Eugenio Coccia

The Time of Physics

Abstract

Space and time are fundamental concepts of the natural sciences. To simplify, we can say that until 1905 they were considered essentially as containers that house substances and bodies whose mutual interactions and movements constituted the real object of study. Einstein's Relativity changed everything. It freed space and time from prejudices that had gone unnoticed over the centuries. He brought them down from the Olympus of the *a priori* and led them to be protagonists of a new physical theory accessible to observation. We report here on the time of physics, and on the path taken by Einstein to arrive to the formulations of Special and General Relativity, which is also the starting point for modern physics theories trying to reconcile General Relativity with Quantum Mechanics.

1 Preamble

April 6, 1922, Paris, Société Française de philosophie. A dispute on the concept of time is underway between two heavyweights: Henri Bergson, philosopher and Albert Einstein, physicist. Bergsonian time speaks of a “human” duration to things. These constitute a “vitalist” time in the sense that they mark subjective durations to life. Einstein point of view is that all we can do with the tools we have—those of science and rational thought—is to mathematically express the mechanics of time, ridding ourselves once and for all of “vitalist” superstitions and contagious metaphysical moods.

2 Galilei

If the role of time as a fundamental variable parameter of natural phenomena was already understood in the late Middle Ages, it is only with Galilei that time becomes a measurable physical quantity. Galilei is the first to give a role to the scientific notion of time—which is included in his *Saggiatore* among the “primary or objective qualities”—and to give it an accurate artificial measure. His observations on the motion of bodies lead him to formulate the principle of inertia: all the bodies to which a force is not impressed remain at rest or, in

case they have an initial speed, continue to move in a uniform rectilinear motion. Galilei gets there by mentally purifying the movement of bodies from the resistance effect of the medium in which they move. The bullets, once set in motion, tend to keep their motion because (this is the important point) the state of motion is a natural state like the state of rest. We can say that before the principle of inertia was formulated there was an asymmetry between movement and stillness: the first had to be motivated and the second not. With Galilei the motion at constant speed becomes natural and equivalent from the physical point of view to the state of rest: the two states even become indistinguishable. This is the deep meaning of the Galileian principle of relativity. Let Galilei speak:

Shut yourself up with some friend in the main cabin below decks on some large ship, and have with you there some flies, butterflies, and other small flying animals. Have a large bowl of water with some fish in it; hang up a bottle that empties drop by drop into a wide vessel beneath it. With the ship standing still, observe carefully how the little animals fly with equal speed to all sides of the cabin. The fish swim indifferently in all directions; the drops fall into the vessel beneath and, in throwing something to your friend, you need throw it no more strongly in one direction than another, the distances being equal; jumping with your feet together, you pass equal spaces in every direction. When you have observed all these things carefully (though there is no doubt that when the ship is standing still everything must happen in this way), have the ship proceed with any speed you like, so long as the motion is uniform and not fluctuating this way and that. You will discover not the least change in all the effects named, nor could you tell from any of them whether the ship was moving or standing still. In jumping, you will pass on the floor the same spaces as before, nor will you make larger jumps toward the stern than toward the prow even though the ship is moving quite rapidly, despite the fact that during the time that you are in the air the floor under you will be going in a direction opposite to your jump. In throwing something to your companion, you will need no more force to get it to him whether he is in the direction of the bow or the stern, with yourself situated opposite. The droplets will fall as before into the vessel beneath without dropping toward the stern, although while the drops are in the air the ship runs many spans. The fish in their water will swim toward the front of their bowl with no more effort than toward the back, and will go with equal ease to bait placed anywhere around the edges of the bowl. Finally, the butterflies and flies will continue their flights indifferently toward every side (Galilei 1967, 186–187)

Galilei really made the bucket which “drop by drop into a wide vessel beneath it” (Galilei 1967, 186) and did not just serve to show the impossibility of establishing “whether the ship was moving or standing still” (Galilei 1967, 187). It was also the working principle of a chronometer capable of measuring time intervals with the precision of a tenth of a second, which he used to study the motion of falling bodies. It is Galilei himself who describes it: “as for the measurement of time, a large bucket full of water was held, attached at the top, which poured a thin stream of water through a thin tube, welded to the bottom” (Galilei 1967, 190).

Assuming a constant flow, time was measured by weighing the water collected in a small glass with the most exact balance.

Galilei will use this system to measure the rolling times of spheres along an inclined plane, a brilliant idea to lengthen the falling times (compared to making bodies fall vertically) and be able to measure them precisely. He was thus able to establish the law of motion of bodies due to gravity, measuring the spaces traveled being between them as the squares of times. He also understood that the pendulum would make a great clock. He arrived there by observing the oscillations of a chandelier fixed to the ceiling of the Cathedral of Pisa. He noticed that the pendulum has a period that depends only on its length, not on the amplitude of the oscillation (at least for small oscillations) nor on the value of the mass hanging from its end. This last experimental fact, due to the equivalence between inertial mass and gravitational mass, will be the basis of Einstein's theory of General Relativity.

