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Between the Time of Physics and the Time of Metaphysics, the Time of Tense Logic?

Sosein, Dasein, Zeitsein

Abstract

According to Russell, Being is divided into atemporal *universals* and temporal *particulars*. But, to the extent that the Antisthene's caballeity pops up in the *topos noeton*, the ancestors of the horse must precede the horse in the sublunar. Thus, Being must be divided into *Sosein*, *Dasein* and *Zeitsein*. Only the Whiteheadian “eternal objects”, such as geometrical forms and colours, are atemporal universals, while caballeity is in the *Zeitsein* together with Conquérant (one among Napoleon's horses). In this ontology, according to the Lautmanian shift, *Spatiotemporality* pertains to the *Sosein* and has a two-fold structure: logical and mathematical. The tense logic of A.N. Prior describes it, while its link with the mathematics of spacetime is fixed by the Boolean kernel in the von Neumann algebra for quantum logic. This speculation will be preceded by a narrative exposition of elementary tense logic.

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1 The Tale of Tense Logic

Tense logic is a variety of *temporal* logic, and temporal logic is a branch of *modal* logic. Thus, in order to give a short history of tense logic, we must start by tracing a short genealogy of the modal logic.¹ The latter has three main moments: the Leibnizian *foundation*, the Kripkean *mathematization* and the creation of *tense logic* by Arthur Prior.

¹ See Blackburn, De Rijke and Venema (2001). *Arthur Prior and Hybrid Logic* is a fascinating paper which focuses on the more promising lines of thought in the area.

1.1 The Leibnizian Foundation

In a draft² written in 1671 or 1672, Leibniz discloses, in Russellian terms, the structural similarity between three versions of the Apulean square AEIO. This is the *Leibnizian Table of Apulean squares*:

<i>Omnis</i>	<i>Nullum</i>	<i>Necessarium</i>	<i>Impossibile</i>	<i>Debitum</i>	<i>Illicitum</i>
<i>Quidam</i>	<i>Quidam non</i>	<i>Possibile</i>	<i>Contingens</i>	<i>Licitum</i>	<i>Indifferens</i>

In symbols (or in English, where symbols are still missing):

$\forall \neg \exists$	$\Box p \neg \Diamond p$	Obligatory Forbidden
$\exists \neg \forall$	$\Diamond p \neg \Box p$	Permitted Optional

This table of similarities has two main consequences. The first is the well-known thesis that a necessary truth is true in all possible worlds.³ In symbols:

$$\Box p \Leftrightarrow \forall w \mid \neg_w p$$

(“Necessarily p iff for each world w it is true at w that p ”)

This Leibnizian analysis of necessity (generalizable to the other modes of the square) is of such significance that it deserves a name. I will call it *Leibniz’s Bridge*. It is a bridge because it connects a left-side modality with a right-side quantifier. In Carnapian terms, modality is the *explanandum* and quantification is the *explanans*. Of course, the quantifier operates in the possible worlds of Molina, dubious entities surrounded by a metaphysical flavour. However, in the view of such logicians as Carnap, quantification theory enjoys a less controversial status than modal logic.

² It was rediscovered by Blanché when he prefaced the *Etudes de logique déontique* of Kalinowski. Gardies relates : “Il a en effet trouvé les textes en question grâce à M. Serres qui dans *Le système de Leibniz et ses modèles mathématiques* cite L. Couturat, lequel résume *Definitio iustitiae universalis*, c’est-à-dire le fragment N. 12, des *Elementa juris naturalis* de l’édition de l’Académie Prussienne des Sciences... publié pour la première fois par A. Trendelenburg dans ses *Historische Beiträge zur Philosophie*” (Gardies 1974, 265 – 275).

³ On the Leibnizian source of this theme, see Kalinowski (1985, 236 – 237) who refers to another text, edited by Couturat in the *Opuscles et fragments inédits* of Leibniz (1903): *Phil.* IV, 3, a, 1 – 4, sans titre, pp. 16 – 24.

The second consequence of the Leibnizian reformulation of Apuleio's square is a generalisation of the concept of modality itself. According to Leibniz, beside the *metaphysical* modalities of possibility and necessity—which are well-known since Aristotle—there are also *deontic* modalities such as duty and rightness. And so, we can say that his generalisation systematizes philosophy with modal logic. Its *Begriffsschrift* is the standard symbolism of the latter: on the Apulean square, $\Box$ and $\Diamond$ are transposed in their ethereal versions $[]$ (“Box”) and $\langle \rangle$ (“Diamond”), so that the subalternation obtained is:

$$\begin{array}{c} [] \\ \downarrow \\ \langle \rangle \end{array}$$

It may be rephrased as: “the diamond is in the box”.

