Introduction

This book presents a group description of chemical elements considered as states of a quantum system. Atoms of different elements are viewed as elements of a vector space associated with an irreducible unitary representation of some symmetry group. The elements of the group are carried into each other by operators defined on this group. This approach fundamentally differs from the traditional application of group theory to atomic physics, where a symmetry group is used to carry into each other different excited states of the same atom. Besides, we describe heavy particles (hadrons) as representations of unitary groups. This approach follows the model proposed independently by Gell-Mann and Néeman and developed later in the works of Okubo, Gürsey and Radicati, Pais and other authors.

The book “Unitary symmetry theory” by Yu. Rumer and A. Fet was published in 1970, in which these ideas were discussed systematically. While working on the book, Rumer had an idea to apply the same approach to the system of chemical elements. After getting over some difficulties, we laid down a group systematization of elements in our joint work (Rumer and Fet, 1971). Already in this article, we emphasized that the approach implies considering symmetry as related to the atom as a whole rather than to its electron shells only. As a group symmetry, we chose a two-dimensional covering group of SO(4) group. In 1972, B. Konopel’chenko extended the symmetry group up to the conformal group (Rumer and Fet, 1971). Barut (1972) and Barrondo and Novaro (1972) independently described the symmetry from the viewpoint of electron shells. Apparently, this clear differentiation between the symmetry of the elements and the traditional shell model helped us to achieve a clearer understanding of the obtained classification. In particular, I suggested a group description of chemical affinity in Fet (1974, 1975), and a mass formula for atoms in Fet (1979a, 1981). While refining the classification, we noticed, apparently for the first time, that the properties of elements change with some regularity. In particular, each p-, d-, and f-family is distinctly divided into two subfamilies where the properties of the elements change by different laws. The results were compared with experimental data by Byakov et al. (1976) and later, with additional material, by Sorokin.

Discussions of the method in scientific articles usually imply a sufficient knowledge of Lee groups. However, physicists and chemists usually meet in practice only representations of the three-dimensional rotation group, as it was elaborated by Wigner and Weil in the late 1920s. That is why I wanted to write a book that will be useful for this broad audience rather for a narrow circle of theorists. The work met serious difficulties since the available literature on Lee groups did not provide easy understanding of the ideas of symmetry which have acquired much importance in the Physics these last decades. What we have in this area are either systematic treatises for expert mathematicians or manuals for physicists which imitate these treatises or offer a number of separate applications to special cases. Therefore, I tried to state the main concepts of group theory and Lee algebras briefly, and only as far as it is
necessary to understand the symmetry of particles assuming that the reader has physical or chemical education and some basic knowledge of quantum mechanics. Group theory is explained from the very beginning, starting from the definition of a group. However, it would be desirable for the reader to have some preliminary idea of using rotation group for classification of atomic and molecular energy spectra. Group methods are introduced as necessary and are always illustrated by physical examples. Especially, the physical background of spin and unitary spin is discussed in detail. Therefore, the presentation of the material is “genetic” and inductive rather than deductive, that is, the main attention is paid to the origin and meaning of the concepts rather to the formal exactness of the constructions. In particular, mathematical rigor, though followed when possible, is not an end in itself. We cite only those theorems which are constantly used further, their meaning is discussed in detail but the proofs are usually omitted as well as cumbersome calculations which are cited, when necessary, in the Appendices.

First chapter of this book is an introduction to quantum chemical group methods and starts with the traditional applications of the rotation group to the hydrogen atom. Then a classical, though not broadly known, work of Vladimir Fock (1935) is discussed to demonstrate how hydrogen energy spectrum can be obtained from a four-dimensional rotation group. Since the work is very brief and not readily available nowadays, its most important part is considered in detail in Appendix A.

Second chapter shows how the observables of a quantum system can be obtained from its symmetry group. Thereby, symmetry groups are considered from the viewpoint of modern physics which implies that a quantum system is defined by the representation of a symmetry group. In particular, we discuss the relation between the Hamiltonian of the system and Casimir operators of its symmetry group.

Third chapter introduces the main ideas of Lee groups and algebras using physical background of the first two chapters. Each concept is illustrated with examples and is justified by detailed explanations.

Fourth chapter introduces the concepts of spin and isotopic spin, and explains the ideas of unitary symmetry. The discussion here, similar to the other chapters, implies the knowledge of simplest quantum mechanical concepts than some special literature. The concluding part of the chapter presents the main principles of classifying particles (or states) in a quantum theory and, I believe, contains some novelty when introducing a common principle of identifying similar particles in the multiplets of a symmetry group.

Chapters five and six present the main content of the book. They introduce the symmetry of the system of chemical elements illustrated by tables and graphs to compare the theory with experimental data.

I am very grateful to S. Prishchepionok who greatly contributed to improve the algebraic part of the book, and to N. Sorokin who did much work with the experimental material. In particular, Sorokin noticed that the suggested group classification provides good description of those properties which are associated with chemical
reactivity. Lately, this viewpoint was confirmed by Zhuvikin and Hefferlin (1983) who used our group method to explain regularities in the properties of diatomic molecules systematically studied by R. Hefferlin et al. (1979), Hefferlin and Kuhlman (1980), and Refferlin and Kutzner (1981). Unfortunately, I learned about these results after the book had been finished, and so the material was not included in this book.

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