

ERWIN TEGTMEIER

The Ontological Problem of Order

1. Three Views of Relations and the Problem of Order

The ontological problem of order arises with relations. If there were only properties and no relations it would not arise. While a property belongs in each case to one thing only, a relation has in each case more than one relatum and these relata come or, at least, seem to come in various orders. Hence a relation can be said always to hold in a certain direction or sense, as Russell calls it. The two-term relation 'earlier than' (simple quotes refer to things, properties and relations, not to words) e.g. holds between an event a and an event b, which is different from the case of b occurring earlier than a. In the first case the relation holds from a to b, in the second from b to a. Now, the problem of order in ontology is to account for that difference of direction. The problem is most pressing if one compares relational cases which differ merely in direction, i.e. in which the same relation holds between the same relata as in our example.

The problem of order is no traditional problem. It was not discovered before Russell. And even Russell paid attention to it only temporarily in a manuscript published only posthumously in 1984. So, Gustav Bergmann had to rediscover it and independently the present writer. It is no accident that the problem was noticed in ontologies with facts as complexes and relational universals. We will see that after we have distinguished and compared three ontological views of relations.

The first, held by Aristotle and the later Brentano, is that relations are properties belonging to one thing only though with respect to another thing. The second, held by Ockham, Locke and Meinong, is that relations are internal to the relata and grounded on qualities, i.e. non-relational properties of them. The relata are taken as consisting of qualities. The third view to be considered is the Russellian of relations as many-placed universals which are not derived from properties and are not internal but external to the relata. Russellian relations are connected with things by facts, i.e. by complexes with relations and relata as constituents.

What solution does each view offer to the problem of order in our example? The solution of the property-view is very easy. In the first case, a certain property ('earlier') belongs to event a with respect to event b and in the second it belongs to event b with respect to event a. Thus, this view implies that in reality there is no direction from one relatum to the other and no order of the relata, if only because in both cases no more than one thing, one relatum is involved. The ontological analysis of our example offered by the internality view is a bit more complicated. Things are assumed to have temporal qualities. Then the temporal relation 'earlier' between the two events is founded on these qualities. If events a and b had occurred in a different order they would have had different temporal qualities but the relation between those would not change. Since a relation is grounded on and determined uniquely by the qualities, there is ontologically only one possibility. Given two qualities, e.g. temporal qualities, there can be only one relation. From the standpoint of the internality view, this holds not merely for relations which seem symmetrical like proximity or similarity but also of seemingly asymmetrical relations like the spatial part-whole-relation. Since the latter relation is grounded on the places of the part and the places of the whole, another relation or the holding of the relation in another direction is *ontologically* impossible. But the possibility of different cases of the same relation and the same relata is a precondition for order and direction and also for the symmetry and asymmetry of relations. This is not realised by those who speak of the asymmetry of the connection between thing and property or subject and predicate while conceiving of it in such a way that it is fundamentally impossible for a property to have a thing or a predicate to have a subject. Asymmetry presupposes that a reversal of the relata is possible though not actual. The asymmetry of a relation is defined by the general condition that it must never hold in both directions. Hence the opposite of a given relational case must make sense, must be thinkable and ontologically possible. If there can be only one case with respect to a given two-place relation, given two relata there is no direction and no order. Hence, the internality as well as the property view imply the denial of direction and order of relations. Therefore, advocates of those views rightly saw no problem here. Whether their views of relation are problematic in other respects is another matter. Also, how they will account for graded and quantitative dimensions and series without assuming order?

2. Russell's Solution

With Russellian relations and facts as complexes the problem of order arises as soon as one tries to unpack ontologically the metaphorical talk of places of relata and directions and as soon as one adheres to the principle that a phenomenological difference such as that between event a coming before event b and event b being before event a must be reflected in the ontological analysis. In the example, the ontological analysis starts from two relational facts with *prima facie* the same constituents, the same relational universal and the same relata. Hence, to account for the difference, additional entities have to be assumed. Russell assumes positions which relata occupy in relational complexes (Russell 1913, Part III Chap.1). While earlier (in the *Principles of Mathematics* e.g.) he did not go beyond metaphorical talk, this time he does and the first step is to categorise the entities introduced. Positions are categorised as relations which hold between each relatum and the respective relational complex. The second step is to describe the content of the introduced entities. Russell takes the positions of relata not to be general order positions but to be specific to the relation relating the relata in the complex. In the case of Russell's example, the temporal sequence of two tones a and b, the relation of a to the relational complex is not that it is the first relatum but that it has the earlier-position and the relation in which b stands to the complex is not that it is the second relatum but that it has the later-position. Russell stresses “that these relations do not essentially put one term *before* the other, as though the relation went *from* one term *to* another.” And he adds that “this only appears to be the case owing to the misleading suggestions of the order of words in speech or writing.” (Russell 1913, p.88).

