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Static Characteristic of Self-compensated Hydrostatic Bearing

Abstract: The influence of design parameter on the static characteristic of a self-compensated hydrostatic bearing was investigated in this article. This paper is obtaining the relationships of worktable displacement, load capacity, and static stiffness by using flow continuity equations of a self-compensated hydrostatic bearing. The results reveal that the appropriate range of design parameters for self-compensated hydrostatic bearing can obtain the maximum stiffness. We provided and verified a new design parameter which is restriction island height ratio can improve static performance of self-compensated hydrostatic bearing. Further, we also discuss the influence of viscosity ratio for bearing.

Keywords: Self-compensated Hydrostatic bearing, Load capacity, Static stiffness, Restriction island height ratio, Viscosity ratio

Nomenclature

$A_i$, $A_{\text{eff}}$, practical area ($m^2$), effective area ($m^2$) and dimensionless effective area $\overline{A}_i = A_{\text{eff}} / A_i$, $i = 1, 2$

$h_0$ initial film thickness or bearing clearance (mm)

$e$, $\varepsilon$ worktable displacement (mm) and dimensionless worktable displacement $\varepsilon = e / h_0$

$K$, $K_p$ static stiffness, dimensionless static stiffness and dimensionless static stiffness parameter

$P_i$ working pressure ($N/m^2$), $i = 1, 2$

$P_s$ supply pressure ($N/m^2$)

$P_i$, $P_s$ pressure ratio $\overline{P}_i = P_i / P_s$, $i = 1, 2$

$Q_{ri}$, $Q_{hi}$ flow rates of restriction part and oil pad part ($m^3/s$), respectively, $i = 1, 2$
Hydrostatic bearing is widely used in high precision mechanism and machine for the advantage of low friction, high stiffness, high accuracy, and long life. But the pressure in the hydrostatic bearing is generated by external pumps, and compensation devices are necessary to regulate flow into the recesses. Rippel [1] presented a general manual for designing hydrostatic bearings. In his manual, the single pad and multiple pads with various geometries were provided but only capillary, orifice, and constant-flow devices for the compensation of recess pressures are considered. Rowe [2] has introduced the mechanism design and written elementary theory of control valves for designing hydrostatic bearings. However, who mainly related the relationships between static characteristics of various bearing types and contact restrictions belong to capillary, slot, orifice, flow control valves, and constant flow pump; pressure-sensing valves.

The self-compensated hydrostatic bearing doesn’t need additional restrictors, so it can avoid manufacturing errors and reduce costs. Bassani [4] persented that a self-regulated hydrostatic opposed-pad bearing is highly efficient in a constant pressure system. He shown its performance is comparable to that of a conventional opposed-pad hydrostatic bearing compensated with flow control valves, and is superior compensated with orifices or capillaries. Slocum et al. [5] presented a self-
compensated, water-hydrostatic bearings for precision machine linear motion axes has been developed and tested. The bearing has many benefits of hydrostatic bearings systems and has less sensitive to manufacturing errors than are traditional hydrostatic bearing systems. Kane et al. [6] proposed a hydrostatic bearing with angled-surface self-compensation. The bearing consists of several round parts which can be easily manufactured and assembled and achieves high stiffness and accuracy. Zuo et al. [7] presented a newly designed self-compensating bearing, which is combining the traditional self-compensation and Kane et al.’s self-compensation [6], and based on the principles of flow balance equations derived its design formula to obtain optimal stiffness and optimal condition. Kang et al. [8] investigate the static stiffness of the hydrostatic bearing compensated with membrane-type restrictor and self-type compensation. We obtained the static stiffness due to self-compensation can be larger than that due to constant restrictors and also larger than that due to double-action membrane-type restrictor with dimensionless deformation coefficient and restriction ratio of bearing to restrictor departing from critical values.

In this paper, we design a new parameter which is the restriction island height ratio can improve static performance and obtain optimal design range. It is the critical technology for mechanical engineers to facilitate the design and application of this type hydrostatic bearing.

2 Theory of Self-compensated Hydrostatic Bearing

2.1 The Design Principles for Self-compensated Hydrostatic Bearing

Self-compensated hydrostatic bearing is belonging to the closed-type bearing, which configured as shown in Fig. 1. This bearing is composed of oil pad part and restriction part. A group bearing has two oil pads and two restriction islands that position are opposite. One is loading side and another is backing side. The design of restriction part must be higher than the oil pad part. The pressures in backing side can reduce load capacity of bearing, but can increase the stiffness to resist the loads. The design principle of restriction part is composed of oil inlet hole, connecting hole, Restriction Island, and Restriction Island surround as shown in Fig. 1(c).

The majority of supply oil flow into connecting hole and only the less flow into surround. If designed improperly such as when the restriction island made higher than the restriction island surround, it can make the most oil overflow surround without flowing into connecting hole. Then, it will make the backing side failure and extremely consume pump energy. So, the restriction island always higher than the restriction island surround, but the surround can easy collide machine when it's
too high. Where $\xi$ is the restriction island height ratio, defined as the ratio between the restriction island height difference and initial film thickness or bearing clearance ($h_0$). Its negative values indicate higher than the surface of oil pad, oppositely positive values indicate lower, and absolute value equals dozens of to hundreds of $\mu m$. $\xi h_0$ is the height difference of Restriction Island as shown the symbol in Fig. 1(b).

