V$ is a $\mathcal{V}B_{m,n}$-natural affinor on $J^{[r]}E$.

4. The main result

The main result of the present paper is the following classification theorem.

THEOREM 2. Let $r \geq 1$, $m \geq 2$ and $n \geq 1$ be integers.

(a) Any $\mathcal{V}B_{m,n}$-natural affinor $B$ on $J^rE$ is a constant multiple of the identity affinor $\text{Id}$.

(b) Any $\mathcal{V}B_{m,n}$-natural affinor $B$ on $J^r_vE$ is a linear combination with real coefficients of $\text{Id}$ and $U$. 

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Any $\mathcal{VB}_{m,n}$-natural affinor $B$ on $J^r E$ is a linear combination with real coefficients of $\text{Id}$ and $V$.

To prove Theorem 2 we need the following lemma.

**Lemma 1.** Let $r, m, n$ be as in Theorem 1. Let $B$ be a $\mathcal{VB}_{m,n}$-natural affinor on $F^r E$ such that $B \circ J^r X = 0$ for any linear vector field on $E$. Then $B = 0$.

**Proof of Lemma 1.** It is sufficient to show that $B = 0$ over $0 \in \mathbb{R}^m$. We fix a basis in the vector space $F^r_0 \mathbb{R}^{m,n}$.

**Step 1.** $B$ is of vertical type. Consider

$$T \pi \circ B : (TF^r \mathbb{R}^{m,n})_0 \cong \mathbb{R}^m \times F^r_0 \mathbb{R}^{m,n} \times F^r_0 \mathbb{R}^{m,n} \rightarrow T_0 \mathbb{R}^m.$$

Using the invariance of $B$ with respect to the fiber homotheties we deduce that $T \pi \circ B(a, u, v) = T \pi \circ B(a, tu, tv)$ for any $u, v \in F^r_0 \mathbb{R}^{m,n}$, $a \in \mathbb{R}^m$, $t \neq 0$. Then $T \pi \circ B(a, u, v) = T \pi \circ B(a, u, 0)$ for $u, v, a$ as above. But $(a, u, 0) = \mathcal{F}^r(a^i \frac{\partial}{\partial x^i}) u$. Then $T \pi \circ B(a, u, 0) = 0$ because of the assumption of the lemma. Then $B$ is of vertical type.

**Step 2.** $B = 0$. Consider

$$pr_2 \circ B : (VF^r \mathbb{R}^{m,n})_0 \cong F^r_0 \mathbb{R}^{m,n} \times F^r_0 \mathbb{R}^{m,n} \rightarrow F^r_0 \mathbb{R}^{m,n},$$

where $pr_2 : (VF^r \mathbb{R}^{m,n})_0 \cong F^r_0 \mathbb{R}^{m,n} \times F^r_0 \mathbb{R}^{m,n} \rightarrow F^r_0 \mathbb{R}^{m,n}$ is the projection onto the second factor. Using the invariance of $B$ with respect to the fiber homotheties we deduce that $pr_2 \circ B(a, u, v) = tpr_2 \circ B(a, u, v)$ for $a, u, v$ as in Step 1. Then $pr_2 \circ B(a, u, v)$ is a linear combination of the coefficients of $u$ and $v$ (with respect to the obvious basis in the vector space $F^r_0 \mathbb{R}^{m,n}$) with coefficient being smooth maps in $a$ because of the homogeneous function theorem. On the other hand, since $B$ is an affinor $B(a, u, v)$ is a linear combination of the coefficients of $a$ and $v$ with coefficient being smooth functions in $u$. We see that $pr_2 \circ B(a, u, 0) = 0$ by the same reason as in Step 1. We also see that $(0, v, v) = \mathcal{F}^r L_v$, where $L$ is the Liouville vector field on $\mathbb{R}^{m,n}$, and consequently $B(0, v, v) = 0$ because of the assumption of the lemma. Hence $B(a, u, v) = 0$ for all $a, u, v$ as above. □

**Proof of Theorem 2.** Lemma 1 says that a $\mathcal{VB}_{m,n}$-natural affinor $B$ on $F^r E$ is uniquely determined by the vector fields $B \circ J^r X$ for linear vector fields $X$ on $E$. On the other hand $B \circ J^r X$ is a $\mathcal{VB}_{m,n}$-natural linear operator lifting linear vector fields on $E$ into $F^r E$. Using Theorem 1 (a) we know that $B \circ J^r X = aJ^r X$. Hence $B = a\text{Id}$. This complete the proof of Theorem 2 for $F^r E = J^r E$. Using Theorem 1 (b) we complete the proof of Theorem 2 for $F^r E = J^r E$. Using Theorem 1 (c) we complete the proof of Theorem 2 for $F^r E = J^r E$. □
5. Some versions on the main result

We say that an affinor $B : TE \to TE$ on a vector bundle $E$ is linear if $B(X)$ is for any linear vector field $X$ on $E$.

