

Preface and personal acknowledgments

Finishing this book was a challenging exercise!¹ Let me start differently than usual. First of all, I wish to thank De Gruyter and notably Nadja Schedensack as well as the managing editor of this book series, Ulrich Langer, for this opportunity and their endless patience and professional cooperation. When I proposed the concept in April 2016, I had a *tranquille*² life as a Post doc at the Radon Institute of Computational and Applied Mathematics (RICAM) in Linz, Austria. Originally, this monograph was envisaged as part of the habilitation procedure as final qualification for a university career. I am sure that I would have achieved the goal of completing this book if I had stayed longer at RICAM.

Suddenly, however (luckily as I can say today!), I was overtaken by events: I got an offer as *maître de conférences* from École Polytechnique in Palaiseau (near Paris), France, and soon after a (permanent) professorship position at the Leibniz University Hannover in Germany. I was incredibly lucky to get involved in so many activities since then. I never wanted to miss all of these activities. But there is one task I did not pursue as I should have: this book! In Spring 2020, I found the most valuable treasure of our times in my profession: time! Time for both my children³ and this book.⁴

I shall now briefly introduce the main topic, which is to discuss recent developments in mathematical modeling, design of algorithms, and numerical simulations in multiphysics variational phase-field fracture propagation. Variational modeling of fracture has been influential in several communities since 1998 when the first paper by Gilles Francfort and Jean-Jacques Marigo appeared.

The simplified prototype structure of a phase-field model for fracture propagation is: Find vector-valued displacements $u : B \rightarrow \mathbb{R}^d$ and a scalar-valued, smoothed, indicator function $\varphi : B \rightarrow \mathbb{R}$ such that

$$\begin{aligned} -\nabla \cdot (\varphi^2 \nabla u) &= f, \\ |\nabla u|^2 \varphi - \varepsilon \Delta \varphi - \frac{1}{\varepsilon} (1 - \varphi) &\leq 0, \\ \partial_t \varphi &\leq 0, \\ \left[|\nabla u|^2 \varphi - \varepsilon \Delta \varphi - \frac{1}{\varepsilon} (1 - \varphi) \right] \partial_t \varphi &= 0. \end{aligned}$$

1 But I should add: When I was in the “flow” of thinking and writing, it has been a great, great pleasure!

2 *tranquille* (French) $\hat{=}$ carefree.

3 Rather I should say: time for “family” including my wife, but she would not totally agree, I guess.

4 With the virtual summer semester start, my “free time” also had to be shared preparing online lectures, too. Needless to say that I did this also with great pleasure.

Therein, ε is a positive regularization parameter and we have appropriate boundary conditions on the outer boundary ∂B , and some initial data. The inequality $\partial_t \varphi \leq 0$ is an irreversibility constraint and states that the fracture cannot heal.

Multiphysics extensions include models in porous media, thermal effects, coupling to pressures and fluid flow inside the fracture, to name a few. Let us denote the information coming from the other physics by g , g_D and g_N . These enter as coupling terms into coefficients, additional operator terms or right-hand side information, e. g., $f := f(g)$. Or data is given on the fracture surface $\partial \mathcal{C} \subset B$ via kinematic and dynamic coupling conditions, namely

$$\begin{aligned} u &= g_D & \text{on } \partial \mathcal{C}, \\ \sigma n &= g_N & \text{on } \partial \mathcal{C}, \end{aligned}$$

where $\sigma := \varphi^2 \nabla u$ and n is the outward pointing normal vector on $\partial \mathcal{C}$. Such interface conditions will play a key role throughout this monograph.

As previously mentioned, this book project was originally planned as habilitation thesis (the traditional European university qualification exam) either at Johannes Kepler Universität Linz (Austria) or at École Polytechnique in Palaiseau (France). I got involved in phase-field fracture in February 2013 with Andro Mikelić and Mary F. Wheeler when I was a Post doc at the Institute for Computational Sciences and Engineering (ICES; now Oden Institute) at the University of Texas at Austin (USA). Later, I concentrated more on phase-field fracture multiphysics extensions that include inter alia fluid-structure interaction, a field that I knew from my Master's thesis,⁵ PhD⁶ and early Post doc time in Heidelberg.

Consequently, this book contains not only interdisciplinary topics because I consider multiphysics that intrinsically connect difference fields, but it is also international in the truest sense of the word by assembling research in four different countries: USA (2012–2014), Austria (2014–2016), France (2016–2017), Germany (2012, 2015, and again since October 2017).

