Fuzzy Waste-Load Allocation Model: Application to a Case Study

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ABSTRACT

A fuzzy optimization model, developed earlier to address uncertainty due to imprecision in management goals in river water quality management problems, is applied to a case study in this paper by addressing uncertainty due to randomness implicitly through Monte Carlo simulations. Municipal and industrial effluents, characterized by their Biochemical Oxygen Demand (BOD) loads are considered as point sources of pollution and the Dissolved Oxygen (DO) level at specified checkpoints in the river stretch is taken as the single water quality indicator. Conflicting goals related to water quality and fraction removal levels are expressed as fuzzy sets with appropriate membership functions in the model. The fuzzy optimization model is solved for a number of simulated sets of inputs to obtain optimal solutions, thus generating a large set of optimal objective function values and associated decisions. This information is used to obtain waste-load allocation decisions for a specified level of reliability.

KEYWORDS

water quality management, waste-load allocation, fuzzy optimization, Monte-Carlo simulations, uncertainty modeling, decision making.

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1. INTRODUCTION

Waste-load allocation (WLA) models play an important role in planning for water quality control in a river system. Such models incorporate pollutant transport models within the framework of multiobjective optimization models. In general, solutions of such waste-load allocation models yield decisions on optimal treatment levels for various dischargers, keeping the water quality criteria in view. Recent developments on waste-load allocation models are found in Takyi and Lence (1999), Fujiwara et al. (1988), Sasikumar and Mujumdar (1998), Sasikumar and Mujumdar (2000), and Mujumdar and Sasikumar (2002). Waste-load allocation problems are characterized by uncertainties due to both randomness and imprecision. Uncertainty due to randomness arises mainly from the random nature of the input variables of water quality-simulation model, and that is due to imprecision or fuzziness is associated with setting up the water quality standards and goals of Pollution Control Agencies (PCA) and dischargers. Considering either only randomness or only fuzziness in water quality-management models may not accurately address the underlying risk. From a practical point of view, some variables should be modeled more appropriately as fuzzy variables instead of treating them as deterministic variables or random variables.

In this paper, the fuzzy waste-load allocation model (FWLAM) developed by Sasikumar and Mujumdar (1998) is applied to a real case study by addressing uncertainties due to randomness of water quality influencing variables like river flow, effluent flow, temperature, reaction rate, etc. and fuzziness in management goals for obtaining fraction removal levels. Figure 1 shows the overall organization of the model. The objectives of the pollution control agency and dischargers remain the same as those discussed in Sasikumar and Mujumdar, and are discussed in this paper. The optimization model is solved for a large number of input variables generated through simple data generation techniques, to arrive at optimal decisions for a given level of reliability of water quality. A main objective of the work presented in this paper is to demonstrate the applicability of the FWLAM to a real case study by considering the random variations in stream-flow, effluent flow, and temperature that most influence water quality at a location. The following two sections summarize the FWLAM of Sasikumar and Mujumdar (1998).

Fig. 1: Organization of the Fuzzy Waste Load Allocation Model

2. DESCRIPTION OF THE RIVER SYSTEM

A general river system is considered for developing the FWLAM. In addition to the uncontrollable sources of pollution, the river system includes a set of dischargers who are allowed to release the pollutants into the river after removing some fraction of the pollutants. A common practice of the pollution control agency to ensure acceptable water quality condition is to check the water quality at a finite number of checkpoints in the river. The water quality at a checkpoint is described by means of some indicators called water quality indicators. The dissolved oxygen (DO) is an example of a water quality indicator. The concentration of a water quality indicator at a checkpoint is affected by the controllable as well as the uncontrollable sources of pollutants in the river system, and it is obtained using a water quality transport model that determines the spatial distribution of the pollutants in the river system. In a water quality management model, the concentration of the water quality indicator may be expressed as a function of the fraction removal levels, which constitute the decision variables of the optimization problem.

The relevant components of the system are identified as sets. Set Q_L represents the collection of water quality checkpoints in the river system. Set D_M is the collection of dischargers (e.g., industries, municipal dischargers etc.). Set P_N is the collection of the pollutants in the system (e.g., point sources of the Biochemical Oxygen Demand (BOD), a mixture of toxic pollutants, etc.). Set T_P is the collection of uncontrollable sources of pollutants in the system (e.g., BOD addition due to runoff and scour in a stream). Set V_I is the collection of water quality indicators (e.g., dissolved oxygen and toxic pollutant concentration). A water quality indicator is indexed as i (e.g., $i=1$ may correspond to DO, $i=2$ to phosphorus content etc.). A pollutant is assumed to affect one or more water quality indicators in the set V_I . The pollution control agency specifies a desirable level, $c^D_{i,l}$, and minimum permissible level, $c^L_{i,l}$ of the water quality indicator i at the checkpoint l . The following section focuses on FWLAM.