3 Newton

Isaac Newton was born in 1642, the year Galilei died. In what framework of space and time does Newton's mechanics operate? This is his thought, reported in *Principia Mathematica* (1687):

1. Absolute, true, and mathematical time, in and of itself and of its own nature, without reference to anything external, flows uniformly and by another name is called duration. Relative, apparent, and common time is any sensible and external measure (precise or imprecise)³ of duration by means of motion; such a measure—for example, an hour, a day, a month, a year—is commonly used instead of true time. 2. Absolute space, of its own nature without reference to anything external, always remains homogeneous and immovable. Relative space is any movable measure or dimension of this absolute space; such a measure or dimension is determined by our senses from the situation of the space with respect to bodies and is popularly used for immovable space, as in the case of space under the earth or in the air or in the heavens, where the dimension is determined from the situation of the space with respect to the earth. Absolute and relative space are the same in species and in magnitude, but they do not always remain the same numerically. For example, if the earth moves, the space of our air, which in a relative sense and with respect to the earth always remains the same, will now be one part of the absolute space into which the air passes, now another part of it, and thus will be changing continually in an absolute sense (Newton 1999, 408–409).

Newton thus expresses his conviction of an absolute space and time “without reference to anything external”. It is a conception that seems to correspond to our elementary intuition. This was not the view implicitly accepted in previous

centuries, which was instead that of Aristotle. According to Aristotle, where there is nothing, and nothing moves to indicate the passage of time, space and time simply do not exist. For Newton, on the other hand, space also exists even where there is nothing to indicate a reference and time exists even if there is nothing that moves showing its unfolding. According to Newton, in space, bodies could remain *absolutely* in one of the following states: at rest, in straight and uniform motion, in accelerated motion. This last condition was clearly recognized with the appearance of fictitious forces, such as centrifugal forces, and evident effects, such as the crushing of the Earth at the poles or the rotation of the pendulum's swing plane. Leibniz's critique (which Ernst Mach took up centuries later in a deeper way and stimulating Einstein's mind on this problematic) was based on the fact that in this way absolute space assumed the role of cause of physical phenomena, regardless of the effect of other bodies. However, as far as Newtonian time is concerned, no one could complain, it was so natural that time would flow imperturbably at the same pace for everyone and everywhere.

4 Maxwell

Newtonian physics worked and the scientists' attention shifted to other fields. Thermodynamics allowed us to understand the relationships between energy, heat and mechanical work, making the first industrial revolution possible in the 18th century. Then came the century of electricity and magnetism, and new extraordinary things happened. Someone was needed to translate the amazing fact that electricity, magnetism and light were manifestations of the same reality into mathematical equations: electromagnetism. This was James Clerk Maxwell, and his four equations are among the most famous in the history of modern physics. In a sense, Maxwell represents for electromagnetism what Newton was for mechanics. There was in fact something in Maxwell's equations, a detail, which will prove to be fundamental for modern physics: the speed of propagation in vacuum of the electromagnetic signals that appears in his equations is a constant, therefore independent of the characteristics (such as position and velocity) that it is found to have who emits the signal or who receives it. This constant is indicated by all with the letter *c*, from the Latin *celeritas*, and is equal with a good approximation to 300 thousand km/s. But we know from Galilei that the concept of speed has relative meaning, its measurement depends on the state of motion of the observer. There is therefore no absolute speed, identical for everyone. Now one came out instead! How was it possible that the speed

of light was constant regardless of the observer's motion, and therefore did not add to or subtract from other speeds?

It is at this point in the story that an unknown employee of the Bern Patent Office intervenes. It will find the solution and change our concepts of space and time. In Einstein's Relativity, the spatial and temporal intervals will not be rigid and will depend on the observer, but with one impassable limit: in the phenomena of nature, the cause always precedes the effect. The principle of causality imposes a temporal order on events, and this order cannot change with the observer. To guarantee this order there is the non-overcoming of the speed of light in vacuum.

5 Einstein

Einstein faces the dilemma between Galilei's relativity and Maxwell's electromagnetism. Could they both be correct? That is, could on the one hand the stillness and uniform motion be indistinguishable and on the other hand there be a constant speed in all reference systems? Einstein's answer was: yes, they could. But there was a price to pay: two fundamental physical quantities, linked to elementary mental categories, space and time, had to be completely rethought. Let's see how, by performing a thought experiment, a *Gedankenexperiment*.

We postulate that both—Galilei with his relativity and Maxwell with his constant speed of light—are right. Let's try to imagine time and space measurements performed in reference systems moving relative to each other. For this we mentally build a clock able to show in a simple way the effects of relative motion between reference systems. The measurement of time has always been based on the counting of periodic phenomena. The first examples were the rotation of the Earth until finding the Sun in the same position (the day) or a complete revolution of the Earth around the Sun (year) and their appropriate subdivisions in equal intervals. Then came the pendulums and mechanical oscillations, finally in modern times the atomic oscillations to give ever better precision to the time measurements. Let's build a clock using the light: we count the oscillations of light reflected back and forth between two mirrors facing a fixed distance L . The speed of light, by definition is c , so the time it takes to go from one mirror to another is $T = L/c$. A complete oscillation back and forth will take double time, equal to $2L/c$. It is a watch like any other, has a constant period and is able to measure the passage of time.