![Diagram of the self-compensated hydrostatic bearing](image-url)

**Fig. 1:** The self-compensated hydrostatic bearing
2.2 Flow, Static Load Capacity and Stiffness for The Self-compensated Hydrostatic Bearing

![Diagram of self-compensated hydrostatic bearing system](image)

Fig. 2: Self-compensated hydrostatic bearing system

The self-compensated hydrostatic bearing as shown in Fig. 2. The flow through gap of oil pad is equal to through gap of restriction island can be determined

\[ Q_{r1} = \frac{\pi(P_2 - P_s)(h_0 + \xi h_0)^3}{6 \mu_{ri} \ell n(R_2 / R_i)} = Q_{h1} = \frac{P_1(h_0 - e)^3}{12 \mu_{hi}} \gamma_1 \]  
\[ Q_{r2} = \frac{\pi(P_1 - P_s)(h_0 - e + \xi h_0)^3}{6 \mu_{r2} \ell n(R_2 / R_i)} = Q_{h2} = \frac{P_2(h_0 + e)^3}{12 \mu_{hi}} \gamma_2 \]  

where \( Q_{ri} \) is flow rate of restriction part, \( Q_{hi} \) is flow rate of oil pad part, \( \mu_{ri} \) is viscosity of restriction part, \( \mu_{hi} \) is viscosity of oil pad part, \( \gamma_i \) is dimensionless geometric constants, in which the subscript 1 denote the loading side of bearing, and the subscript 2 denote the backing side of bearing. \( R_i \) is radius of connecting hole, \( R_3 \) is radius of restriction island, \( R_3 \) is radius of restriction island surround, and \( e \) is worktable displacement.
Eqs. (1) and (2) describe the function of oil film thickness due to the recess pressure and loading caused of flow variation, therefore, they divided respectively by $P_s h_0^3 / 12 \mu r_1$ and $P_s h_0^3 / 12 \mu r_2$ to obtain the dimensionless flow equations of the loading side and backing side. They can be expressed as

$$
\overline{\mu}_1 \zeta_1 \overline{P}_1 (1 - \varepsilon)^3 = (1 + \varepsilon + \zeta)^3 (1 - \overline{P}_1)
$$

(3)

$$
\overline{\mu}_2 \zeta_2 \overline{P}_2 (1 + \varepsilon)^3 = (1 - \varepsilon + \zeta)^3 (1 - \overline{P}_2)
$$

(4)

Where $\zeta_i = \gamma_i \ln (R_2 / R_1) / 2 \pi$ the restriction parameter of oil pad is, $\overline{\mu}_i = \mu_i / \mu_{hi}$ is the viscosity ratio, and $\varepsilon = e / h_0$ is the dimensionless worktable displacement. Transposing Eqs. (3) and (4) can obtain the working pressure ratios of self-compensated hydrostatic bearing as

$$
\overline{P}_1 = (1 + \varepsilon + \zeta)^3 \left[ (1 + \varepsilon + \zeta)^3 + \overline{\mu}_1 \zeta_1 (1 - \varepsilon)^3 \right]^{-1}
$$

(5)

$$
\overline{P}_2 = (1 - \varepsilon + \zeta)^3 \left[ (1 - \varepsilon + \zeta)^3 + \overline{\mu}_2 \zeta_2 (1 + \varepsilon)^3 \right]^{-1}
$$

(6)

The pressure ratio which obtained by the simultaneous equations of dimensionless flow is multiplying the dimensionless effective area to get dimensionless load capacity. Thus, the dimensionless load capacity of self-compensated hydrostatic bearing is

$$
\overline{W} = \overline{P}_1 \overline{A}_1 - \overline{P}_2 \overline{A}_2 = \overline{A}_2 \left[ \alpha \overline{P}_1 (\varepsilon) - \overline{P}_2 (\varepsilon) \right] = \overline{A}_2 \overline{W}_p
$$

(7)

Thus, the dimensionless load capacity parameter is

$$
\overline{W}_p = \frac{\alpha (1 + \varepsilon + \zeta)^3}{(1 + \varepsilon + \zeta)^3 + \overline{\mu}_1 \zeta_1 (1 - \varepsilon)^3} - \frac{(1 - \varepsilon + \zeta)^3}{(1 - \varepsilon + \zeta)^3 + \overline{\mu}_2 \zeta_2 (1 + \varepsilon)^3}
$$