During these times, I am grateful to many people and circumstances, which might all be summarized as: “There is a window of opportunity in life” (Mary F. Wheeler).⁷ And I was lucky enough to have several such *windows* (in terms of topics and colleagues/professors) during my Post doc time (Heidelberg/Germany, Austin/USA, Linz/Austria) and early Assistant Professor period (Palaiseau/France). I owe my deep-

⁵ Master's thesis in 2008 with Franz-Theo Suttmeier, University of Siegen.

⁶ PhD (Dr. rer. nat.) in 2011 with Rolf Rannacher, Heidelberg University.

⁷ Thanks to Thomas Carraro, my former office mate in Heidelberg, who took the initiative to call me on my cell phone on Friday, February 17, 2012, for a truly spontaneous first-time meeting with Mary Wheeler at IWR Heidelberg. Very likely without him, I would have missed this “window.”

est appreciations to Rolf Rannacher,⁸ Thomas Richter,⁹ Jeremi Mizerski, Winnifried Wollner, Mary F. Wheeler, Andro Mikelić,¹⁰ Boris Vexler, Ulrich Langer who mentored my Post doc periods. Then becoming assistant professor at École Polytechnique, I want to thank Grégoire Allaire who supported me.

Becoming a professor in October 2017, I got again involved in short time in various projects. These include the excellence cluster PhoenixD (headed by Uwe Morgner), the German-French IRTG proposal, and the French-German Doctoral College Sophisticated Numerical and Testing Approaches (in particular with Udo Nackenhorst and Amélie Fau), DFG-SPP 1748 (including the GAMM article [446] and editing together with Jörg Schröder a special issue in the GAMM-Mitteilungen), DFG-SPP 1962, DAAD-Peru, DAAD-IIT Indore, collaboration with the Institut für Kontinuumsmechanik (Peter Wriggers, Fadi Aldakheel), member as young research scientist in the German excellence application, and the Computing Future course. I just feel extremely flattered about all the opportunities.

There are many others whom I cannot all name in person here, i. e., office mates, co-authors, collaborators in writing joint software, colleagues in joint proposals, and those organizing conferences with me. Various types of collaborations are influential in this monograph and have a fruitful impact for which I am grateful. Special thanks go the Sanghyun Lee with whom I had enriching programming sessions at UT Austin. Likewise pleasure I had with Timo Heister over Thanksgiving in November 2013 in Austin and June 2015 in Linz.

Of course, I want also to thank my current IfAM professor colleagues, Sven Beuchler, Joachim Escher, Marc Steinbach, and Christoph Walker at the Institute of Applied Mathematics (IfAM) and, needless to say, specifically the members, former members, and associated members of my group “Wissenschaftliches Rechnen” for help and joyful discussions. Thanks to: Carmen Gatzen, Roswitha Behrens, Katrin Mang, Philipp Thiele, Sebastian Kinnewig, Denis Khimin, Nima Noii (2018–2020), Meng Fan (exchange, 1 year), Mats Brun (exchange, 3 months), Amirreza Khodadadian, Rajendra Choudhari, Daniel Jodlbauer (RICAM Linz), and Bernhard Endtmayer (RICAM Linz).

I also thank Bernhard Endtmayer, Daniel Jodlbauer, Sebastian Kinnewig, Katrin Mang, Philipp Thiele, and Diva Hurtado for suggestions and proofreading (parts of)

8 Thanks, Rolf, for giving me, as you said literally, the full education (in German: Vollausbildung) in your numerical analysis group in Heidelberg. I learned from you all basic ingredients to pursue a university career. Thank you very much for this wonderful opportunity!

9 Thomas was my actual mentor during my PhD and early Post doc time and he did a great, great job. In 2012, we shared a lecture that finally resulted in the book [381].

10 Meetings with Andro Mikelić have been just incredibly extraordinary, and demonstrated how mathematical PDE analysis and numerical simulations yield fruitful results. During Andro’s sabbatical in the first half of 2013 at UT Austin, we had nearly every day discussions resulting in [318] and several follow-up publications. This experience has been influential to me and was an outstanding opportunity being a young Postdoc.

this monograph. Furthermore, I appreciate Katrin's work for doing some of the tikz graphics and for allowing me to take some materials from our joint lecture notes.

Almost finished, I thank my parents-in-law H  l  ne and Henri for all the support as well as my families in Germany, France, and our (unofficial) Texan grandmother Laura.

Lastly, I thank my wife Marie-C  cile for her dedication to explore together (four!) different countries, cultures, and horizons. It holds true what an African proverb says:

If you want to go fast, you go alone, but if you want to go far, you go together.

Vielen Dank! Thank you very much! Merci beaucoup!

Austin, USA

Linz, Austria

Versailles, France

Hannover, Germany

June 2020

Thomas Wick