We deliver it to a friend who takes a train, equipped with glass walls for observation convenience, which travels with constant speed compared to us who stay still at the station. For our friend on the train, the clock will continue to

tick with the same rhythm it had when it was at rest in the station –do you remember Galilei? Any measurement has the same result in rest as in uniform rectilinear motion. Suppose that the train passes in front of us proceeding in a straight and uniform motion with speed v . Looking through the glass wall of the wagon we then see the clock, held by our friend in a vertical position, working while it is in motion with respect to us. We observe that for us the light to go from one mirror to another takes a longer, oblique path D than the vertical path L that our friend sees, since for us the two faces of the mirror move horizontally with the speed of the train during the light oscillation. And since light always travels at speed c (as Maxwell tells us), the longer travel distance we see will result in a longer time taken by light to travel from one mirror to the other on the clock, compared to that measured by our friend. See Fig. 1.

Fig. 1: Left: a simple clock which counts the oscillations, the ticks, of light reflected back and forth between two mirrors facing a fixed distance L . Right: the same clock as seen by an observer respect to which the clocks moves in a straight and uniform motion with speed v . For the observer the light takes a longer, oblique path to go from one mirror to another since for him the two faces of the mirror move horizontally with speed v during the light oscillation. And since light always travels at speed c , the longer travel distance will result in a longer time taken by tick, as measured by the observer.

The time measurements performed in the two reference systems in motion with respect to each other are not the same. For us, the time marked by the clock on the train is dilated with respect to the time our friend measures. It is a logical and astonishing consequence of the constancy of the speed of light in any frame of reference and of Galilei's principle of relativity. If the clock were mechanical, instead of using light, would we have the same result? Yes, of course! In fact, if our friend had also brought a mechanical clock with him, it could only have been always in synchrony with the one made with light, otherwise he would distinguish the state of motion from that of rest and would violate Galilean relativity. So from the station I could only see the same time dilation on both clocks.

Let's calculate how much time dilates. Let's say that the elementary interval that light takes us to go from one mirror to the other passes from the value $T = L/c$ measured by our friend on the train to the value $T' = D/c$ measured by us at the station. To know what T' is, just apply the Pythagorean theorem. The path to go from one mirror to another is $L = cT$ for my friend who sees the mirrors facing each other, while for us it is the hypotenuse $D = cT'$ of the right triangle whose sides are L and vT' –which is horizontal displacement of one of the two mirrors during the time T' (v is the speed of the train). Try to do the math to believe, the time measured for us will be:

$$T' = T \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = T \gamma$$

The factor, which is usually indicated with the Greek letter γ , is a number greater than 1, because the speed of the train v is certainly less than that of light. From here follows the higher value of T' compared to T . The clock in motion goes γ times slower, in short.

For any reasonable value of train speed v , the γ factor is so close to 1 that it is practically impossible to measure the difference between T and T' with the clocks we deal with in our daily practice. The factor γ becomes significantly greater than 1 only at very high speeds and would become infinite if the train went at the speed of light (an impossible event). It goes without saying that accurate measurements actually made in the years following the formulation of Einstein's theory, from those with aircraft equipped with atomic clocks to those made with satellites and in elementary particle physics laboratories, will fully confirm these calculations and this vision of reality. Of course for our friend on the train, we are moving at speed v relative to him. So, if we also had a watch identical to his, in a completely symmetrical way it would be our time interval to be measured by him as longer than his. There is no contradiction. To meet and compare our clocks, one or both of us must stop being an inertial system. Our friend would have to stop and go back for example, this would break the symmetry of the situation.

The surprises don't end here. Let's try to measure a space interval. We mark two points at distance L along the station platform. When the train passes at speed v , we measure the time it takes to sight the two points, that is to travel along the stretch L . The resulting time T will naturally be equal to L/v .

The same measurement is performed by our friend on the train. Keep in mind that the station is now in motion with respect to him with speed v and that his clock is slower than ours by a factor γ . We conclude that the time he measures to pass the stretch that he will call L' will be $T' = L'/v$. But since we

know that $T' = T\gamma = (L/v)\gamma$, it follows that the distance L' measured by our friend will be reduced with respect to L by a factor γ :

$$L' = \frac{1}{\gamma} L$$

For the observer on the train space has contracted. Also in this case, the effect is extremely small if we do not involve very high speeds.

In summary, we have two observers in rectilinear and uniform motion that perform different measurements for the same intervals of time and space.

These differences are in a certain sense the price to pay so that the laws of physics are the same in all reference systems in relative rectilinear and uniform motion and therefore so that observers cannot distinguish the state of rest from that of uniform motion.

The first experiment that spectacularly demonstrated time dilation was done by studying cosmic rays. These consist primarily of high-energy particles from space (especially protons) that bombard the earth's atmosphere, generating other particles because of the impact and often real showers of particles that reach the ground. Among these secondary particles there are muons, similar to electrons but 200 times heavier and, above all, unstable: they tend to decay into other particles in a few millionths of a second. They are mostly generated in the upper atmosphere, and given their short lifetime, they would not have the time to reach the ground. Instead they get there! Since they travel at speeds close to those of light, their lifetime as measured by us here on Earth is considerably dilated, so actually they live longer and have the time to arrive to the ground.

As Bruno Rossi, protagonist of these measurements but above all one of the protagonists of Italian physics, said: nature has made available to us watches that travel at speeds that we cannot make ours reach.

