Preface to the first edition

For a long time disciplines such as astronomy, celestial mechanics, the theory of heat, and the theory of electromagnetism counted among the most important sources in the development of mathematics, continually providing new problems and challenges. The main mathematical facts resulting from these fields typically crystallized in linear ordinary or partial differential equations.

In the early 20th century Heisenberg's discovery of a rule showing that the measurement of the position and momentum of a quantum particle give different answers, depending on the order of their measurement, led to the concept of commutation relations, and it made matrix theory find an unexpected application in the natural sciences. This resulted in an explosion of interest in linear structures developed by what we call today linear algebra and linear analysis, among them functional analysis, operator theory, C^*-algebras, and distribution theory. Similarly, quantum mechanics inspired much of group theory, lattice theory, and quantum logic.

Following this initial boom Wigner's beautiful phrase talking of the "unreasonable effectiveness of mathematics" marks a point of reflection on the fact that it is by no means obvious why this kind of abstract and sophisticated mathematics can be expected at all to make faithful descriptions and reliable predictions of natural phenomena. If indeed this can be appreciated as a miracle, as he said, the degree of flexibility that mathematics allows in its uses is perhaps at least a remarkable fact. Feynman has pointed to the different cultures of using mathematics when he remarked that "if all of mathematics disappeared, physics would be set back by exactly one week." While mathematics cannot compete with physics in discovering new phenomena and offering explanations of them, physics continues to depend on the terminology, arsenal, and discipline of mathematics. Mathematical physics, which is part of mathematics and therefore operates by its rules and standards, has set the goal to understand the models of physics in a rigorous way. Mathematical physicists thus find themselves at the borderline, listening to physics and speaking mathematics, at best able to use these functionalities interchangeably.

If one of the imports of early quantum mechanics has been the realization that the basic laws on the atomic scale can be formulated by linear superposition rules, another was a probabilistic interpretation of the wave function. This connection with chance, amplified and strongly advocated by the Copenhagen school, was no less revolutionary than the commutation relations. Richard Feynman has discovered a second connection with probability when he offered a representation of the state of a particle in terms of averages over all of its possible histories from one point in space-time to another. While this was an instance seriously questioning his dictum quoted above and his use of mathematics was in this case very problematic, the potential lying in the description advanced in his work turned out to be far reaching. The mathemati-

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cian Mark Kac was the first to show that this method could be made sense of and was eminently viable.

Our project to write the present monograph has grown out of the thematic program *At the Interface of PDE, Self-Adjoint Operators and Stochastics: Models with Exclusion* organized by the first named author at the Wolfgang Pauli Institute, Vienna, in 2006. The initial concept was a smaller-scale but up-to-date account of Feynman–Kac-type formulae and their uses in quantum field theory, which we wanted to dedicate to the person all three of us have much to thank to, both scientifically and personally, Herbert Spohn. At that time we secretly meant this as a present for his 60th birthday, and we are glad that we are now able to make this hopefully more mature tribute on the occasion of his 65th birthday!

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