Russell thus retracts earlier statements mentioned at the beginning, though it is not very clear what entities he retracts because those statements were rather metaphorical. However, he is convinced now that order does not exist in relational facts, that there is no order of the relata in them. The reason given (and this is also the point where his later view clearly and definitely differs from his earlier view) is that he now takes it to be “so obvious as to be undeniable” that there are no inverse relations and no respective facts, that e.g. the sentences “x is before y” and “y is after x” refer to the same fact (Russell 1913, p.87). Earlier he had assumed that they stand for two different facts which merely imply one another (Russell 1903, §219). Russell's argumentation against an order of relata now seems to be that to the order of relata signs there does not correspond anything in

reality since the sentences with the opposite order of relata signs, “ $R(x,y)$ ” and “ $R'(y,x)$,” where R' is the inverse of R , refer nevertheless to the same fact. An accompanying conclusion would be, of course, that “ R ” and “ R' ” do not represent either. Moreover, even in Russell's view the order of the relata symbols in “ $R(x,y)$ ” represent something, though not the order of relata, namely in comparison with “ $R(y,x)$ ” the holding of certain positional relations.

As against Russell I do not take it to be obvious that “ x before y ” and “ y after x ” mean the same fact. I agree that in this case there are not two different facts. But I suppose, without claiming obviousness for it, that only the first sentence represents a fact while the second is based on a fictitious inverse relation and its truth conditions are parasitic on the first fact. Thus it is possible to hold that the order of the relata signs represents the order of the relata without having to admit inverse relations. And the odd conclusion, that whether the order of the relata signs in a sentence does or does not have a referent depends on the sentence it is compared with, is avoided.

Russell's later solution to the order problem seems to me fundamentally to be a return to the property view of relations (reducing a relation to two properties), which Russell meant to overcome. Officially, his relational facts continue to consist of an n -place relational universal, such as temporal succession and n relata, to which are added now n facts connecting each relatum by a different positional relation with the main relational fact. But the positional relations contribute what should be the content of the relation in the main relational fact. For x to have the before-position in a certain relational fact and y the after-position amounts to y succeeding x temporally. Hence, the positional relations make the relation which they allegedly merely accompany in fact superfluous. A two-place relation e.g. is thus substituted by a pair of relations which turn out on closer inspection to be nothing but properties.

According to the property view “ x before y ” stands for x having the before-property with respect to y and “ y after x ” for y having the after-property with respect to x . Russell relates the before- and after-positions to a relational complex containing x and y . But wouldn't it be more meaningful to say that x has the before-position with respect to y and y the after-position with respect to x rather than relating these positions to the relational fact? Russell's relapse to an older view of relations is, I think, an inevitable consequence of his rejecting a general ordering of relata and his

attempt to solve the problem of order by specific relations of the same content as the relation of which the relata are relata, adding that the order problem requires a distribution of the entity intended to solve it over the relata.

In connection with Russell Herbert Hochberg (Hochberg 1987, p.440ff.) offered a solution of the problem of order which assumes two ordering relations between each relatum and the respective relational fact. These relations are represented linguistically by "being the first relatum of", "being the second relatum of" etc. Russell's solution is very unsatisfactory also insofar as it bases the difference between two facts on facts which have the facts to be differentiated and analysed already as constituents. Hochberg avoids that difficulty and offers an analysis in which the relata do not stand in the ordering relations to the finished complex but to a complex having the same constituents but no order. This solution seems to me unacceptable, too, since it introduces complexes which cannot be facts (having the same constituents as certain relational facts, but not being completed to form such facts) and whose nature and category is unclear. Moreover, the presumed facts of which these unordered complexes are constituents cannot be facts either. It is no fact, it is simply not true that a certain relatum is first or second etc. relatum of a complex if that complex is not ordered. Besides, both solutions open an infinite regress basing order on relational facts which also need an order of the relata. The regress is to be seen as a difficulty though it need not be vicious.