However, Special Relativity, and indeed Galilean relativity itself, requires the process to be symmetrical. If we traveled astride the muon, there would be no time dilation, so how would we get to the ground? The answer is simple: in this case we would see the Earth with its atmosphere layer coming towards us at very high speed, there would be an evident contraction of the lengths and the atmospheric layer would appear thinner to us. So our muon actually arrives to ground. The relativity of time intervals has among its consequences that of making the concept of simultaneity also relative. To explain it, let's go back for a moment to the example of the train passing through the station, while we stand still on the platform. Our friend on the train stands in the middle of the wagon with his arms outstretched towards the two opposite end walls of

the wagon with a laser pointer in each hand and simultaneously sends two pulses of light to the two ends of the wagon, where two flashes are ready to signal its arrival. The two ends of the train are equidistant to him and since the light goes at speed c in both directions, it is obvious that he will see the two flashes simultaneously. Standing still at the station instead we see the two pulses of the laser traveling, always at speed c , towards moving walls. One wall meets the impulse, shortening its path, the other moves away from the impulse, lengthening its path. Therefore, we will see the impulse that goes against the first wall arriving before the one on the second wall. Two simultaneous events for one observer are not simultaneous for another in uniform relative motion with respect to the first.

If we imagine that we can reach speeds close to those of light, we encounter so-called “paradoxes”, which in reality are not such, even if they seem so. The most famous is that of the twins. Two twins in their twenties decide to separate, one stays on Earth and the other embarks on a “high-speed” spaceship for a long round trip during which his vehicle will reach speeds close to c . After 10 years, the traveling twin returns home to celebrate their thirtieth birthday with his brother but finds his 80-year-old brother who in the meantime has had children and grandchildren. It is a situation in perfect agreement with the theory of Relativity.

The paradox lies in the fact that from the point of view of the traveling brother (from his reference system) it is the brother who remained on Earth who moved away with the whole planet at sidereal speed and then rejoined the spaceship. It should then be the brother on Earth who remains younger! How is the paradox resolved?

The solution lies in the lack of symmetry of the situations experienced by the twins. Only the one who took the spaceship accelerated to reach the very high speed, then decelerated to go back, accelerated again to reach high speed in the opposite direction and then finally must decelerate to land back on Earth. When all the calculations are made, taking into account also the theory of General Relativity that we will explain shortly, we see that these differences are decisive.

From all these considerations, the principle of Special Relativity emerges in its simplicity and elegance: All the laws of nature, formulated in a specific reference system, remain valid in completely identical form, when referring to another reference system which is moving of rectilinear and uniform motion with respect to the first.

The term “Special Relativity” refers to the scope of validity of the theory, limited only to cases of rectilinear and uniform motions.

6 Spacetime

Space and time in Einstein's Relativity are no longer independent entities; they change with the same factor of proportionality in passing from one inertial reference system to another. One expands and the other contracts. It is as if they were partial aspects of a more general reality. Separately they do not give us an adequate representation of physical phenomena, we must consider them part of a more general physical property in which they are indissolubly united. What is this property?

A great mathematician, Hermann Minkowski, who had been Einstein's professor at the Zurich Polytechnic, understood that time must be considered a fourth dimension of the same quality as the three spatial dimensions, with the difference that must be regarded mathematically as an imaginary dimension, and can therefore be represented geometrically with a coordinate similar to the other three spatial coordinates.

From then on, the description of events will be associated with a new representation of reality: four-dimensional spacetime. The transformation from one reference system to another will correspond to a rotation in four dimensions in which all four coordinates can change. The distance between two events in spacetime will remain invariant, just as rotations in two or three dimensions keep the distance between two points in space unchanged.

Einstein's Theory of Relativity is ultimately the geometric theory of spacetime and can be seen as the last stage in the evolution of human thought on space and time. Aristotle considered three-dimensional space a hierarchical and differentiated entity, with absolutely privileged places. Galilei and Newton made space and time homogeneous and undifferentiated but they have kept their independent and absolute character. Einstein also drops these latter privileges: space does not have the same magnitude for everyone, time does not flow in the same way for everyone.

7 General Relativity

In the spacetime of Special Relativity free bodies and light rays move in a straight line. Euclidean geometry applies. Spacetime is therefore limited to being the theater where gravity acts as an additional and indispensable element to the explanation of phenomena, without however interacting and modifying the theater itself. According to General Relativity, instead, space, time and gravity are not three distinct entities, but something inextricably linked. Gravity is

nothing more than the effect of the curvature of space-time due to the presence of matter and energy.

Before delving into more in-depth explanations, and understanding the path that led Einstein to what has been called the most beautiful theory ever formulated, let's get to the heart of General Relativity in a few lines. Let's imagine two two-dimensional beings who think they live on one plane, a universe with only two dimensions, completely flat. The two protagonists of this story propose to carry out an experiment with an outcome that they consider obvious: to stand at a certain distance and move, at the same speed, along parallel lines, then occasionally remeasuring the distance that separates them they expect to always find the same distance. Therefore, both leave, being careful to go at the same speed, and proceed along the direction perpendicular to the one that unites them, thus believing that they move according to the dictates of Euclidean geometry along two lines that will never meet. After a while they stop and measure the distance between them. They discover with surprise that they have approached.