1.2 The Kripkean Mathematization

With respect to modal logic, we must distinguish between its axiomatization and its *mathematization*. The well-known axiomatization of modal logic was laid down in its main lines by C.I. Lewis in 1932 (Lewis and Langford 1932). But the *mathematization* of modal logic is another story. This concept was introduced by Robert Goldblatt (2005) and the understanding of its true meaning requires a philosophical elucidation.

The mathematization of modal logic has at least two births. The first, as we shall see, is probably due to Arthur Prior. The second, instead, is due to Saul Kripke,⁴ the philosopher who provided its canonical formulation. The cornerstone of the latter is a *Kripke-model* or K-model. A Kripke-model M must be written in two main versions. The original (Kripke 1963)⁵ one is:

$$M = \langle \langle W, P \rangle, V \rangle$$

⁴ Here we cannot discuss the question of priority between authors. In addition to the already mentioned paper by Goldblatt on the evolution of mathematical modal logic, see Copeland (2002). And it is enough to say that the mathematizations of Prior and Kripke are independent from each other. The publication of the results of the young Kripke's work has been postponed because of bad advices from his academic masters.

⁵ Original version but not in literal terms. The essential point is the difference between this original version, with its philosophical significance, and the new one which has vanished in the rarefied air of abstract formalism.

In this version of the model M , W is a set of possible worlds w , P is the relation of *relative possibility* between possible worlds, and V is the so-called “function of valuation” which takes the proposition p of a language L as argument and the subsets of W where p is true as values.

From a theoretical point of view, in Kripke’s model the two members have a very different status (Blackburn, De Rijke and Venema 2001, Df 1.19, 17). The couple $\langle W, P \rangle$, i.e., the *frame* of the model, gives the mathematical picture of its ontology, while the valuation function V gives the additional information endowing this ontology with a logic. In Husserlian terms, which are derived from Aristotle, one can say that the frame is the *Ontology* of the model and the valuation is its *Apophantics*. In the frame, usually, there is no relation of “relative possibility”, but a relation of *accessibility*. This, at least, is what Peter Geach suggested to Prior in a letter of 1960. Yet, relative possibility and accessibility are the converse of one another: w has access to w' if and only if w' is possible relatively to w .

Therefore, since modal logic has evolved, we may write Kripke’s model in the following terms:

$$M = \langle \langle E, R \rangle, V \rangle$$

In this new version E is *any* non-empty set, whose members are not only the old “worlds” of Molina or Leibniz, but as well “points”, instants, words, or whatnot; and R is *any* binary relation. This acceptance defines an *abstract* or “formal” Kripke model.

Now, when modal logic is endowed with a Kripke model, what is the effect of this model on those concepts of modal logic illustrated by the definition of necessity that we find in Leibniz? The definition of necessity becomes

$$M \models_v \Box p \Leftrightarrow \forall w \ w P v \rightarrow M \models_w p$$

(“In M it is true at v that necessarily p iff for all w , if w is possible relatively to v , then in M it is true at w that p ”)

In other words, a *footbridge* is added on the right side of Leibniz’s bridge. It conditions or *controls* the relevance of possible worlds in the definition of a modality as a constraint imposed on relative possibility, so that the relation R of an abstract model can be said to be a relation of “parametrization”.

Now, why is this footbridge or parametrization a *mathematization* of modal logic? A crucial example will help us to answer this question. Yet, in order to find it, we have to return to the second modal discovery made by Leibniz, i.e., the structural similarity between the metaphysical modalities, such as possibility and necessity, on the one hand, and the deontic modalities, such as permission

and obligation, on the other. Nevertheless, this structural similarity does not go without restrictions.