3. FUZZY WASTE-LOAD ALLOCATION MODEL

The fuzzy decision Z for the water quality management problem is defined using the concept of fuzzy decision proposed by Bellman and Zadeh

(1970) and the fuzzy multiobjective optimization technique developed subsequently (Zimmermann, 1978). For a general fuzzy optimization problem, the decision space is defined by the intersection of the fuzzy goals and constraints. In the context of a waste-load allocation problem, the goals of the pollution control agency (PCA) and the dischargers are viewed as imprecisely stated conflicting goals, and are treated as fuzzy goals. The goal of the PCA (E_{il}) is to make the concentration level, c_{il} , of the water quality indicator i at the checkpoint l as close as possible to the desirable level, c_{il}^D , so that the water quality at the checkpoint l is enhanced with respect to the water quality indicator i , for all i and l . The goal of the discharger (F_{imn}) is to Make the fraction removal level x_{imn} as close as possible to the aspiration level x_{imn}^L for all water quality indicators i , effluent dischargers m , and pollutants n , so that the effluent treatment cost is minimum. These goals can be viewed as imprecisely defined conflicting goals and can be represented as fuzzy sets. The arguments of the membership functions of the fuzzy sets E_{il} and F_{imn} are c_{il} and x_{imn} respectively. The fuzzy set corresponding to the decision Z is then given as:

$$Z = \left(\bigcap_{i,l} E_{il} \right) \cap \left(\bigcap_{i,m,n} F_{imn} \right) \quad (1)$$

From the definition of intersection of fuzzy sets, the membership function corresponding to the decision Z is given by:

$$\mu_Z(X) = \underset{i, l, m, n}{\text{Minimum}} \left[\mu_{E_{il}}(c_{il}) \mu_{F_{imn}}(x_{imn}) \right] \quad (2)$$

where X is the space of alternatives, $\mu_Z(X)$ is the membership function of the fuzzy set Z , $\mu_{E_{il}}(c_{il})$ is the membership function of the fuzzy set E_{il} and $\mu_{F_{imn}}$ is the membership function of the fuzzy set F_{imn} . The corresponding optimal decision, X^* , is given by:

$$\mu_Z(X^*) = \lambda^* = \max_{X \in Z'} [\mu_Z(X)] \quad (3)$$

Fig. 2: Membership Function for Goal, E_{il}

The space of alternatives X is restricted by precisely defined constraints known as crisp constraints, such as physical limits on the concentration levels, technologically feasible range of fraction removal levels etc. For the goal of the PCA E_{il} , the desirable level c_{il}^D , for the water quality index i at checkpoint 1 is assigned a membership value of 1. The minimum permissible level, c_{il}^L , is assigned a membership value zero. The general form of membership function for fuzzy goal E_{il} is then expressed (Figure 2) as:

$$\mu_{E_{il}}(c_{il}) = \begin{cases} 0 & c_{il} < c_{il}^L \\ \left[\frac{c_{il} - c_{il}^L}{c_{il}^D - c_{il}^L} \right]^{\alpha_{il}} & c_{il}^L \leq c_{il} \leq c_{il}^D \\ 1 & c_{il} \geq c_{il}^D \end{cases} \quad (4)$$

The membership function may be interpreted as the variation of satisfaction level of the pollution control agency with respect to the water quality indicator concentration, c_{il} . The satisfaction of the pollution control agency increases as the concentration level, c_{il} , of the water quality indicator i at the checkpoint 1 approaches to the desirable limit, c_{il}^D , starting with the minimum permissible level, c_{il}^L . The exponent α_{il} defines the shape of the membership function and is decided by the decision maker.