At first they believe they were wrong, they repeat the experiment several times, but they always get the same result. Eventually they give up and come to the following conclusion: "When we move in that direction, there is a force that attracts us". They do not discuss the premise of living on a plane and decide to introduce a force to explain what they observe. They reason *à la Newton*. What Einstein would conclude instead is that perhaps the two characters are not on a plane, but on a spherical surface: if the geometry is changed, their approach will no longer be explained by the addition of a further ingredient to flat spacetime—that is the force of gravity—but simply with the fact that starting from the equator and actually going in a direction perpendicular to that which unites them, the two travel along two meridians. Their approach is therefore completely natural: it is a consequence of the curved geometry in which they live. There is no need to introduce any force.

The moral is: in the universe there is no force of gravity, all bodies move free but in a curved spacetime. This very simple example is perhaps able to grasp the meaning of the revolution introduced by Einstein, adding that is the presence of matter (and energy) to generate the curvature. We are facing a completely different conception of the universe from that of Newton. We do not live in a Euclidean space: in the region near matter, around the Sun for example, we can think that there is a kind of *depression*, as if the Sun were a heavy lead sphere placed on an elastic carpet. According to Einstein, a planet revolves around the Sun like a ball thrown around the lead sphere, forced by the curvature of the carpet. It's not just a different way of explaining the same predicted effects of Newton's law. From Einstein's vision arise equations capable of explaining new phenomena, with revolutionary consequences for the Universe, its origin and its destiny.

8 The Einsteinian Path

After the formulation of Special Relativity, the problem remained of understanding how to ensure that the laws of physics did not remain valid only in inertial frames of reference but in any frame of reference however in motion. That is, how to formulate a general theory of relativity. In this sense, Special Relativity had made no progress with respect to Galilean relativity. These are the words of Einstein:

All the previous considerations [on space and time in pre-relativistic physics and on the theory of Special Relativity] are based on the hypothesis that inertial systems are equivalent [...] but that they are privileged, with regard to the formulation of natural laws, over reference spaces in different states of motion. The attribution of such a privilege to particular states of motion is not justified [...] either by particular properties of bodies, or by the concept of motion; this privilege must therefore be considered as an independent property of the space-time continuum. The principle of inertia, in particular, seems to lead us to attribute physically objective properties to the space-time continuum. Just as from the Newtonian point of view it was necessary to make the two hypotheses: *tempus est absolutum, spatium est absolutum*, so from the point of view of the special theory of Relativity we must say: *continuum spatii et temporis est absolutum*. In the latter hypothesis, *absolutum* means not only ‘physically real’, but also ‘independent in its physical properties, having a physical effect, but not in turn influenced by physical conditions’ (Einstein 1923, 59).

Einstein will succeed in the intent to free physics from absolute space and time, but after many years of effort. The starting situation of General Relativity was much more complicated than that of Special Relativity. The latter had behind its “claim” a series of experimental facts established over many years of research on electromagnetism and the compelling agonizing problem to be solved of a constant speed in all reference systems. The general theory, on the other hand, had behind it only a subtle observation to explain and some epistemological dissatisfactions. The observation was the anomaly of the precession of the perihelion of Mercury, the planet closest to the Sun. The point of Mercury’s elliptical orbit closest to the Sun (the perihelion) moved year after year by a very small but still too large amount to be explained with Newton’s theory. So much so as to hypothesize the disturbing presence of another small planet, baptized Vulcan—because being close to the Sun it should have been very hot—, which instead was not found.

A first dissatisfaction was due to the fact that Newton’s theory did not predict a propagation speed of gravity, or rather this speed was implicitly infinite. Conversely, it was evident from Maxwell’s equations of electromagnetism, that the influence of an electric charge on another distant one does not occur instan-

taneously, that is, with infinite speed, but at the speed of light. So, if an electric charge is moved from its position, the information reaches the second after a certain time. On the other hand, in Newton's theory if a mass is moved, all the others, however far away, "know" it immediately. Information in Newtonian gravity travels at infinite speed. Gravity was incompatible with Special Relativity, and Einstein thought of getting his hands on it. Then there was another cloud in the sky of Einstein's mind, and it was a disturbing experimental fact already highlighted by Galilei and Newton: the inexplicable coincidence of inertial mass with gravitational mass. This explains why all bodies reach the ground together if dropped from the same height, and why the period of the pendulum does not depend on the mass hanging at its end. (We are naturally neglecting light objects that can be affected by friction with the air).

All this was well known but it gave Einstein a lot to think about. In fact, the role that mass has in the equation $F = ma$ is to express the reluctance to change the body state of motion, that is, it is a measure of the body's inertia, let's call it the inertial mass. The role that mass plays in the Newtonian law of gravitation is instead another. Here the masses appear in the role of attracting and being attracted to other masses. The similarity of Newton's law to Coulomb's law of electric force, where electric charges play the same role, suggests that the masses here appear as gravitational "charges", sources of an attractive field. The mass m of the same body in Newton's equation therefore deserves a different name, that of gravitational mass. These two masses, which play different roles for the same body, turns out to be equivalent, or the acceleration of gravity on Earth to have the same value for all bodies would not be justified (it is experimentally verified with great precision). The equivalence of inertial mass and gravitational mass, according to Einstein, was not accidental and it had a profound meaning, which until then had eluded natural philosophers.

