Contents

Preface — V

Acknowledgment — IX

1	Introduction: why fractional derivatives? — 1
2	Main results — 45
3	Boundary behavior of solutions of time-fractional equations — 51
3.1	Sharp boundary behavior for the time-fractional eigenfunctions — 51
3.2	Sharp boundary behavior for the time-fractional harmonic
	functions — 53
4	Boundary behavior of solutions of space-fractional equations — 57
4.1	Green representation formulas and solution of $(-\Delta)^s u = f$ in B_1 with
	homogeneous Dirichlet datum — 57
4.1.1	Solving $(-\Delta)^s u = f$ in B_1 for discontinuous f vanishing near ∂B_1 — 57
4.1.2	Solving $(-\Delta)^s u = f$ in B_1 for f Hölder continuous near ∂B_1 — 62
4.2	Existence and regularity for the first eigenfunction of the higher order
	fractional Laplacian — 63
4.3	Boundary asymptotics of the first eigenfunctions of $(-\Delta)^s$ — 70
4.4	Boundary behavior of <i>s</i> -harmonic functions — 83
5	Proof of the main result — 87
5.1	A result which implies Theorem 2.1 — 87
5.2	A pivotal span result towards the proof of Theorem 5.1 — 88
5.3	Every function is locally $\Lambda_{-\infty}$ -harmonic up to a small error, and completion of the proof of Theorem 5.1 — 113
5.3.1	Proof of Theorem 5.1 when <i>f</i> is a monomial — 113
5.3.2	Proof of Theorem 5.1 when <i>f</i> is a polynomial — 116
5.3.3	Proof of Theorem 5.1 for a general <i>f</i> — 117
Α	Some applications — 119

Bibliography — 123

Index — 129