Fig. 3: Membership Function for Goal, F_{imn}

The fraction removal level, x_{imn}^L , corresponding to the aspiration level of the discharger m with regard to x_{imn} is assigned a membership value of 1. The maximum acceptable level, x_{imn}^M , is assigned a membership value of zero. The membership function for the fuzzy goal F_{imn} (Figure 3) is expressed as

$$\mu_{F_{imn}}(x_{imn}) := \begin{cases} 1 & x_{imn} \leq x_{imn}^L \\ \left[\frac{x_{imn}^M - x_{imn}}{x_{imn}^M - x_{imn}^L} \right]^{\beta_{imn}} & x_{imn}^L \leq x_{imn} \leq x_{imn}^M \\ 0 & x_{imn} \geq x_{imn}^M \end{cases} \quad (5)$$

This membership function may be interpreted as the variation of satisfaction level of the discharger m in treating the pollutant n to control the water quality indicator i in the river system. The exponent β_{imn} defines the shape of the membership function, and is decided by the discharger m . Using these membership functions for the fuzzy goals E_{il} and F_{imn} , the fuzzy waste-load allocation model is expressed as follows:

$$\text{Maximize } \lambda \quad (6)$$

subject to

$$\left[(c_{il} - c_{il}^L) / (c_{il}^L - c_{il}^D) \right]^{p_{il}} \geq \lambda \quad \forall i, l \quad (7)$$

$$\left[(x_{imn}^M - x_{imn}) / (x_{imn}^M - x_{imn}^L) \right]^{\beta_{imn}} \geq \lambda \quad \forall i, m, n \quad (8)$$

$$c_{il}^L \leq c_{il} \leq c_{il}^D \quad \forall i, l \quad (9)$$

$$\max [x_{imn}^L, x_{imn}^{MIN}] \leq x_{imn} \leq x_{imn}^{MAX} \quad \forall i, m, n \quad (10)$$

$$0 \leq \lambda \leq 1 \quad (11)$$

The model is a multiobjective formulation maximizing minimum satisfaction level (λ). In the fuzzy constraints (7) and (8), the goals of PCA and dischargers respectively are made greater than or equal to λ , to formulate this MAX-MIN model. The lower and upper bounds of water quality indicator i at the checkpoint l are fixed as permissible (c_{il}^L) and desirable level (c_{il}^D), respectively as set by PCA in constraint (9). The bounds of fractional removal level x_{imn} of the pollutant n from the discharger m to control the water quality indicator i in the river system, is given by constraint (10). The aspiration level and maximum fractional removal level acceptable to the discharger m with respect to x_{imn} are represented as, x_{imn}^L and x_{imn}^M , respectively. The PCA imposes minimum fractional removal levels that are also expressed as the lower bounds, x_{imn}^{MIN} in constraint (10). The concentration of water quality indicator c_{il} in constraints (7) and (9) is determined using a water quality simulation model. In the present study, DO is used as an water quality indicator. Randomness in the DO concentration is addressed implicitly in the model through Monte-Carlo simulations.

4. MONTE CARLO SIMULATIONS

The variables influencing the DO concentration at a check point, viz., streamflow, effluent flow, reaeration rate, deoxygenation rate, sedimentation coefficient, BOD addition by resuspension of sediments, oxygen reduction by benthic demand, oxygen production by photosynthesis, flow travel time, temperature, and BOD loading are all random variables. Due to randomness in the streamflow, there are random variations in the velocity of flow, travel

time, reaeration rate, and deoxygenation rate. The effluent quantity and emission concentration of the effluent into the river system tend to vary due to random fluctuations in plant operation. Due to variation in the temperature, the saturation dissolved oxygen concentration and reaction coefficients vary considerably. It is, therefore, difficult to determine the distribution of the concentration level, c_{il} , which is a complex function of the random variables. Uncertainty due to randomness is included implicitly in the model through Monte-Carlo simulation. To limit the number of random variables in the computations, only the three most important variables that influence water quality indicator, c_{il} , viz., streamflow, effluent flow, and temperature are treated as random. To account for randomness due to these variables, a large number of data is generated using a simple generation model of the type:

$$y_k = \mu_y + \sigma_y \varepsilon_k \quad (12)$$

where, y_k is the k^{th} generated value of the variable Y , μ_y is the observed mean, σ_y is the observed standard deviation of the variable Y , and ε_k is the noise term that accounts for randomness. When a Gaussian distribution is used for Y , ε_k is a zero mean Gaussian random variate.

For each set of the generated values of streamflow, effluent flow, and temperature, the other variables, such as travel time, reaeration rate, deoxygenation rate, and saturation DO levels are computed using functional relationships, available in literature. For the case study, the effect on the DO concentration due to BOD addition by resuspension of sediments, oxygen reduction by benthic demand, and oxygen production by photosynthesis is neglected, as it is relatively less significant compared to that due to other processes considered.