There were two factors that made Einstein's journey difficult. While the mathematical language of Special Relativity was already well developed, the development of General Relativity will require a new effort in understanding the relationship between physics, non-Euclidean geometries and absolute differential calculus. As Einstein said, compared to this problem, the original theory of Relativity is child's play. Moreover, while Special Relativity was soon understood by his colleagues and he had immediately found authoritative supporters, such as Max Planck, for the development of General Relativity Einstein found himself alone. Planck himself, having listened to Einstein's state of the art of his efforts while working on his new theory, paternally advised him to let it go: he would not be successful and in any case no one would take him seriously. But Einstein certainly couldn't let go, he had the happiest thought of his life:

When, in 1907, I was working on a review article on Special Relativity for the *Jahrbuch der Radioaktivität und Elektronik*, I also had to try to modify the Newtonian theory of gravitation in order to make its laws compatible with Special Relativity. [...] It was then that I had the happiest thought of my life, in the following form. The gravitational field has only a relative existence, analogous to the electric field generated by magnetoelectric induction. In fact, for an observer who falls freely from the roof of a house, there is no gravitational field—at least in the immediate vicinity. In fact, if the observer drops bodies, they remain in a state of rest or uniform motion with respect to him, regardless of their particular chemical or physical nature (obviously neglecting the air resistance). Consequently, the observer has the right to interpret his state as a ‘state of rest’. Thanks to this idea, that very unique experimental law according to which, in a gravitational field, all bodies fall with the same acceleration, suddenly acquired a profound physical meaning. [...] The independence of the acceleration of falling from the nature of bodies, well known experimentally, is therefore a solid argument in favor of the extension of the postulate of Relativity to coordinate systems in non-uniform motion with respect to one other (Einstein 2002, 135–136)

In this writing there is enlightenment. The magical moment equivalent to what Newton must have felt when he thought the Moon was falling to Earth. Einstein realizes that the principle of equivalence of inertial mass and gravitational mass is a fundamental empirical fact and he erects it as the foundation of its construction.

The starting point, as we read in Einstein’s passage from 1907, is another *gedankenexperiment*, whose protagonist is a man who throws himself off the roof. Years later it will be described by him as “the Elevator Experiment” (in 1907 there were obviously few elevators). This experience illustrates Einstein’s happiest idea and reveals the extraordinary equivalence between inertia and gravity. A stationary system in a gravitational field is equivalent to an accelerated system. A free-falling system in a gravitational field is equivalent to an inertial system. This is the new formulation of the equivalence principle, which Einstein transformed into a fundamental principle of physics.

9 Gravitation and Geometry

To understand how it is possible to use Einstein’s principle of equivalence to arrive at General Relativity, let’s proceed to a new *Gedankenexperiment*.

Imagine we are observing a glass-walled elevator free-falling in a gravitational field. Inside the elevator there is a friend of ours who experiences the freedom of being in an inertial reference system, it is as if he were stationary and far from any gravitational field. He has a laser pointer in his hand and shoots the light beam horizontally, towards a side wall as shown in Figure 2. Naturally he will see the light go straight towards the wall, any other behavior of the

light beam would indicate the non-inertiality of his system. But how do we, standing still on the ground, see the same scene?

For us, the elevator and the laser are falling and accelerating. We see the beam of light actually “follow” the cabin and fall with it. The trajectory of light will therefore be the combination of a horizontal motion at the constant speed of light (proportional space and times) and an accelerated vertical motion (spaces proportional to the square of times). The result is a curved motion for the light beam, precisely a parabolic motion, like that of projectiles in a gravitational field. The conclusion is astounding: in a gravitational field the trajectory of light is not a straight line.

Fig. 2: The different trajectories of a light beam as seen by a person inside an inertial reference system constituted by a glass-walled cabin (left), and by an observer who sees the same cabin free falling in a gravitational field (right). In a gravitational field the trajectory of light is not a straight line.

Einstein knew well that in Euclidean space, where the Pythagorean theorem holds, the shortest line connecting two points (called the geodesic line) is a straight line. In inertial systems, for example, Euclidean geometry applies and the geodesic line is a straight line.

The thought experiment described above suggested to Einstein that in the presence of a gravitational field, space-time is no longer Euclidean and geodesic lines are no longer straight lines. Einstein’s thoughts can be expressed like this: if everything that happens in space-time senses the presence of gravity, it means that gravity modifies space-time. What if what we call gravity is nothing more than the visible manifestation of the curvature of spacetime? In other words,

what if curved spacetime was gravity? This is how the fundamental message of the new theory is outlined: the presence of mass and energy modifies the geometry of space-time, curving it. The curved space-time in turn tells bodies and light how to move: following its geodesic lines, which will no longer be straight lines.