Indeed, when the Apulean square is decorated with metaphysical modalities, we find in its A-I side, that is between the necessity $\Box p$ and the possibility $\Diamond p$, the halfway house of the naked p :

$$\begin{array}{c} \Box p \\ \downarrow \\ P \\ \downarrow \\ \Diamond p \end{array}$$

This writing presupposes the law $\Box p \rightarrow p$ (*a necesse ad esse valet consequentia*) according to which, if Cicero is necessarily Tullius Marcus, then Cicero is Tullius Marcus. But if we resort to deontic modalities, things change. From the fact that Babbitt has the duty to pay taxes, it doesn't follow that Babbitt pays taxes, because metaphysical and normative modalities have a different meaning. But this fact has also a *mathematical* accompaniment. If the relation of parametrization P or R is compelled to be *reflexive*, then we have wPw and so, by applying our previous definition of necessity, we now obtain

$$\vdash_v \Box p \Rightarrow \vdash_v p$$

("If it is true at v that necessarily p , then it is true at v that p ")

Nonetheless, there is no corresponding principle for the deontic modalities.

Now, reflexivity, as well as transitivity, symmetry (etc.) are *mathematical* features of a relation. So that, if one controls the modal laws using these mathematical attributes of the parametrizing relation R , modal logic becomes *mathematical modal logic* in the sense of Goldblatt.

1.3 The Creation of Tense Logic by Arthur Prior

Arthur Norman Prior was the main founder of temporal logic and the creator of tense logic. Overall, its work can be broken down into three major moments-movements: 1. an anachronistic encounter between Diodorus Chronos and St. Thomas Aquinas; 2. a mathematization of temporal logic and 3. the encoding of tense logic.

1.3.1 St. Thomas Aquinas Meets Diodorus Chronos

The foundation of temporal logic coincides with the erection of an *Apulean column* containing a *pedestal* (I) and a *capital* (A). Prior founds the pedestal I of temporal logic in Aquinas definition of past and future: the *Thomas Aquinas Theorem* of temporal logic:

Past is what has been present and future is what will be present (Est enim praeteritum quod fuit praesens, futurum autem quod erit praesens)

More profoundly this definition means that the present appears in the three “parts” of time: in the past as past *present*, in the present as *present* and in the future as future *present*. As a result of this, only two symbols will be required on the pedestal position I: *Pp* for “It is past that *p*” and *Fp* for “It is future that *p*”; for the present time, the simple *p* will be enough, since its illustrations will be, for example, “It rains” in order to report a *present* fact equally reported in the *p* of *Pp* and *Fp* according to the Thomas Aquinas theorem.

Here a remark is imperative. If a census of the symbols is taken as the criterion for a census of the modalities, then each post on the Apulean column will require, in temporal logic, two modalities, such as *Pp* and *Fp*. The capital A of temporal logic has been designed by taking a definition of necessity given by Diodorus Chronos and remedying its hemiplegia.

Thanks to Boethius we know the four definitions of the four modalities given by Diodorus Chronos. Each one fits with a position of the Apulean square AEIO. But before Prior’s work, faced with these Diodorean definitions, the historians of logic were *comme une poule qui a trouvé un couteau*. Among these definitions, in fact, we find for example this definition of necessity:

Necessary is what, being true, will not be false.

If we put this Diodorean definition of necessity at the top A of our AI column, what we obtain as capital of the column is only a half-Janus which, camped in the present, can only see the future. And all Diodorean modalities are affected by this semi-blindness. Yet, from Prior’s point of view, this means that Diodorus has provided only one half of the couple required in position A. Thus, the Diodorean capital must be completed with the Aquinate pedestal in order to obtain on its top a well-balanced Janus-logic. The required symbols are *Hp* and *Gp*, with the mnemonics “it always *Has* been the case” and “it is always *Going* to be the case”; in French: *Auparavant* et *Dorénavant*.

When all this is done, we get the complete Column of Temporal Logic, with its Aquinate pedestal and its Diodorean capital:

	Hp	p	Gp
Auparavant	p	p	Dorénavant

$$\begin{array}{c} | \\ | \\ | \\ Pp \quad p \quad Fp \end{array}$$

One of the laws of this logic is $Pp \rightarrow Gp$ (“Whatever has happened will always have happened”).

1.3.2 The Mathematization of Temporal Logic

On 27th August 1954, in his Presidential Address at the second New Zealand Congress of Philosophy (held in Wellington) Prior unveiled a calculus which, in its published version of 1958, contains the following formula:

$$\vdash_t Gp \Leftrightarrow \forall u (t < u \rightarrow \vdash_u p)$$

(“It is true at t that dorénavant p iff for all u , if t is followed by u , then it is true at u that p ”)

Since in this formula the truth of p at u is conditioned by the relation of succession ($<$) between t and u , thanks to it Prior accomplished the *mathematization* of both temporal and modal logic. In other words, building a *Prior’s bridge* for temporal logic as a case of the Leibnizian bridge built for modal logic, Prior adds on the right side of the latter the *footbridge* of the “before-after relation” in its function of parametrization.