(8)

where $\alpha = \overline{A}_1 / \overline{A}_2$ is the effective area ratio of loading side and backing side and $\beta = \zeta_1 / \zeta_2$ is the restriction parameter ratio of both pads. Differentiating Eqs. (5) and (6) with respect to dimensionless worktable displacement ($\varepsilon$) give
The dimensionless static stiffness of hydrostatic bearing is supported the load variation with respect to the eccentricity variation per unit which is defined as follows

\[
\bar{K} = \frac{d\bar{W}}{d\bar{e}} = A_2 \left( \frac{d\bar{P}_1}{d\bar{e}} - \frac{d\bar{P}_2}{d\bar{e}} \right) = A_2 \left( \alpha \frac{d\bar{P}_1}{d\bar{e}} - \frac{d\bar{P}_2}{d\bar{e}} \right) = A_2 \overline{K_p}
\]

(9)

Thus, the dimensionless static stiffness parameter is

\[
\overline{K_p} = 3\zeta (2 + \xi) \left\{ \frac{\alpha \beta \bar{\mu}_1 (1 - \bar{e})^2 (1 + \bar{e} + \bar{\xi})^2}{\left[ \bar{\mu}_1 \beta \zeta (1 - \bar{e})^3 + (1 + \bar{e} + \bar{\xi})^3 \right]^2} + \frac{\bar{\mu}_2 (1 + \bar{e} + \bar{\xi})^2}{\left[ \bar{\mu}_2 \zeta (1 + \bar{e})^3 + (1 - \bar{e} + \bar{\xi})^3 \right]^2} \right\}
\]

(10)

Using Eqs. (8) and (12) can obtain the relationships of dimensionless worktable displacement (\(\bar{e}\)), dimensionless load capacity parameter (\(\bar{W}_p\)), and dimensionless static stiffness parameter (\(\overline{K_p}\)).
3 Analysis Results and Discussions

Fig. 3: The influence of design parameters $\alpha$ and $\beta$ at $\xi = 0$ and $\kappa_i = 1$. (Left: $\varepsilon$ versus $\bar{y}$; Right: $\bar{y}$ versus $\kappa_i$)
We analyzed the static characteristics of a self-compensated hydrostatic bearing by four dimensionless parameters which are effective area ratio ($\alpha$), restriction parameter ratio ($\beta$), viscosity ratio ($\mu_i$) and restriction island height ratio ($\xi$). In Figs 3 to 5, the left is dimensionless worktable displacement ($\varepsilon$) versus dimensionless load capacity parameter ($\bar{W}_p$), and the right is dimensionless load capacity parameter ($\bar{W}_p$) versus dimensionless static stiffness parameter ($\bar{K}_p$). The results as shown in Fig. 3 that the load capacity and stiffness of bearing both are proportional to effective area ratio. For raising load capacity and stiffness of a self-compensated hydrostatic bearing, the design on effective area is better than the restriction parameter. In Fig. 3, the $\zeta_2$ decreases as the $\bar{K}_p$ increases at low $\bar{W}_p$, but the $\zeta_2$ decreases as the
$K_p$ decreases at high $W_p$. When $\alpha > 1$ and $\zeta_2 < 10$, the $W_p$ ranges are smaller and smaller because there has negative displacements. But $\alpha$ can obvious improve the static performance.

In Fig. 4, the restriction parameter can make the viscosity to have obvious influence. When restriction parameter is more than or equal to 1, the viscosity can make worktable displacement to increase and load capacity to decrease. The stiffness of bearing is not affected by viscosity when high load, but the viscosity can make stiffness to better in specific range of load. Fig. 5 divided into the four zones A, B, C, and D which can know the influences of restriction parameter. For zone of A, it is the op-
timal operation range because the variation of displacement is lesser when load increased. When $\zeta_2$ is increasing, the zone of A can have more $\xi$ to design and the changes of displacement reduced. When the restriction island height ratio is less than 0 for any restriction parameters, we can get the best stiffness of bearing. The $\overline{K_p}$ has maximum at $\overline{W_p} = 0.4 \sim 0.6$. In Fig. 5, we can know the optimal $\overline{K_p}$ is in the zone of A and close to maximum $\overline{K_p}$ is the best design.

4 Conclusion

The influence of design parameters on static characteristics of self-compensated hydrostatic bearing is discussed. The effective area ratio and the restriction parameter ratio both can make the stiffness and load capacity to raise, especially effective area ratio can be multiplied. The viscosity is affected by restriction parameter. When the restriction parameter increase, the load capacity and stiffness of bearing and the viscosity ratio are inversely proportional. Only when the restriction parameter is too big or too small, the viscosity can make load capacity and stiffness to better. When the restriction parameter increase, the restriction island height ratio can make the load capacity and stiffness of hydrostatic bearing to better. Only when the restriction parameter is too big or too small, the viscosity can make load capacity and stiffness to better. But the load capacity available range is decreased with restriction island height ratio increasing. The analysis result can get the restriction parameter can affect available values of restriction island height and the influence of effective area on static performance is better than the restriction parameter. For the best stiffness of bearing, the viscosity and restriction parameter have a collocation relationship and the restriction island height must be taller than oil pad.

Acknowledgement: The authors greatly appreciate the support of this work by Grants NSC 104-2221-E-033-061-MY3 from the National Science Council Taiwan R. O. C.

References