3. Set Theory and Bergmann's Solution

For many philosophers set theory is some kind of ontology. They will wonder what the ontological problem of order is all about. From an ontological point of view to think of relations as sets of ordered n-tuples may not be very convincing (it is part of what Bergmann called „dead end nominalism“) but it may, nevertheless, be promising to include n-tuples in relational facts to ground the order of relata. Hochberg (Hochberg 1981, p.233ff.), for a time, took this to be a satisfactory ground. To see whether they furnish a satisfactory solution, let us look at the n-tuples more closely. The usual identity conditions for them presuppose order rather than defining it or indicating its source. Apparently, there is neither a constituent of the n-tuple nor an entity connected with it in another way to

order them. Hence, the alleged order of it has no ground and simply is not there. It is a mere fiction permissible to the mathematician but not to the ontologist. The mathematician represents and symbolises the n-tuple as ordered without being concerned with the nature or ground of that order.

Now, set theorists themselves felt uneasy about the n-tuples because they are complex yet no sets. Thus they have been replaced or rather shown to be replaceable in principle by certain sets which serve the same purposes. These so-called definitions of ordered n-tuples by Wiener and Kuratowski introduce entities which are unordered and normal sets. The ordered pair $\langle a, b \rangle$ e.g. is replaced in Kuratowski's definition by the pair-set $\{\{a\}, \{a, b\}\}$.

While it would have made sense to take ordered n-tuples to be constituents of relational facts and relations as attributes of them, the corresponding unordered pair sets (according to Wiener or Kuratowski) would certainly be misplaced as constituents. It seems impossible to think of a two-place relation as holding between its first relatum and the class of both its relata. Similarly for relations with more than two places. It would also be obviously wrong to think of the relation as an attribute of the Kuratowski-Set of its relata. There seems to be no way to make sense of a relational fact with a Kuratowski-Set as one and a relation as the other constituent.

Thus, the ideas of Wiener and Kuratowski offer no immediate solution to the ontological problem of order. Only if they are transposed ontologically is there a chance that they will. That is what Gustav Bergmann did (Bergmann 1992, Chap.III). In his late ontology he adds to his categorial inventory the category of diads. Diads are similar to facts in being complex and corresponding to sentences. Yet, the sentences corresponding to diads all express diversities between two entities. In Bergmann's middle ontology diversity is no entity at all. In his late ontology it has neither become a relation nor a fundamental connector like exemplification, though he advocated the latter alternative temporarily. Rather, diversity is a complex consisting of the two diverse entities and of nothing else. When one grants ontological status to diversity, one has to face the consequence that diversity is iterating infinitely, that there are diversities of diversities etc. (this is one of the objections against ontologising diversity). However, Bergmann takes advantage of the iteration of diversity to solve the problem of order. The diversities of diversities furnish entities structurally similar to Kuratowski-Sets. Instead of the pair set $\{\{a\}, \{a, b\}\}$ Bergmann has the diversity between a and the

diversity of a and b. He symbolises the latter thus: $\langle a, \langle a, b \rangle \rangle$, employing the corner which set theory uses to represent ordered n-tuples, though he points out that diads are not ordered. Applying Bergmann's analysis to our example of the two events a and b, we get on the one hand a relational fact with the diversity between a and the diversity of a and b as constituents and on the other hand a relational fact with the diversity between b and the diversity of a and b as constituents. Insofar as the task was to account by ontological analysis for the phenomenological difference between the two cases, the problem is solved by Bergmann's analysis. But does this analysis make sense? Bergmann is aware of the phenomenological distance, as he calls it, i.e. the distance of his analysis to the phenomenological data. While phenomenological presentation may indeed not be the indisputable criterion of adequacy, an ontological analysis has at least to make sense. It does not suffice to have a perspicuous and syntactically well-organised symbolisation. I can make sense of the exemplification of a property by an individual thing as building on the diversity of property and thing (i.e. I can make sense of Bergmann's late analysis of nonrelational facts) and also of diversity as connecting entities into a complex (because something is stated about the diverse entities together and the conception of diversity as separating is based merely on a spatial metaphor). But I cannot make sense of the suggestion that the exemplification of a two-place relation is built on the diversity between it and the diversity between its first relatum and the diversity of both its relata. Only the diversity between the relata and between them and the relation seems to me to be involved at all.

