Preface to the second edition

The *relaxation* method in optimization and variational calculus has remained an active topic after the first edition was published more than two decades ago, as it is documented by a dozen of new specialized monographs.¹⁹ This second, revised and substantially expanded edition reflects in particular few dozens of new papers of mine (and my coauthors)²⁰ together with hundreds of other relevant references, without claiming completeness. The main enhancement can be specified as follows:

First, the background generalities (Chapter 1) has been rewritten or extended at several spots. The next two chapters have been enhanced in particular by inverse systems of convex compactifications (Sect. 2.6 and 3.3e-f), coarse polynomial convex compactifications (Sect. 3.3.d), and by a higher-order approximation using quasi-interpolation (Sect. 3.5.d).

Many changes have been done in Chapter 4: The Filippov-Roxin existence theory has been applied both to optimal control of system governed by ordinary-differential equations (for what it was originally devised) and also for other optimization problems. In addition to optimal control of systems governed by ordinary-differential equations where now also some computational illustration has been added (Sect. 4.3e,f), also optimal control of differential-algebraic systems has been presented. An illustration of usage of the maximum principle is there for adaptive numerical approximation. The elliptic and parabolic equations are now more general, not relying on potentiality, and systems of such equations being considered, using also simpler arguments in the proofs. As a certain illustration, the Navier-Stokes system describing incompressible fluid flows is taken. In the elliptic equations, optimal control in coefficients (optimal material design) in certain stratified materials has been included in Chapter 4, too. An incremental formula is illustrated in some situations, leading to convexity of the relaxed problems. The state constraints are treated also in the qualified way. Beside the conventional single-criteria optimization, the multicriteria Pareto/Slater concept is applied at some spots, allowing also an interpretation as cooperative games.

Variational problems in Chapters 5 and 6 have enjoyed particularly vast number of new results emerged in relaxation during past decades, and only selected ones are recorded in this new edition. On top of it, scalar variational problems (Chap. 5) have been illustrated also by relaxation in ferromagnetic materials.

A newly added Chapter 8 addresses the evolution on convex compactifications, exploiting both compactness and convex structure governed by gradient flow involving

¹⁹ Since the first edition in 1997, the relaxation theme has been published in the monographs [171, 187, 192, 268, 314, 601, 603, 605, 622, 623], or also [33, Chap. 4,11], [113, Chap. 3–6], [320, Chap. 8], and [643, Chap. 4,7].

²⁰ Specifically, it concerns the articles [89, 104, 106, 141, 179, 453, 459–461, 520, 536, 543, 545, 619, 675–678, 681–684, 686, 688, 689, 693–702, 784].

also a dissipation potential, as devised in an older article [668] for quadratic potentials. This is addressed both on the abstract level and on specific applications for evolution of oscillations (microstructure) in ferroic materials or concentrations (slip bands) in perfectly plastic materials.

Simultaneous usage of convexity and compactness of the relaxed problems is illustrated, beside non-cooperative game theory, now also on a feasibility/controllability via fixed-point theorems and on existence of periodic solutions in evolution problems.

Moreover, the notation has been occasionally slightly modified to be more conventional or to gain a better logic. And the (rather too much) abstract topological concept of nets has been suppressed in favor to the more conventional concept of sequences, relying on an assumption of separability of spaces of test functions in particular in Chapters 4-8. Also, beside the mentioned enhancements of the 1st edition, some reduction of the presentation has occasionally been applied.

Some differences likely also have occurred because all the final files from the 1st edition which included all galley proofs and language corrections, kept exclusively by the Publisher/typesetting company, were unfortunately unprofessionally annihilated in the meantime and no further language corrections have been executed in the old and the new text in this second edition.

This new, extended edition also benefits from the classes on “Selected parts from optimization theory” which I had during 1993-2010 at Charles University in Prague and reflects also some recent research²¹. Special thanks are to Miroslav Hušek for advising me the old general-topological constructions of inverse systems and threads, and to my coauthors Sören Bartels, Barbora Benešová, and Martin Kružík for providing the computational calculations and figures added to this new edition. Particular thanks are to Dr. Apostolos Damialis, the former Acquisitions Editor for Mathematics in W. de Gruyter in Berlin, who initiated this 2nd edition in 2017.

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