As an illustration of this function, let’s take the formula $Fp \rightarrow FFp$. Is it a law of temporal logic? Yes, if, generally, $t < u \rightarrow \exists v (t < v \ \& \ v < u)$.

Now this condition of *density* can be presented either as an attribute of the parametrizing *relation* ($<$), or as an attribute of the *set* on which this relation plays (here it is a set of temporal moments). This shows that, in a Kripke model $\langle \langle E, R \rangle, V \rangle$, the frame $\langle E, R \rangle$ which is denoting the ontology of the model, enjoys a strong unity because the mathematical attribute (such as density), that is the reason of its inclusion in the alleged model, can be attributed either to E or to R .

1.3.3 The Emergence of Tense Logic

Among Prior’s bridges, let’s now consider the following one:

$$\vdash_t Pp \Leftrightarrow \exists u (u < t \ \& \ \vdash_u p)$$

("It is true at t that p is past iff there is a u which precedes t
and it is true at u that p ")

Here we must remember that, according to McTaggart (1908), any event E can be grasped in two series: it can be said to be *future*, *present* and *past*—the three attributes of the A-series—and it can be temporally related with other events with respect to which it can be said to come *before*, be *contemporary* with or come *after*—the three relations of the B-series. Now, as we can see if we consider, for example, the succession of births of Homer, Milton and Borges, there is no change in the B-series but only in the A-series. Only in the latter, in fact, is a future event present and then becomes past. Then, McTaggart's distinction decides the fate of the metaphysics of time: if the B-relations prevail, we are in the timeless world of Parmenides, Spinoza and Bradley; if the A attributes prevail, we live in the moving world of Heraclitus, Leibniz, Bergson, Alexander, Whitehead or Prior.

Prior thematized this metaphysical watershed in his temporal logic. In the last equivalence, in fact, we find that, on the left side, the A attribute P of McTaggart has become a *modal* one, and that, on the right side, the B relation $<$ of McTaggart has become a *parametrizing* relation of mathematical modal logic. So, we are no more in the situation of the Kripkean models $\langle\langle E, R \rangle, V \rangle$ where any binary relation R was doing the job of the parametrizing one. In the temporal logic of Prior, in other words, not only must " $<$ " have its ordering meaning, but this relational meaning has the same *philosophical* and *metaphysical* relevance given to it by McTaggart.

Arthur Prior is the logician who has introduced the metaphysics of time into temporal logic. But this contamination is only the first step of his revolution. In fact, from Leibniz to Carnap (and his followers), Leibniz's bridge has been conceived as a path connecting left-shore modalities with right-shore quantifications. Yet, if we follow Prior, Leibniz's bridge becomes a *palindrome* that can be crossed in both directions, with different results. If we go from the A attributes on the left shore to the B relations on the right shore, we are in temporal logic. But if we go from the B relations to the A attributes as modalities, we enter into (pure) *tense logic*.⁶

⁶ See Dumoncel (2018).

2 The Pythagorean Table of Physics

Physics has its Pythagorean table⁷ (Table T):

	Space-time
c	Space, Time and Matter
h	Mechanical Action :
	$mv \times d \quad mv^2 \times t$

This table represents space-time in its two main physical forms. It is a double-entry table where the abscissa opens to the general notion of space-time and the ordinate superposes both a c and an h line. Of course, the letters c and h refer to Relativity and Quantum Mechanics: c is Einstein's constant and h is the Planck constant. In short, this Pythagorean table shows that the 20th century's revolutionary physics is fully *spatio-temporal* or, more exactly, *doubly* spatio-temporal: there is a *relativistic* spatio-temporality and a *quantum* spatio-temporality. The first is notorious; the second needs to be made explicit. What is quantized in Quantum Mechanics is, in fact, the mechanical *action* and the mechanical action can be expressed in both a *spatial* and a *temporal* form. Thus, a quantized action is a link between time and space: a fact which is quite ironical with respect to the history of the development of physics.