4. A Solution with Ordering Forms

The solution which I regard as the most satisfactory and which is my own (Tegtmeier 1992, Chap.V) draws its inspiration not from set theory but like Russell's from the phenomenological data. Unlike Russell, however, I do not solve the problem by additional entities of the category thing (namely by relational universals), rather I assume additional entities of the category form (which are to be distinguished from literal forms of bodies). Forms are much more dependent entities than things (i.e. either individuals or universals) and facts. They depend on things, if they are forms of things, and on facts, if they are forms of facts. Forms of facts are e.g. exemplification, which forms atomic facts, or conjunction, which forms molecular facts. Forms of things are e.g. individuality and two-place

universality of the first order. They determine the subcategory of a thing. Like literal forms of bodies, members of the ontological category of form are not constituents of what they form. Their connection with what they form is closer than that between constituent and complex and analogous to what the mathematicians call idempotency. A thing together with its form is the thing and nothing else. Now, there is a kind of forms which I would call secondary forms because they form an entity already formed as a whole. Negation is such a secondary form since it forms form with respect to atomic facts, which have already the form of exemplification. The entities grounding the order in relational facts (but also the order of the constituents of molecular facts), the ordinator, as I named them, belong to the secondary forms. In relational facts they form things which are preformed as individuals or as universals of a certain type. Ordinator are firstness, secondness, thirdness and fourthness. I assume that there are not more ordinator since it seems to me that there are no underived relations with more than four places. My ontological analysis of our example would be this: there are two relational facts with the same constituents, the relation 'earlier' and the event a and the event b falling under the category of individual. The difference between the two cases grounds on a having the form of firstness in the first relational fact and not having it in the second or on b having the form of secondness in the first and not in the second relational fact. I would claim that the ordinator are presented to us in perception, that we see e.g. in the first case a as first relatum and the b as second relatum (this is no idealistic but a realistic seeing-as). Naturally, ordinator are not perceived separately but in connection with the fact as a whole. If order thus presents itself in the relational facts it follows that it cannot be derivative. It cannot derive from an ordering of ordinator in a series. One has to see that ordinator themselves are not ordered, rather they are order.

Ordinator are not familiar and not particularly plausible, indeed, they seem somewhat ad hoc. To assess and appreciate them one has to consider the alternatives in an ontology with the categories of things, facts, and forms (because this is the theory into which the concept of ordinator belongs). Things divide into particulars, properties, and relations. Correspondingly, there are the alternatives of assuming ordering particulars, ordering properties, and ordering relations. According to the first alternative our example $E(ab)$ (the event a occurring earlier than the event b) would be analysed by assuming ordering particulars p_1 and p_2 ,

which could be called relata-places. A relation T (takes the place) would have to connect these places with the relata in the relational facts $T(a,p_1)$ and $T(b,p_2)$. The T-facts are either inside or outside the E-fact. If the former holds $E(a,b)$ is actually $E(T(a,p_1),T(b,p_2))$. If the latter holds $E(a,b)$ forms a conjunction with $T(a,p_1)$ and $T(b,p_2)$. The assumption of T-facts inside the E-fact has two grave difficulties: first, the relation E (earlier) would not have a and b as relata but the two T-facts, which is discordant with the phenomenon $E(a,b)$ given to us in perception. And second, if T-facts are taken to have ordered relata, it leads into an infinite regress since each T-fact needs another T-fact to base the order of its relata. To assume unordered T-facts would be rather ad hoc and would make it *ontologically* necessary, i.e. very fundamental, that T connects places to particulars of other kinds but not to other places. The alternative assumption that T-facts are outside the E-fact leads to grave difficulties, too. First, in addition to the infinite regress for T-facts, the question arises what fact $E(a,b)$ is in the conjunction $E(a,b)\&T(a,p_1)\&T(b,p_2)$, since the order of its relata grounds on additional facts. Can $E(a,b)$ be a relational fact if it has no order of itself? The second difficulty is logical. The conjunctive analysis of the order of relata permits false conclusions from true premises. By the law of adjunction the true premises $E(a,b)\&T(a,p_1)\&T(b,p_2)$ and $R(a,b)\&T(a,p_2)\&T(b,p_1)$, where R be some relation which holds between a and b in the opposite direction, logically imply $E(a,b)\&T(a,p_2)\&T(b,p_1)$, i.e. that b is earlier than a, which, naturally, is not the case.

