Preface

This is volume 2 of our trilogy on invariant differential operators. In volume 1 we presented our canonical procedure for the construction of invariant differential operators and showed its application to the objects of the initial domain – noncompact semisimple Lie algebras and groups.

In volume 2 we show the application of our procedure to quantum groups. Similarly to the setting of volume 1 the main actors are in duality. Just as Lie algebras and Lie groups are in duality here the dual objects are the main two manifestations of quantum groups: quantum algebras and matrix quantum groups. Actually, quantum algebras typically are deformations of the universal enveloping algebras of semisimple Lie algebras. Analogously, matrix quantum groups typically are deformations of spaces of functions over semisimple Lie groups.

Chapter 1 presents first the necessary general background material on quantum algebras and some generalizations as Yangians. Then we present the necessary material on \( q \)-deformations of noncompact semisimple Lie algebras. Chapter 2 is devoted to highest weight modules over quantum algebras, mostly being considered Verma modules and singular vectors. The latter is given for the quantum algebras related to all semisimple Lie algebras. Chapter 3 considers positive energy representations of noncompact quantum algebras on the example of \( q \)-deformed anti de Sitter algebra and \( q \)-deformed conformal algebra. In Chapter 4 we consider in detail the matrix quantum groups. Many important examples are considered together with the quantum algebras which are constructed using the duality properties. In many cases we consider the representations of quantum algebras that arise due to the duality. In Chapter 5 we consider systematically and construct induced infinite-dimensional representations of quantum algebras using as carrier spaces the corresponding dual matrix quantum groups. These representations are related to the Verma modules over the complexification of the quantum algebras, while the singular vectors produce invariant \( q \)-difference operators between the reducible induced infinite-dimensional representations. This generalizes our considerations of volume 1 to the setting of quantum groups. These considerations are carried out for several interesting examples. In Chapter 6 we continue the same considerations for the invariant \( q \)-difference operators related to \( GL_q(n) \). Finally, in Chapter 7 we consider representations the \( q \)-deformed conformal algebra and the deformations of various representations and hierarchies of \( q \)-difference equations related in some sense to the \( q \)-Maxwell equations. Each chapter has a summary which explains briefly the contents and the most relevant literature. Besides, there are bibliography, author index, and subject index. Material from volume 1, Chapter N, formula n is cited as (I.N.n).

Note that initially we planned our monograph as a dilogy; however, later it turned out that the material on quantum groups deserves a whole volume, this volume.
Volume 3 will cover applications to supersymmetry, the AdS/CFT correspondence, infinite-dimensional (super-)algebras including (super-)Virasoro algebras, and ($q$-) Schrödinger algebras.

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