Leibniz, as it is well-known, has made of continuity one of the most important principles of nature (*Natura non facit saltus*) and something like a quantized action, precisely to the extent that it is a discrete action, seems to refute this principle. Yet, Leibniz is also the philosopher-physicist who saw mechanical action (the so-called "moving-action") as the heart of physics. This fact is duly registered by Martial Gueroult in his *Leibniz. Dynamique et Métaphysique* (1934). Here, Gueroult has also argued that Leibniz was aware of the *spatio-temporal* character of mechanical action. What he called "action mortice", in fact, is "the product of the amount of movement multiplied by the space travelled or of the living force multiplied by time" (Gueroult, 1967, 50, our translation). So that, for the momentum and the energy, the best *Begriffsschrift* is the Leibnizian couple $\langle mv, mv^2 \rangle$.

Now, let us consider Heisenberg's relations⁸ of indeterminacy transliterated in this Leibnizian *Begriffsschrift*:

⁷ I have presented a first version of this thesis when I was invited by Elie During to give a lecture in his seminar series on physics and philosophy at Diderot University.

⁸ Where d shortens "distance" as a coordinate of space.

$$(H1) \Delta mv \times \Delta d \geq h$$

$$(H2) \Delta mv^2 \times \Delta t \geq h$$

These relations can be viewed as a simple corollary of our Pythagorean table: H1 stating the *spatial* Heisenberg relation, H2 its *temporal* mate. But this fact can also work as the occasion for a reflection: which account does Heisenberg give for the two members in each relation? What is *common* to d and t on the one hand and to mv and mv^2 on the other? In brief, what is the *ratio essendi* of the spatial *or* temporal factor on the one side, *and* of the energy *or* momentum factor on the other?

With regard to this issue, Leibniz provides, if not the concepts, at least the vocabulary. As Gueroult explains: “If extension is considered in time, intensity is force; if it is considered in space, intensity is speed”⁹ (Gueroult, 1967, 130, our translation). Here we have *the Leibnizian watershed of Nature*: in Nature, space and time are the *extensive* components; momentum and energy are the *intensive* ones.

This Leibnizian vocabulary will receive its full conceptual expression when it will be housed in Lautman’s philosophy of science (Dumoncel 2008) and in its systematic and speculative network. Lautman, in fact, is a direct heir of Plato and it is well known that Plato, trying to give a blueprint of his allegory of the Cave, traced an analogical Line between the visible and the intelligible worlds which undergoes a second analogical division. Thus, the scale of beings *or* realms we obtained has the following four steps:

By means of Lautmanian transposition, we obtain the *Lautman* scale:

Ideas

⁹ Here, in mv , since the mass m is constant, the velocity v suffices to represent mv as a variable factor.

From a philosophical point of view, we find that, on the Lautman Line, the mathematical and the physical floors are already given and defined by their well-known scientific status. What is problematic are the floor of the “Ideas” and the floor of Natural philosophy. The former overcomes what concerns our present¹⁰ but the latter, according to Lautman, plays a crucial role in our theory of time. In the Lautman scale, Natural philosophy works in fact as a *lock* between Mathematics and its application in Physics. And it is here that the Leibnizian watershed of Nature plays a key-role: we leave mv and mv^2 , as well as *energy* and *momentum*, to the physicist, keeping for the philosopher only the Leibnizian couple of *extensive* and *intensive*. This is the *Lautmanian sampling* of Natural Philosophy.

3 The *Zeitsein* in its Diagram

From Plato to Russell, passing through Leibniz, metaphysics has been crudely dualist. In the exsanguinous language of Russell, metaphysics boils down to the ϕx and the a to be substituted to the x in ϕx . So, in the vocabulary of Nicolăi Hartmann, Being is split in the *Sosein* of the “universals” ϕx and the *Dasein* of the “particulars” a . Yet, symbolic logic has been constructed by mathematicians, and in mathematics the typical a is a number, again in the *Sosein*. Conversely, the *Sosein* is supposed to include the caballeity.

Darwin has changed all this. To the extent that caballeity sits in the *Sosein*, the ancestors of the horse and the horses must previously pop up in the *Dasein*. According to Whitehead’s vision, only geometrical forms and colours are “eternal objects”. Then, the *Zeitsein* may be defined as the union of the *Dasein* and of the quiddities which, in the *Sosein*, are derived from the evolution at work

¹⁰ Deleuze has understood this point well. Barot has given a good account of it in his book on Lautman (Barot 2009), discussed in the symposium on Lautman (Marquis 2010). For more on the topic we must refer to our *Cavaillès et Lautman*, again in our scriptorium.

in the *Dasein*. The *Zeitsein* requires its *Übersichtlich Darstellung* in the form of Figure 1, p. 341.

