Introduction

This book may be understood as a continuation to previous expositions devoted to applications of the Nevanlinna theory to complex differential equations, see, e. g., the books by Jank and Volkmann [113] and by the second author [120], to the uniqueness theory of meromorphic functions, see the book by Yang and Yi [235], and to complex difference equations, see the book by Chen [29]. Looking at the combinations of various difference operators such as the shift operator $f(z+c)$, the basic difference operator $f(z+c) - f(z)$, and the q-difference operator $f(qz)$, combined with the derivatives of $f(z)$, this book is the first monograph introducing applications of Nevanlinna theory to the value distribution theory of complex delay-differential polynomials and complex delay-differential equations of various types.

The book is organized in twelve chapters. Most of the results presented here may be found either in the respective original references or are to be found in a closely related form at least. The basic notations and facts of Nevanlinna theory are included in Chapter 1, most proofs and details being omitted. As to this background material, the reader may consult three key monographs in Nevanlinna theory by Gol'dberg and Ostrovskii [61], Hayman [90], and Jank and Volkmann [113]. Difference Nevanlinna theory, including difference variants of the logarithmic derivative lemma, the second main theorem in the difference setting, see, e. g., [83], and difference- and delay-differential forms of the Clunie lemma will also be introduced in Chapter 1. In the short Chapter 4, Wiman–Valiron-type results in the differential, difference, and q-difference setting are included for the convenience of the reader, the proofs again being omitted.

Chapter 2 contains a number of difference- and delay-differential variants of some classical results treating zeros of complex differential polynomials, zeros of complex difference, and, in particular, delay-differential polynomials of various types. Chapter 3 then offers uniqueness results, related to value sharing, of meromorphic functions, presenting delay-differential variants of classical results.

Chapter 5 then offers some results in first- and higher-order complex delay-differential equations. Also, comparisons are given to similarities and nonsimilarities while looking at various properties of complex differential and complex delay-differential equations. The next four chapters are then devoted to considering nonlinear complex delay-differential equations. Chapter 6 presents Fermat-type delay-differential equations, first recalling Fermat-type functional and differential equations as the background. Chapter 7 is devoted to delay-differential Riccati equations, again related to essential properties of differential and difference Riccati equations. Chapter 8 then treats Malmquist-type delay-differential equations. Here some important equations have been singled out, with the existence of sufficiently many finite-order and hyper-order less than one meromorphic solutions, considered as a version of the

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Painlevé property. Based on some trigonometric formulas, some related nonlinear complex delay-differential equations are then treated in Chapter 9.

The corresponding investigations related to the q-difference operator $f(qz)$, such as Nevanlinna theory for $f(qz)$, value distribution and uniqueness of q-delay-differential polynomials and q-delay-differential equations, are presented in Chapter 10. Meromorphic solutions to systems of complex delay-differential equations are then included in Chapter 11. To close the contents of this book, some basic results on the periodicity of meromorphic functions are recalled, and the periodicity of complex delay-differential polynomials is considered by making use of the Nevanlinna theory and its difference versions in Chapter 12.