Preface to the second volume of *Model Order Reduction*

This second volume of the *Model Order Reduction* handbook project mostly focuses on snapshot-based methods for parameterized partial differential equations. This approach has seen tremendous development in the past two decades, especially in the broad domain of computational mechanics. However, the main ideas were already known long before; see, e. g., the seminal work by J. L. Lumley, "The structure of inhomogeneous turbulent flows," in *Atmospheric Turbulence and Radio Wave Propagation*, 1967, for proper orthogonal decomposition (POD), and the one by A. K. Noor and J. N. Peters, *Reduced basis technique for nonlinear analysis of structures*, AIAA Journal, Vol. 4, 1980, for the reduced basis method.

The most popular mathematical strategy behind snapshot-based methods relies on Galerkin projection on finite-dimensional subspaces generated by snapshot solutions corresponding to a special choice of parameters. Because of that, it is often termed as a projection-based intrusive approach. A suitable offline-online splitting of the computational steps, as well as the use of hyperreduction techniques to be used for the nonlinear (or nonaffine) terms and nonlinear residuals, is key to efficiency.

The first chapter, by G. Rozza et al., introduces all the preliminary notions and basic ideas to start delving into the topic of snapshot-based model order reduction. All the notions will be recast into a deeper perspective in the following chapters.

The second chapter, by Grässle et al., provides an introduction to POD with a focus on (nonlinear) parametric partial differential equations (PDEs) and (nonlinear) time-dependent PDEs, and PDE-constrained optimization with POD surrogate models as application. Several numerical examples are provided to support the theoretical findings.

A second scenario in the methodological development is provided in the third chapter, by Chinesta and Ladevèze, on proper generalized decomposition, a research line significantly grown in the last couple of decades also thanks to real-world applications. Basic concepts used here rely on the separation of variables (time, space, design parameters) and tensorization.

The fourth chapter, by Maday and Patera, focuses on the reduced basis method, including a posteriori error estimation, as well as a primal-dual approach. Several combinations of these approaches have been proposed in the last few years to face problems of increasing complexity.

When facing nonaffine and nonlinear problems, the development of efficient reduction strategies is of paramount importance. These strategies can require either global or local (pointwise) subspace constructions. This issue is thoroughly covered in the fifth chapter, by Farhat et al., where several front-end computational problems in the field of nonlinear structural dynamics, scattering elastoacoustic wave propagation problems, and a parametric PDE-ODE wildfire model problem are presented.

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In the sixth chapter, by Buhr et al., localized model order reduction is presented. With this approach, the model order reduction solution is constructed via a suitable coupling of local solutions whose support lies within a subdomain of the global computational domain. Applications are provided for multiscale, linear elasticity, and fluid-flow problems.

Last but not least, the final chapter, by Brunton and Kutz, addresses a snapshot-based nonintrusive data-driven method. In particular, dynamic mode decomposition and its Koopman generalization are used to discover low-rank spatio-temporal patterns of activity, and to provide approximations in terms of linear dynamical systems, which are amenable to simple analysis techniques. These methods can be used in a nonintrusive, equation-free manner for improved computational performance of parametric PDE systems.

Several chapters contain instructive descriptions of algorithms that can serve as templates for implementing the discussed approaches in problem-specific environments.

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