

CONTENTS

INTRODUCTION.	vii
I. THE CLASSICAL LINEAR GROUPS.	1
II. TOPOLOGICAL GROUPS	25
III. MANIFOLDS.	68
IV. ANALYTIC GROUPS. LIE GROUPS	99
V. THE DIFFERENTIAL CALCULUS OF CARTAN	139
VI. COMPACT LIE GROUPS AND THEIR REPRESENTATIONS.	171
INDEX	215

THEORY OF LIE GROUPS

I

Some Notations Used in This Book

I. We denote by ϕ the empty set, by $\{a\}$ the set composed of the single element a .

If f is a mapping of a set A into a set B , and if X is a sub-set of B , we denote by $f^{-1}(X)$ the set of the elements $a \in A$ such that $f(a) \in X$. If g is a mapping of B into a third set C , we denote by $g \circ f$ the mapping which assigns to every $a \in A$ the element $g(f(a))$.

We use the signs \cup , \cap to represent respectively the intersection and the union of sets. If E_α is a collection of sets, the index α running over a set A , we denote by $\bigcup_{\alpha \in A} E_\alpha$ the union of all sets E_α and by $\bigcap_{\alpha \in A} E_\alpha$ their intersection. We denote by δ_{ij} the Kronecker symbol, equal to 1 if $i = j$ and to 0 if $i \neq j$.

II. If G is a group, we call "neutral element" the element ϵ of G such that $\epsilon\sigma = \sigma$ for every $\sigma \in G$.

We say that a sub-group H of G is "distinguished" if the conditions $\sigma \in G, \tau \in H$ imply $\tau\sigma\tau^{-1} \in H$.

If $\sigma = (a_{ij})$ represents a matrix, the symbol $|\sigma| = |a_{ij}|$ stands for the determinant of the matrix; $\delta p\sigma$ stands for the trace of the matrix.

If $\mathfrak{M}, \mathfrak{N}$ are vector spaces over the same field K , we call *product* of \mathfrak{M} and \mathfrak{N} , and denote by $\mathfrak{M} \times \mathfrak{N}$, the set of the pairs (\mathbf{e}, \mathbf{f}) with $\mathbf{e} \in \mathfrak{M}, \mathbf{f} \in \mathfrak{N}$, this set being turned in a vector space by the conventions

$$\begin{aligned} (\mathbf{e}, \mathbf{f}) + (\mathbf{e}', \mathbf{f}') &= (\mathbf{e} + \mathbf{e}', \mathbf{f} + \mathbf{f}') \\ a(\mathbf{e}, \mathbf{f}) &= (a\mathbf{e}, a\mathbf{f}) \quad \text{for } a \in K. \end{aligned}$$

III. *Topology.* We call topological spaces only the spaces in which Hausdorff separation axiom is satisfied.

A neighbourhood of a point p in space \mathfrak{B} is understood to be a set N such that there exists an open set U such that $p \in U \subset N$; N need not be open itself.

The adherence \bar{A} of a set A in a topological space is the set of those points p such that every neighbourhood of p meets A . Every point of \bar{A} is said to be adherent to A . We shall make use of the possibility of defining the topology in a space by the operation $A \rightarrow \bar{A}$ of adherence (cf. Alexandroff-Hopf, *Topologie*, Kap. 1).

Intervals. If a and b are real numbers such that $a \leq b$, we denote by $]a, b[$ the open interval of extremities a and b . We set $]a, b] =]a, b[\cup \{b\}$, $[a, b[=]a, b[\cup \{a\}$, $[a, b] =]a, b[\cup \{a\} \cup \{b\}$.