The analysis of our temporal example with ordering properties is analogous to that with ordering particulars. It is simpler because it requires no relation connecting the particulars and the relata. The ordering properties would be exemplified by the relata immediately. But the analogous difficulties, which arise, are a strong evidence against this alternative, too. There remains the relational alternative to which the solution of the order problem belongs which Hochberg proposed starting from Russell. The ordering relations hold either between the relata and their relation or the respective relational fact. In the former case the analogues of the difficulties of ordering particulars and properties arise. There remains the possibility that the relata stand in the ordering relations to the respective relational fact. Let the relations 'first relatum of' and 'second relatum of' be symbolised by C_1 and C_2 , then a being earlier than b is analysed thus: $C_1(a,(E(a,b))\&C_2(b,(E(a,b))\&E(a,b)$. The last conjunct is the fact that a is before b. And if its relata are ordered, this order must be

contained in it. Otherwise it would not be that fact. Hence, the other conjuncts are superfluous as grounds of the order of the relata. If one follows Hochberg's suggestion and substitutes "E(a,b)" in the C-facts by unordered complexes of E, a and b, it will no longer be the case that a is first relatum and b second relatum. The insuperable difficulty is that C-facts stand in the dilemma pointed out already with respect to Hochberg's analysis. They are either useless or non-existent. One can conclude that the alternatives to orderings forms must be ruled out because of grave difficulties. With ordinator one does not get into the difficulties discussed because they are inside the relational facts and yet do not require entities other than the usual relata.

5. Order and Time

The order of relata is easily mixed up with the temporal succession of relata signs in speaking or reading the sentence representing the relational fact of which the relata are constituents (see Tegtmeier 1995). Yet, a temporal succession of two signs is just another relational fact whose relata need a ground of their order, too. Hence, temporal succession cannot be the ground of all order in the world. Nevertheless, order and series, which bases on the order of relata in relational facts, was equated by many philosophers (e.g. Leibniz and Kant) with temporal succession. When we try to apprehend the order of relata we usually fall back on temporal facts, due to our rules of linguistic representation and our stepwise way of more careful apprehension, though we could attend to it in any relational fact. The point to be noted is that we apparently cannot grasp order separately, which, by the way, supports my categorising ordinator as mere forms. To get an idea of order as such we turn to temporal successions because these are used to represent order. Since we cannot get hold of the reality, we put up with the sign. And it is not nearly as easy to keep sign and reality apart as one would think.

Russell takes the standpoint, as was reported already, that we actually confuse language and reality or rather, that we project a structure of language into reality, if we assume an order of relata. But this standpoint undermines itself. It presupposes that relata in facts of temporal succession are ordered or at least in linguistic temporal facts. Yet, linguistic and temporal facts are facts among facts. Russell implies that some relata in

relational facts are ordered. Why shouldn't all other relational fact be ordered in that way, too ?

Russell's and Bergmann's ontological analyses eliminate order from relational facts. And I would not want to appeal to phenomenological data to argue that order is there. It is not a starting point but a result, if my analysis of relational facts is right, that order is basic and neither eliminable nor reducible. I am convinced that this has far-reaching consequences (first of all, for the ontology of time; see Tegtmeier) and that the problem of order has been greatly underestimated.

REFERENCES

- Bergmann,G. 1964 *Logic and Reality*. Madison: University of Wisconsin Press.
- Bergmann,G. 1981 *Notes on Ontology*. *Nous* 15.
- Bergmann,G. 1992 *New Foundations of Ontology*. Madison: University of Wisconsin Press.
- Hochberg,H. 1981 *Logical Form, Existence and Relational Predication*, in: P.A.French et al (eds): *Midwest Studies in Philosophy VI*. Minneapolis: University of Minnesota Press.
- Hochberg, H. 1987 *Russell's Analysis of Relational Predication and the Asymmetry of the Predication Relation*. *Philosophia* 17.
- Russell,B. 1903 *Principles of Mathematics*. London: Allen&Unwin.
- Russell,B. 1913 *Theory of Knowledge*, in: *The Collected Papers of Bertrand Russell*. London 1984: Allen&Unwin.
- Tegtmeier,E. 1990 *Relations and Order*, in M.Sukale (ed) *Sprache, Theorie und Wirklichkeit*. Frankfurt: Peter Lang.
- Tegtmeier,E. 1992 *Grundzüge einer kategorialen Ontologie*. Freiburg: Alber.
- Tegtmeier,E. 1995 *Ein vernachlässigtes ontologisches Problem der Relationslogik*, in: J.Brandl/A.Hieke/P.Simons (eds.) *Metaphysik.Neue Zugänge zu alten Fragen*. Sankt Augustin: Academia.

Tegtmeier,E. 1997 Direction of Time: A Problem of Ontology, not of Physics. In:
J.Faye / U.Scheffler / M.Urchs (eds.) Perspectives on Time. Dordrecht / Boston
/ London: Kluwer.