

Contents

| | |
|--|-----|
| Preface | vii |
| Introductory Remarks: a Guide for the Reader | xi |

PART I: BASIC CONCEPTS

| | |
|--|----|
| §1. Affine Sets | 3 |
| §2. Convex Sets and Cones | 10 |
| §3. The Algebra of Convex Sets | 16 |
| §4. Convex Functions | 23 |
| §5. Functional Operations | 32 |

PART II: TOPOLOGICAL PROPERTIES

| | |
|---|----|
| §6. Relative Interiors of Convex Sets | 43 |
| §7. Closures of Convex Functions | 51 |
| §8. Recession Cones and Unboundedness | 60 |
| §9. Some Closedness Criteria | 72 |
| §10. Continuity of Convex Functions | 82 |

PART III: DUALITY CORRESPONDENCES

| | |
|---|-----|
| §11. Separation Theorems | 95 |
| §12. Conjugates of Convex Functions | 102 |
| §13. Support Functions | 112 |
| §14. Polars of Convex Sets | 121 |
| §15. Polars of Convex Functions | 128 |
| §16. Dual Operations | 140 |

PART IV: REPRESENTATION AND INEQUALITIES

| | |
|--|-----|
| §17. Carathéodory's Theorem | 153 |
| §18. Extreme Points and Faces of Convex Sets | 162 |
| §19. Polyhedral Convex Sets and Functions | 170 |
| §20. Some Applications of Polyhedral Convexity | 179 |
| §21. Helly's Theorem and Systems of Inequalities | 185 |
| §22. Linear Inequalities | 198 |

PART V: DIFFERENTIAL THEORY

| | |
|---|-----|
| §23. Directional Derivatives and Subgradients | 213 |
| §24. Differential Continuity and Monotonicity | 227 |
| §25. Differentiability of Convex Functions | 241 |
| §26. The Legendre Transformation | 251 |

PART VI: CONSTRAINED EXTREMUM PROBLEMS

| | |
|--|-----|
| §27. The Minimum of a Convex Function | 263 |
| §28. Ordinary Convex Programs and Lagrange Multipliers | 273 |
| §29. Bifunctions and Generalized Convex Programs | 291 |
| §30. Adjoint Bifunctions and Dual Programs | 307 |
| §31. Fenchel's Duality Theorem | 327 |
| §32. The Maximum of a Convex Function | 342 |

PART VII: SADDLE-FUNCTIONS AND MINIMAX THEORY

| | |
|---|-----|
| §33. Saddle-Functions | 349 |
| §34. Closures and Equivalence Classes | 359 |
| §35. Continuity and Differentiability of Saddle-functions | 370 |
| §36. Minimax Problems | 379 |
| §37. Conjugate Saddle-functions and Minimax Theorems | 388 |

PART VIII: CONVEX ALGEBRA

| | |
|---|-----|
| §38. The Algebra of Bifunctions | 401 |
| §39. Convex Processes | 413 |
| Comments and References | 425 |
| Bibliography | 433 |
| Index | 447 |