This representation is a doubled cone with its upper sheet and its lower sheet, separated by a plane. The lower sheet represents *time*; the upper sheet with the plane represents *temporality*. At the top of temporality, on the empyrean of metaphysics, we find the *Prototime*: in order to represent it we must imagine a compass rotating in the drawing of a circle.

The center of the circle, which is marked by the puncture of the compass, represents the *protopresent*. The radius of the circle, measured by the opening of the compass, represents the *protopast*. The compass' rotation represents the *protofuture*. The three parts of proto-time are divided into two subsets: the *protopresent* and the *protopast* are united in the *happened* (*advenu*), while the future is *à venir*.

In this construction, we find a first contingency: the circle can be traced either *dextrogyre*, in the sense of the hands on a watch, or *levogyre*, in the sense of the trigonometric circle. This is only a binary contingency: the two possibilities correspond to the two tickings of the *prototime-clock*: if the prototime turns *dextrogyre*, the ticking is *tick-tack*; if it turns *levogyre*, the ticking is *tack-tick*.

Now, the prototime is taken in a "bushing" operation *b*. We can have an intuitive idea of its effect only by analogy. We can imagine prototime by comparing it with an elementary particle that has to go through a Young split: before it crosses it, the path of the particle is straight; when it passes through the Young split, its localisation is determined.

Yet, we shall suppose that, because of this localisation, the velocity-vector of the particle (with its three dimensions, and so with its direction too) is underdetermined. Each state of the particle, the one before and the one after the Young split, corresponds to an effect of the bushing *b*. The first one is a *linearisation* of what has happened: the *protopast* is mapped onto an actualised part of the *protofuture*, so that the *protopresent* gets its chronological standard position: being *after* the *protopast*. The second effect is a *ramification* of the future in a fan of "future contingents".

The result of these two bushing operations on prototime is what I propose to call *Protoduration*:

$$b(\text{prototime}) = \text{protoduration}$$

From this hypostasis forth, one can conceive the genesis of the *Zeitsein* exploiting the affinities between the works of three philosophers: Elisabeth Anscombe,

Fig. 1: Zeitsein

Peter Geach and Arthur Prior,¹¹ remembering that physics also has its demons: Laplace's demon, with his fatal determinism and Maxwell's demon, with his charge of neguentropy. But, since physics gives rights to its demons, we can give the same rights for metaphysics' demons.

With respect to protoduration, the tutelar demon is¹² *Miss Anscombe's one*. Her 1964's paper *Before and After* provides her contribution to tense logic. The title of the paper allows us to define *Miss Anscombe's demon*. Its essence is dipolar, that is made of two different possibilities: being an *angel* and then a *devil*; being a *devil* and then an *angel*. In symbols, *A* stands for the Anscombe tense operator "and after", so that the two main possibilities for her demon are:

angel *A* devil
("he is an angel, and then he is a devil")

devil *A* angel
("he is a devil, and then he is an angel")

Of course, these two possibilities are only two branches of a binary contingency amid the full fan of the contingents-futures opened by the protoduration. Thus, coming from *Temporality*, that is from *Prototime* and *Protoduration*, to *Time*, one has to distinguish between what is written in capitals and what is written in lower case.

In capital letters, we find the Lautman's non-commutativity (Lautman 2006, 269). And so, when Miss Anscombe's demon applies his metaphysical function to natural philosophy, the Leibnizian couple of the *extensive* and *intensive* becomes useful and relevant. The binary contingency will in fact take precisely the form of the two above mentioned natural possibilities:

intensive *A* extensive
or
extensive *A* intensive

11 For the relationship between Anscombe and Prior, see Gardies 1975, 120. It is in a letter to Prior that P.T. Geach defined the Transworlds Airlines *TWA* of his relation of *accessibility* between possible worlds in the same way as Prior did in 1962.

12 Anscombe was already married to Geach when my friend Georges Kalinowski met her at the Manchester *conclave* (*closed colloquium*) where Arthur Prior gathered together the few pioneers of deontic logic; but, as he told me later, he learned on this occasion that, in academic circles, G.E.M. Anscombe must only be named "Miss Anscombe". And it is only under this name that she has a demon.