To find the equation that could mathematically translate these concepts, Einstein had to labor for ten years. We will get there with the approach followed by Tullio Regge. Let's start with Euclid and the axioms of his geometry. The famous fifth postulate asserts the existence of only one parallel to a given straight line passing through a point outside this. From this postulate it follows that the sum of the internal angles of a triangle is 180 degrees. If we draw a triangle on a sphere, however, this is no longer true, just think of a triangle consisting of two meridian arcs and an equator arc. The two base angles by definition are both 90 degrees, their sum is already 180 degrees. The sum of the three internal angles will therefore certainly be greater than 180 degrees. It is shown that if we draw any triangle on the surface of a sphere of radius R , using great circle arcs, which are the geodesic lines on the sphere, the relation holds:

$$\alpha + \beta + \gamma - \pi = \frac{A}{R^2}$$

Where A is the area of the triangle. We can then define a coefficient K as follows:

$$K = \frac{1}{R^2} = \frac{\alpha + \beta + \gamma - \pi}{A}$$

A triangle drawn on a flat sheet, which can be seen as a portion of a sphere of infinite radius, will naturally have $K = 0$. On a spherical surface, wherever the triangle is drawn using geodetic lines, however angles are chosen, and for any area, the value of K will not change, as this coefficient is directly related to the radius of the sphere. For any curved surface, even if not spherical, we can think of drawing very small triangles, calculating the value of the angles and of the very small enclosed area and having K values that change point by point on the surface. In other words, we will have a quantity $K(x)$ which will be a function of the point x on the surface. This quantity is known as Gaussian curvature and has universal significance, depending only on the intrinsic curvature of the surface at that point. Let's take an example of what we mean by "intrinsic". For the surface of a cone or cylinder, K is zero, just like for a plane. In fact, I can make those surfaces by appropriately rolling flat sheets. Instead, let's try to wrap a flat sheet to make a sphere, or a horse saddle surface. Impossible, precisely because there is an intrinsic curvature in the sphere and in the horse saddle.

For a number of dimensions greater than two things get complicated (also due to our inability to mentally visualize objects in more than 3 dimensions). For each point of the surface, we can no longer limit ourselves to a single value of the curvature, as for a two-dimensional surface, and the curvature K will depend on the orientation in multidimensional space of the geodetic lines used to define it. The list of all possible curvatures in a point is called the “Riemann Tensor” from the name of the great German mathematician who first defined it. For a 4-dimensional space, the Riemann tensor has twenty components. Both the Gaussian curvature K and the components of the Riemann tensor are expressed as the inverse of an area. The value of this area is also important because it tells us that if we are considering an area much smaller than that we can neglect the curvature and use Euclidean geometry.

Einstein was looking for the mathematical equation that quantified how the geometry of spacetime was affected by the distribution of matter. He found himself having to learn the language of absolute differential calculus, developed by two great Italian mathematicians, Gregorio Ricci Curbastro and Tullio Levi-Civita. With the latter Einstein had an intense and fruitful correspondence. Einstein said that Levi Civita rode that subject on the horse of true mathematics, while he was forced to go on foot. The equations, in their complete form, represent the conclusion of a splendid climb to what is considered one of the highest peaks reached by human thought. Geometry and matter are represented by quantities that have indices, which take the values from 1 to 4 to signify their possible components in four-dimensional space-time. We must write them:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$G_{\mu\nu}$ contains the components of the Riemann tensor and expresses all the possible curvatures of space-time while $T_{\mu\nu}$ contains the possible distributions of matter and energy. If $T_{\mu\nu}$ is equal to zero, $G_{\mu\nu}$ describes a flat, i.e. Euclidean, spacetime. If we do not consider the indices, the equation can be written in a conceptually simpler way while retaining its essence:

$$K \approx \frac{G}{c^4} \varrho$$

where K is the Gaussian curvature and ϱ is the density of matter and energy.

Of all the symbols of the first equation, the most important is the sign of equality: the curvature of space-time is determined by the mass and energy present in that region. The equations of General Relativity are the analogue, for gravitation, of Maxwell’s equations for electromagnetism, while Newton’s gravitation appears as the analogue of electrostatics: it is valid for slow motions and

low gravitational fields. These are the conditions experienced by bodies on Earth and in the solar system, which is why Newtonian mechanics has not been questioned for centuries.

There are some fundamental differences between electromagnetism and gravitation. One is that electric charges are of two opposite signs, electrons and protons therefore tend to attract each other to form neutral atoms and bodies. The electromagnetic interactions then lose their effectiveness. The gravitational charge, on the other hand, has only one sign and is always attractive. The larger the body that forms, the greater its ability to attract and be attracted. The celestial bodies and structures present in the cosmos are therefore shaped by gravitation. Another difference: photons, quanta of the electromagnetic field, are neutral and are not sources of other electromagnetic fields. Instead, the quanta of the gravitational field, the gravitons, in turn generate gravitational fields. This makes Einstein's equations non-linear and very difficult to solve.

General Relativity teaches us that, unlike what happened in classical physics, space and time are not simple containers of nature but are themselves part of it, intervening in the development of phenomena and being modified by them. They do not constitute an empty theater where reality is staged, they are instead reality themselves, a flexible and deformable continuum, a sort of cosmic jelly that dictates to any form of energy how to travel.

10 The Time of General Relativity

Einstein tested his equations with the problem of the anomaly of the precession of the orbit of Mercury, and was comforted: the right value was coming! He immediately suggested another fundamental test to highlight the difference between Newton's theory and General Relativity. During total eclipses, from the Earth we should see that the stars that are behind and around the Sun occupy a different position than usual, that they "move away" from each other, because the light they emit deviates due to the curvature around the Sun, giving us an apparent different location on the celestial vault. It was the photos taken in 1919 during a total solar eclipse on the Atlantic island of Principe that gave Einstein a planetary notoriety, thanks to the resonance that the news had in the press around the world. The leader of that ocean-going expedition was another original scientist, the Englishman Arthur Eddington. Later, in 1936, Einstein showed that a body of large mass, such as a galaxy, could act as a gravitational lens if it came between a light source and the observer. The image of the source can be seen by the observer multiplied or deformed depending on the arrangement in space of the source, the lens and the observer. If the three are perfectly

aligned, the source can appear as a ring of light, the so-called “Einstein Ring”. These predictions were then promptly verified and the spectacular gravitational lensing technique is now also used to estimate the presence of the mysterious dark matter in galaxies and clusters.

