

Contents

Preface — VII

- 1 Basics of formal power series — 1**
 - 1.1 Basic algebraic operations — 2
 - 1.2 Composition of formal power series with nonunit — 7
 - 1.3 Right distributive law for composition — 13
 - 1.4 The matrix representation of formal power series — 17
 - 1.5 Almost unit formal power series — 22
 - 1.6 Algebraic structure of \mathbb{X} — 35
 - 1.7 Analytic structure of \mathbb{X} — 38

- 2 Calculus of formal power series — 49**
 - 2.1 Formal derivatives of formal power series — 49
 - 2.2 Formal differential equations and their applications — 54
 - 2.3 Ultrametric — 65
 - 2.4 Some topological structure for \mathbb{X} — 69
 - 2.5 The umbral calculus and formal power series — 78

- 3 Applications of formal power series, I — 85**
 - 3.1 Riordan group and formal power series — 85
 - 3.2 Riordan involutions — 92
 - 3.3 Some subgroups of the Riordan group — 93
 - 3.4 Cayley–Hamilton theorem and Fermat’s little theorem — 96

- 4 Applications of formal power series, II — 101**
 - 4.1 Formal power series and some differential equations — 101
 - 4.2 Functional equations and formal power series — 105
 - 4.3 Lagrange inversion — 115
 - 4.4 Difference equations and formal power series — 123

- 5 General composition — 129**
 - 5.1 Introduction — 129
 - 5.2 Matrix representation of the general composition, I — 132
 - 5.3 Coefficients of $f^n(z)$ — 135
 - 5.4 The general composition theorem — 141
 - 5.5 The general chain rule — 150
 - 5.6 The general right distributive law — 154
 - 5.7 Matrix representation of the general composition, II — 156

| | |
|----------|---|
| 6 | Formal analysis and classical analysis — 161 |
| 6.1 | Introduction to the boundary behavior — 161 |
| 6.2 | Boundary behavior and formal analysis — 166 |
| 6.3 | Banach spaces $\mathcal{H}^p(\beta)$ and $\mathcal{L}^p(\beta)$ — 173 |
| 6.4 | General formal logarithm — 179 |
| | |
| 7 | Formal Laurent series — 189 |
| 7.1 | Semiformal Laurent series — 190 |
| 7.2 | Reversed semiformal Laurent series — 199 |
| 7.3 | Basic algebra of formal Laurent series — 206 |
| 7.4 | Composition of formal Laurent series and formal power series — 217 |
| 7.5 | Canonical mapping and dot product of formal Laurent series — 224 |
| 7.6 | Canonical mapping and multiplication of formal Laurent series — 226 |
| 7.7 | Topological spaces of formal Laurent series — 233 |
| | |
| 8 | Iteration and iterative roots — 245 |
| 8.1 | Conjugacy of formal power series in \mathbb{D} — 245 |
| 8.2 | The iteration of formal power series — 264 |
| 8.3 | The iterative roots of formal series in \mathbb{D} — 269 |
| 8.4 | Schröder's problem — 273 |
| | |
| 9 | Formal series and general exponents — 285 |
| 9.1 | Introduction — 286 |
| 9.2 | Formal root series and its algorithm — 287 |
| 9.3 | Power of formal power series with rational exponents — 298 |
| 9.4 | The real exponents of formal power series — 304 |
| | |
| | Bibliography — 315 |
| | |
| | Index — 319 |