

Contents

Introduction.	1
Part I. Towards Topological Field Theory	15
Chapter I. Invariants of graphs in Euclidean 3-space.	17
1. Ribbon categories.	17
2. Operator invariants of ribbon graphs.	30
3. Reduction of Theorem 2.5 to lemmas.	49
4. Proof of lemmas.	57
Notes	71
Chapter II. Invariants of closed 3-manifolds.	72
1. Modular tensor categories	72
2. Invariants of 3-manifolds	78
3. Proof of Theorem 2.3.2. Action of $SL(2, \mathbb{Z})$	84
4. Computations in semisimple categories.	99
5. Hermitian and unitary categories.	108
Notes	116
Chapter III. Foundations of topological quantum field theory.	118
1. Axiomatic definition of TQFT's	118
2. Fundamental properties	127
3. Isomorphisms of TQFT's.	132
4. Quantum invariants.	136
5. Hermitian and unitary TQFT's	142
6. Elimination of anomalies	145
Notes	150
Chapter IV. Three-dimensional topological quantum field theory.	152
1. Three-dimensional TQFT: preliminary version.	152
2. Proof of Theorem 1.9.	162
3. Lagrangian relations and Maslov indices	179
4. Computation of anomalies	186

5. Action of the modular groupoid	190
6. Renormalized 3-dimensional TQFT.	196
7. Computations in the renormalized TQFT	207
8. Absolute anomaly-free TQFT	210
9. Anomaly-free TQFT.	213
10. Hermitian TQFT.	217
11. Unitary TQFT.	223
12. Verlinde algebra	226
Notes	234
Chapter V. Two-dimensional modular functors	236
1. Axioms for a 2-dimensional modular functor.	236
2. Underlying ribbon category	247
3. Weak and mirror modular functors	266
4. Construction of modular functors	268
5. Construction of modular functors continued.	274
Notes	297
Part II. The Shadow World	299
Chapter VI. $6j$ -symbols	301
1. Algebraic approach to $6j$ -symbols	301
2. Unimodal categories.	310
3. Symmetrized multiplicity modules	312
4. Framed graphs	318
5. Geometric approach to $6j$ -symbols	331
Notes	344
Chapter VII. Simplicial state sums on 3-manifolds	345
1. State sum models on triangulated 3-manifolds	345
2. Proof of Theorems 1.4 and 1.7	351
3. Simplicial 3-dimensional TQFT	356
4. Comparison of two approaches	361
Notes	365
Chapter VIII. Generalities on shadows	367
1. Definition of shadows.	367
2. Miscellaneous definitions and constructions	371
3. Shadow links	376

4. Surgeries on shadows 382
 5. Bilinear forms of shadows 386
 6. Integer shadows 388
 7. Shadow graphs 391
 Notes 393

Chapter IX. Shadows of manifolds 394

1. Shadows of 4-manifolds 394
 2. Shadows of 3-manifolds 400
 3. Shadows of links in 3-manifolds 405
 4. Shadows of 4-manifolds via handle decompositions 410
 5. Comparison of bilinear forms 413
 6. Thickening of shadows. 417
 7. Proof of Theorems 1.5 and 1.7–1.11. 427
 8. Shadows of framed graphs. 431
 Notes 434

Chapter X. State sums on shadows 435

1. State sum models on shadowed polyhedra 435
 2. State sum invariants of shadows 444
 3. Invariants of 3-manifolds from the shadow viewpoint. 450
 4. Reduction of Theorem 3.3 to a lemma 452
 5. Passage to the shadow world. 455
 6. Proof of Theorem 5.6. 463
 7. Invariants of framed graphs from the shadow viewpoint. 473
 8. Proof of Theorem VII.4.2 477
 9. Computations for graph manifolds 484
 Notes 489

Part III. Towards Modular Categories 491

Chapter XI. An algebraic construction of modular categories 493

1. Hopf algebras and categories of representations. 493
 2. Quasitriangular Hopf algebras 496
 3. Ribbon Hopf algebras. 500
 4. Digression on quasimodular categories 503
 5. Modular Hopf algebras. 506
 6. Quantum groups at roots of unity 508
 7. Quantum groups with generic parameter. 513
 Notes 517

Chapter XII. A geometric construction of modular categories	518
1. Skein modules and the Jones polynomial	518
2. Skein category	523
3. The Temperley-Lieb algebra	526
4. The Jones-Wenzl idempotents	529
5. The matrix S	535
6. Refined skein category	539
7. Modular and semisimple skein categories	546
8. Multiplicity modules	551
9. Hermitian and unitary skein categories	557
Notes	559
Appendix I. Dimension and trace re-examined	561
Appendix II. Vertex models on link diagrams	563
Appendix III. Gluing re-examined	565
Appendix IV. The signature of closed 4-manifolds from a state sum	568
References	571
Subject index	593