In the above observations we have highlighted the deformations induced by matter on space. But time is also distorted. In the universe, contrary to what was implicitly believed for centuries and what was posed as an assumption by Newton, there is no single time valid for everyone. Time flows differently for every observer, at every point in space-time. In particular it flows more slowly where gravity is strongest. We can make a new example to highlight how this is a necessary consequence of the equivalence principle. Imagine again that we are observing the glass-walled elevator in free fall in a gravitational field. Our friend with a laser pointer is still inside the elevator, ready to do another experiment. The lift is equipped with a sensor on the roof that measures the frequency of the incident light, and is calibrated to emit a powerful flash of light outside if the measured frequency is exactly that emitted by the laser pointer. The frequency of a wave is inversely proportional to the wave period, that is, the time interval between successive peaks. As the elevator falls, our friend shoots the laser light up towards the sensor. Since it is located in an inertial reference, by definition the sensor can only measure the light at the same frequency as that emitted by the laser, and the flash in fact signals the completed experiment.

The same result must also happen for us who see the elevator fall into a gravitational field. But for us the sensor is moving towards the light, because it is in accelerated fall. In these conditions the sensor should measure a higher frequency as a consequence of the Doppler effect. (Given a wave that propagates in space—light, sound, of any kind—if you move towards the wave front, the distance between the peaks is shortened and therefore a higher frequency is perceived. If instead we move away from the wave front, the distance between the peaks increases and the frequency decreases. This is the Doppler effect, and explains for example why the frequency of the sound of an ambulance siren becomes increases as it approaches us and decreases when moving away.) Necessarily, also for us the sensor must register the same frequency as the laser, in fact we have seen the flash. So, there must be an equivalent and opposite effect that cancels the Doppler frequency increase. There is no alternative: the electromagnetic wave rising vertically from a height where gravity is stronger to another where it is weaker must decrease the value of its frequency, or what is the same, increase its period.

This effect is measured and is called gravitational red-shift. The historical reason for the name “red shift” is that red light is at the lowest frequencies in the visible spectrum. The key experiment to demonstrate this was conducted in 1960 by

Fig. 3: Left: in an inertial system the frequency of a beam emitted by a laser pointer does not change in its travel towards the roof, as would be shown by a flash emitted by an appropriate detector tuned at the same frequency. Right: The same phenomena must be seen by an observer in a gravitational field looking the system free falling. But for this to be true, the Doppler effect due to the accelerating roof, which would increase the detected frequency as depicted and would prevent the flash emission, must be perfectly balanced by an opposite effect decreasing the frequency. This is the gravitational red-shift.

physicists Pound and Rebka, using not the light of a laser but the gamma radiation (high frequency electromagnetic waves) of a radioactive source and exploiting the difference in height (22.6 meters) between the basement and the top floor of a Harvard University building. The electromagnetic oscillations going upwards actually decreased their frequency.

This effect has extraordinary implications for the speed at which time passes. A regular sequence of peaks, like any other periodic phenomenon, can be taken as a measure of time, the peaks of the wave being nothing more than the ticking of the clock.

We can therefore conclude that time does not flow in the same way in regions of space placed at different values of the gravitational field. Our biological time is marked by biochemical phenomena that involve—all—electromagnetic signals. It is not only the artificial clocks, the ones we build, that keep time slower or faster, *we ourselves are clocks* and we will age faster by being where gravity is weakest, while we will live longer staying where the gravitational field is most strong. It is evident that these effects are not like those of Special Relativity. In that case the effects were reciprocal: in the station we see the clock on the train go slower as well as the observer on the train sees ours go slower than his. In General Relativity there is no this symmetry. We on the ground floor will see

those on the top floor age faster than us, and on the top floor they would see us younger.

The knowledge of the relativistic effects on time finds a spectacular application in GPS, the Global Positioning System, used by the navigators of our cars and by smartphones. The system depends on about twenty satellites, equipped with atomic clocks, which orbit the Earth at about 20 km in height.

The system works by converting the travel times of radio signals exchanged between satellites and a terrestrial receiver, for example our car, into distances, thus allowing us to precisely locate its position. But it is necessary that the clocks of the satellites and of our car are synchronized, and this can only happen if we take into account the effects of Special Relativity and General Relativity. Due to Special Relativity, since satellites are moving relative to our car, their clocks are delayed by 7 millionths of a second compared to ours every day. However, since they are in a region where the gravitational field is weaker, they move forward by 46 millionths of a second every day. The net effect is that satellite clocks go 39 millionths of a second faster every day than ours on the ground. If this difference were not corrected, the daily error on the location of our car would be more than 10 km.

We make use of Einstein's Relativity every day. Einstein's equation turns out to be a gold mine. Over time, other precious nuggets have been extracted from this mine such as black holes, singularities, gravitational waves, the expanding universe.

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