Contents

Notations — **x**

1 Introduction — 1

Part I

2 Some fixed point theorems — 5
2.1 Introduction — **5**
2.2 Banach–Caccioppoli theorem — **5**
2.3 Brouwer's theorem — **6**
2.4 Schauder's theorem — **10**

3 Preliminaries of real analysis — 13
3.1 Introduction — **13**
3.2 Nemitski's composition theorem — **13**
3.3 Marcinkiewicz spaces — **18**
3.4 Appendix — **22**

4 Linear and semilinear elliptic equations — 24
4.1 Introduction — **24**
4.2 The Lax–Milgram and Stampacchia's theorems — **24**
4.3 Linear equations — **27**
4.4 Some semilinear monotone equations — **28**
4.5 Sub and supersolutions method — **31**
4.6 Appendix — **34**

5 Nonlinear elliptic equations — 37
5.1 Introduction — **37**
5.2 Surjectivity theorem — **37**
5.3 The Leray–Lions existence theorem — **40**

6 Summability of the solutions — 47
6.1 Introduction — **47**
6.2 Preliminaries — **48**
6.3 Sources in Lebesgue spaces — **51**
6.4 Sources in Marcinkiewicz spaces — **56**
6.5 Sources in divergence form — **58**

7 H^2 regularity for linear problems — 61
7.1 Introduction — **61**
7.2 Preliminaries — **61**
7.3 $H^2(\Omega)$ regularity of the solutions — **63**

8 Spectral analysis for linear operators — 67
8.1 Introduction — **67**
8.2 Eigenvalues of linear elliptic operators — **67**
8.3 Applications to some semilinear equations — **74**
8.4 Appendix — **82**

9 Calculus of variations and Euler's equation — 84
9.1 Introduction — **84**
9.2 Direct methods in the calculus of variations — **84**
9.3 Euler equation — **86**
9.4 Summability of minimizers of integral functionals — **90**
9.5 The Ekeland variational principle — **95**
9.6 Appendix — **102**

Part II

10 Natural growth problems — 105
10.1 Introduction — **105**
10.2 A problem with bounded solutions — **106**
10.3 A problem with unbounded solutions — **112**

11 Problems with low summable sources — 121
11.1 Introduction — **121**
11.2 A priori estimates — **123**
11.3 Distributional solutions — **130**
11.4 The linear case: a different proof — **130**
11.5 Entropy solutions — **132**
11.6 A comparison between entropy solutions and distributional solutions — **137**
11.7 Measure sources — **138**
11.8 The regularizing effects of a lower order term — **140**
11.9 T-minima — **143**
11.10 Appendix — **148**

12 Uniqueness — 150
12.1 Introduction — **150**
12.2 Monotone elliptic operators — **150**
12.3 A nonmonotone elliptic operator — **152**
12.4 A uniqueness result for measure sources — **154**

13 A problem with polynomial growth — 157
13.1 Introduction — **157**
13.2 Existence results — **158**

14 A problem with degenerate coercivity — 166
14.1 Introduction — **166**
14.2 The case $0 < \theta < 1$ — **167**
14.3 The case $\theta > 1$: existence and nonexistence — **172**
14.4 The regularizing effects of a lower order term — **175**

Bibliography — **187**

Index — **191**