Lautman asks to himself: is it possible to describe, in the womb of mathematics, a structure which would be like a first drawing of the temporal form able to qualify the sensible phenomena? (Lautman 2006, 277). Such a structure may be named *mathematical prototemporality*. Yet, our thesis is that there is also a *logical* prototemporality, and that *logical* prototemporality is the core of tense logic.

In lower case we register the fact that Elisabeth Anscombe married Peter Geach. And P.T. Geach tells us that: “Smith committed seven burglaries, then a murder and then he was hanged” (Geach 1957, 71). But in the *Zeitsein* Geach’s Smith becomes a concrete hypostasis of the two tickings in the prototime-clock. In miss Anscombe’s *Begriffsschrift*:

school A prison
 (“he was at school, and then in prison”)
 prison A school
 (“he was in prison, and then at school”)

Now, our *Zeitsein* is displayed with all its hypostasises. Therefore, we can insert on its scale, each one in its proper place, the main laws of tense logic, also underlining the paradigmatic value that Prior attributed to them.¹³ At the top, that is on the level of Prototime, we find the definition of the modality *Always* given by Prior. It is *not* defined by an instants-quantification (“Always *p*” = for all instants *t* it is true at *t* that *p*) but as a conjunction of temporal *modalities*:

Always *p* = $\text{Df } p \wedge Hp \wedge Gp$
 (“Always *p* means by definition that *p* and auparavant *p* and dorénavant *p*”)

At the level of Protoduration Prior gives the following definition:

$t < t' = \text{Df } @_t Ft'$
 “*t* is before *t'*” means by definition that at *t* it is future that *t'*”

Here, we see that, on the Prior palindrome, the Carnapian order between *explanandum* and *explanans* is inverted: now, the *explanandum* is “before”, i.e., in McTaggart’s terms, a B relation of the B-series; while the *explanans* is *future*, i.e., an A attribute of the A series. The latter has become a tensed modality.

At the level of Time, we find *Findlay’s law*: the emblematic law of temporal logic which is also the first from an historical point of view. It was enunciated by

¹³ For a detailed exposition and explanation of this point, see Dumoncel (2018).

J.N. Findlay in a footnote of his paper on time (Findlay 1941). And, when Prior read it, he found the *impetus* to investigate both temporal and tense logic. Here is Prior's formulation of Findlay's law:

$$p \vee Pp \vee Fp \rightarrow FPp$$

("If p or it is past that p or it is future that p , then it is future that it is past that p ")

Comparative Conclusion

Faced with the question "What is time?", the *Zeitsein* thesis needs to be specified in the light of both Elie During's and Etienne Klein's papers. Elie During has enunciated a new *Tertium non datur*: "There is no third time" between natural time and psychological time. But, in spite of the title of the present paper, the *Zeitsein* thesis is compatible with During's *tertium non datur*.

Yet, the thesis of During seems to entail a plain answer to Etienne Klein's question: "Who is entitled to talk about time?" If there is no third time in addition to physical and psychological time, then the answer to the question is that nobody is entitled to talk about time, except for physicists such as Julian Barbour and psychologists such as Marc Wittmann.

However, we may sustain a more balanced conception. This is because, in During's terms, there is a "natural philosophy" stemming from contemporary physics". "Natural philosophy" is a sort of constant in the development of physics from Newton to Louis De Broglie and Albert Lautman. In the Lautman scale, as we have seen, natural philosophy finds its place, as a halfway house between mathematics and physics. Moreover, when Lautman took into account von Neumann's algebra, he recognized that the mathematical stratum of his Platonic scale is a *logico-mathematical* edifice. From Klein's viewpoint, the logical layer included in the logico-mathematical edifice defines exactly the place of *tense* logic.

Within temporal logic, tense logic runs the risk of being reduced to the logic of our phenomenal time. But this is a misunderstanding. The first mathematization of modal logic coincides, in the work of Arthur Prior, with the birth of temporal logic as a paradigm for modal logic and, in this mathematization, the frame represents the *ontology* of the modal model. Besides, since temporal logic is a kind of modal logic, the contribution of logic in answering the question "What is time" primarily concerns the part of temporal logic where the modal involvement (in Quine's sense) reaches its maximum point: *tense* logic.

Thus, if time is indisputably a feature of nature, the word *becoming*, as C.D. Broad remarked, reminds us that Time is primarily a feature of *Being*. And this is what the concept of *Zeitsein* shows.

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