**Proposition 1.** Let $B$ be a $\mathcal{VB}_{m,n}$-natural affinor on $F^r E$ (resp. $F^r E^*$, $(F^r E)^*$, $(F^r E^*)^*$). Then $B$ is linear.

**Proof.** Observe that a vector field $X$ on a vector bundle $E$ is linear iff $(b_t)_* X = tX$ for $t \neq 0$, where $b_t$ is the fiber homothety on $E$.

Observe also that $F^r b_t$ is the fiber homothety on $F^r \mathbb{R}^{m,n}$ if $b_t$ is the fiber homothety on $\mathbb{R}^{m,n}$.

Let $X$ be a linear vector field on $F^r \mathbb{R}^{m,n}$. Then $(F^r b_t)_* (B(X)) = t B(X)$ because of the invariance of $B$ with respect to $b_t$. Then $B(X)$ is a linear vector field on $F^r (\mathbb{R}^{m,n})$.

Similar method we use for $F^r E^*$, $(F^r E)^*$ and $(F^r E^*)^*$.

There is a natural involution (dualization) $()^* : \mathcal{VB}_{m,n} \to \mathcal{VB}_{m,n}$, $E \to E^*$, $f \to (f^{-1})^*$. Given a linear vector field on a vector bundle $E$ we have the dual linear vector field $X^*$ on $E^*$ such that if $f_t$ is the flow of $X$ then $(f_t^{-1})^*$ is the flow of $X^*$.

**Lemma 2.** Let $B : TE \to TE$ be a linear affinor on a vector bundle $E$. Then there is one and only one linear affinor $B^* : TE^* \to TE^*$ on the dual vector bundle $E^*$ such that $B^*(X^*) = (B(X))^*$ for any linear vector field $X$ on $E$.

**Proof.** We use local vector bundle coordinate argument. If

$$B = a^i_j(x) dx^i \otimes \frac{\partial}{\partial x^j} + b^k_{ik}(x) y^k dx^i \otimes \frac{\partial}{\partial y^k} + c^k_{sk}(x) dy^k \otimes \frac{\partial}{\partial y^s},$$

then

$$B^* = a^i_j(x) dx^i \otimes \frac{\partial}{\partial x^j} + b^k_{ik}(x) v^k dx^i \otimes \frac{\partial}{\partial v^k} + c^k_{sk}(x) dv^k \otimes \frac{\partial}{\partial v^s},$$

where $(x^i, y^k)$ are vector bundle coordinates on $E$ and $(x^i, v^k)$ are the dual vector bundle coordinates on $E^*$. □

Using Proposition 1 and Lemma 2 one can easily deduce from Theorem 2 the following versions of Theorem 2.

**Theorem 3.** Let $r \geq 1$, $m \geq 2$ and $n \geq 1$ be integers.

(a) Any $\mathcal{VB}_{m,n}$-natural affinor $B$ on $J^r E^*$ is a constant multiple of the identity affinor $Id$.

(b) Any $\mathcal{VB}_{m,n}$-natural affinor $B$ on $J^r E^*$ is a linear combination with real coefficients of $Id$ and $U$.

(c) Any $\mathcal{VB}_{m,n}$-natural affinor $B$ on $J^r E^*$ is a linear combination with real coefficients of $Id$ and $V$. 

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THEOREM 4. Let $r \geq 1$, $m \geq 2$ and $n \geq 1$ be integers.

(a) Any $\mathcal{VB}_{m,n}$-natural affinor $B$ on $(J^r E)^*$ is a constant multiple of the identity affinor $\text{Id}$.

(b) Any $\mathcal{VB}_{m,n}$-natural affinor $B$ on $(J_v^r E)^*$ is a linear combination with real coefficients of $\text{Id}$ and $U^*$.

(c) Any $\mathcal{VB}_{m,n}$-natural affinor $B$ on $(J[v]^r E)^*$ is a linear combination with real coefficients of $\text{Id}$ and $V^*$.

THEOREM 5. Let $r \geq 1$, $m \geq 2$ and $n \geq 1$ be integers.

(a) Any $\mathcal{VB}_{m,n}$-natural affinor $B$ on $(J^r E^*)^*$ is a constant multiple of the identity affinor $\text{Id}$.

(b) Any $\mathcal{VB}_{m,n}$-natural affinor $B$ on $(J_v^r E^*)^*$ is a linear combination with real coefficients of $\text{Id}$ and $U^*$.

(c) Any $\mathcal{VB}_{m,n}$-natural affinor $B$ on $(J[v]^r E^*)^*$ is a linear combination with real coefficients of $\text{Id}$ and $V^*$.

References


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