By solving the optimization model repeatedly with the generated values of the input variables, a large number of X^* and λ^* is obtained. Using this information, a distribution of the DO concentrations at all checkpoints is obtained. This set of optimal solutions forms the basis for prescribing fraction removal levels corresponding to a specified exceedence probability of DO at a checkpoint. For a specified level of exceedence probability, optimal satisfaction level, λ^* , the corresponding vector of decision variables, X^* , and resulting water quality levels in the river system are obtained from the set of optimal solutions.

5. CASE STUDY APPLICATION

An application of the model is illustrated through a case-study of Tunga-Bhadra River system shown schematically in Figure 4. The Tunga-Bhadra River is a perennial river formed by the confluence of Tunga and Bhadra rivers, both tributaries of the Krishna River, in southern India. The river has two other tributaries, the Kumadavati and Haridra rivers. The river network is discretized into 10 reaches depending on the river morphology and river environment. The river receives the waste loads from eight major effluent points. Details of the data and the uncertainty information of the basic variables are taken from the Central Water Commission (CWC), the Karnataka State Water Resources Development Organisation (KSWRDO), and the Karnataka State Pollution Control Board (KSPCB).

The optimization model is solved for 2000 sets of input variables, generated through Monte Carlo simulations, for this case study. There were 241 infeasible combinations out of the 2000 considered. The solution of the optimization problem gives the optimal fraction removal levels, optimum satisfaction level, and DO concentration at the checkpoints in the system. By using these solutions, it is possible to determine the distribution of the DO levels at each checkpoint and the minimum DO concentration in the river system. The results for the example set of simulated data shows (Table 1) that the fraction removal level for the dischargers 2, 3, and 7 are very high compared to other dischargers.

TABLE 1

Fractional removal levels for the example simulation

Discharger(m) (1)	Effluent Flow (m^3/s) (2)	Fractional Removal Level (3)
D_1	0.450	0.350
D_2	0.727	0.814
D_3	0.318	0.746
D_4	0.027	0.350
D_5	0.025	0.350
D_6	0.134	0.503
D_7	0.981	0.814
D_8	0.894	0.350

Fig. 4: Schematic Diagram of Tunga-Bhadra River System

TABLE 2

Optimal DO levels resulting for the example simulation

River Reach (1)	c_{ii}^L mg/L (2)	c_{ii}^D mg/L (3)	Minimum DO mg/L (4)
1	6.0	7.84	7.20
2	4.0	7.82	5.04
3	4.0	7.96	5.04
4	4.0	7.95	5.09
5	6.0	7.77	7.13
6	6.0	7.77	7.73
7	6.0	7.77	7.19
8	6.0	7.84	7.01
9	6.0	7.73	6.94
10	6.0	7.77	6.46

The minimum DO levels resulting for this example set of simulated data are given in Table 2. The DO concentration in the river reaches 2, 3, and 4 are close to minimum permissible levels showing critical water quality condition in these reaches. This is because of the low flow in the river Bhadra, along with high BOD concentration in the effluent of discharger 2. The maximized value of the minimum satisfaction level, in the system is 0.2627, for this simulation.

Variation of the DO concentration at a critical checkpoint 30 is analyzed. The exceedence probabilities of the DO levels are estimated from the simulation results. Figure 5 shows the variation of DO concentration at checkpoint 30. In the figure, the exceedence probability, $\alpha = 0.95$ which corresponds to a DO of 7.0 mg/L, implies that 95% of times the DO concentration at checkpoint 30 is greater than or equal to 7.0 mg/L. To achieve this DO concentration, the corresponding fraction removal levels are shown in Figure 6. Figure 6 indicates that, as the exceedence probability decreases, the satisfaction level of the system decreases as the corresponding DO level and the fractional removal levels increase. This feature may be used by the decision-makers in selecting the best choice of management decisions.

Fig. 5: Variation of DO levels at Checkpoint 30

Fig. 6: Fraction Removal Levels for DO control at Checkpoint 30

6. CONCLUSIONS

An application of the fuzzy waste-load allocation model developed earlier by Sasikumar and Mujumdar (1998) to a case study is presented in this paper. In this analysis, the input uncertainty is implicitly considered through Monte Carlo simulations. Linear fuzzy membership functions are considered for the goals of the dischargers and the pollution control agency. Results of the model application will be useful for the decision makers in choosing the best compromise solution of fraction removal levels, at specified levels of reliability.

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