LOCALIZATION AND PATTERN OF DEFORMATION
IN THERMO-VISCO-PLASTIC MATERIAL

Yilong Bai,* Chemin Cheng and Zhong Ling

*Laboratory for Non-Linear Mechanics of Continuous Media
Institute of Mechanics, Chinese Academy of Sciences
Beijing 100080, China*

ABSTRACT

Homogeneous shear deformation in thermo-visco-plastic material may become unstable due to thermal softening. Moreover, it was shown that the coupled dissipative work rate causes the deformation zone to contract. The balance of dissipative work rate and heat diffusion, which otherwise tends to extend the deformation zone, leads to quasi-static narrow shear band. Generally speaking, two kinds of diffusion are involved, namely heat and momentum diffusion. Within the same time scale, the length scale of heat diffusion is usually much less than that of momentum transport in metals. This difference in diffusion rates may govern the pattern of adiabatic shear band.

道生一，一生二，二生三，三生万物

—— < Dao > Chap. 43.

According to ancient literature and legends, Lao Zi, the great Chinese philosopher and author of " Dao ", lived between 400 B.C. and 300 B.C.. So, Aristotle (384 ~ 322 B.C.) and Lao Zi, two great thinkers of the time, thousands of miles apart between the west and the east, both put forward far-reaching thoughts on modern science.

The quotation from " Dao " has been interpreted for years in the following way. Dao unifies the universe. A system should consist of two opposite counterparts. Interaction between the two creates new thirds, which eventually produce complexity. The theme of nonlinearity and pattern making in materials coincides with and deepens the quoted statement surprisingly well. In this paper, we intend to clarify the concept with a concrete example. It is shown how a trivial homogeneous shear deformation in thermo-visco-plastic material gives rise to localization owing to the coupling of two opposite factors: dissipative work rate and heat diffusion. Furthermore, pattern of localized deformation, i.e. a network of shear bands, may be produced as a result of interplay between two kinds of diffusion: heat and momentum diffusion, which take place simultaneously in thermo-visco-plastic deformation.

Brief Review of Studies on Shear banding

One-dimensional simple shear in thermo-visco-plastic material is usually described in the following way^[1,2]

$$\rho \cdot \frac{\partial \dot{\gamma}}{\partial t} = \frac{\partial^2 \tau}{\partial y^2} \quad (1.1)$$

$$\rho \cdot c \cdot \frac{\partial \theta}{\partial t} = \beta \cdot \tau \cdot \dot{\gamma} - \lambda \cdot \frac{\partial^2 \theta}{\partial y^2} \quad (1.2)$$

$$\tau = \tau(\gamma, \dot{\gamma}, \theta) \quad (1.3)$$

$$\dot{\gamma} = \frac{\partial v}{\partial y} \quad (1.4)$$

$$y = -\frac{b}{2}, \quad \frac{\partial \theta}{\partial y} = 0, \quad v = -\frac{V}{2} \quad (1.5)$$

$$y = \frac{b}{2}, \quad \frac{\partial \theta}{\partial y} = 0, \quad v = \frac{V}{2} \quad (1.6)$$

If wave motion is ignored, a uniform initial condition is usually prescribed.

Equations (1.1) and (1.2) are laws of momentum and energy conservation respectively, β indicates the portion of plastic work rate converted into heat and ranges between 0.85-0.95^[3]. For simplicity, β will be taken as 1 and omitted later in the paper. Formula (1.3) is the general form of thermo-visco-plastic constitutive relation, whereas expressions (1.5-1.6) describe adiabatic and constant velocity boundary conditions. Although these boundary conditions by no means represent the real situation, they do represent a typical case with the least number of parameters. Moreover, unlike the isothermal boundary, it includes no externally imposed inhomogeneity and can help reveal the intrinsic motivation for the pattern of shear deformation.

The problem (1.1-1.6) has a trivial homogeneous but unsteady solution

$$\left\{ \begin{array}{l} \dot{\gamma}_h(t) = -\frac{V}{b} \\ \gamma_h(t) = \dot{\gamma}_h \cdot t \\ \theta_h(t) = \frac{1}{\rho \cdot c} \int_0^t \tau_h(t) \cdot \dot{\gamma}_h dt + \theta_h(0) \\ \tau_h(t) = \tau(\dot{\gamma}_h t, \dot{\gamma}_h, \theta_h(t)) \end{array} \right. \quad (1.7)$$

For example, in linear thermo-viscous material

$$\tau = \tau_c \left(1 + \frac{\dot{\gamma}}{\dot{\gamma}_c} - \frac{\theta}{\theta_c} \right) \quad (1.8)$$

the unsteady homogeneous solution is

$$\begin{cases} \dot{\gamma}_h = -\frac{V}{b} \\ \tau_h = \tau_c \cdot \exp\left(-\frac{\tau_c}{\rho \cdot c \cdot \theta_c} \cdot \dot{\gamma}_h \cdot t\right) \end{cases} \quad (1.9)$$

It has been observed that the uniform shearing can not hold for ever and may lead to strain localization. Linear perturbation^[2,3] and relative linear perturbation analysis^[4] have given the conditions of instability and localization of the shearing.

More interesting is the formation of shear band. Assuming a linear thermo-viscous constitutive relation (1.8) and zero deformation rate outside the shear band, $\dot{\gamma} = 0$, Bai et al^[5] have found an essential mechanism governing shear banding. Suppose $y = 0$ is the center of shear band, then

$$\dot{\gamma} \begin{cases} = 0 & y \geq \delta(t) \\ \neq 0 & y < \delta(t) \end{cases} \quad (1.10)$$

Under quasi-static assumption, τ remains uniform throughout the shear band and integration of the energy equation (1.2) gives

$$\begin{aligned} \text{const} &= \int_0^{\delta(t)} \dot{\gamma}(t, y) dy = \int_0^{\delta(t)} \dot{\gamma}_c \left(\frac{\tau}{\tau_c} + \frac{\theta}{\theta_c} - 1 \right) dy \\ &= \frac{\dot{\gamma}_c}{\theta_c} \int_0^{\delta} \theta(t, y) dy - \frac{\dot{\gamma}_c \cdot \theta(t, \delta(t))}{\theta_c} \cdot \delta \end{aligned}$$

Differentiation of the expression with respect to t leads to

$$\begin{aligned} 0 &= \theta(t, \delta) \cdot \dot{\delta} + \int_0^{\delta} \frac{\partial \theta}{\partial t} dy \\ &\quad - \left[\frac{\partial \theta(t, \delta)}{\partial t} \cdot \delta + \frac{\partial \theta(t, y)}{\partial y} \right]_{y=\delta} \cdot \dot{\delta} \cdot \delta + \theta(t, \delta) \cdot \dot{\delta} \end{aligned}$$

so that

$$\dot{\delta} = \frac{\tau \cdot \dot{\gamma}_a + \lambda \cdot \left[\frac{1}{\delta} \int_0^\delta \frac{\partial^2 \theta}{\partial y^2} dy - \frac{\partial^2 \theta}{\partial y^2} \Big|_\delta \right]}{\rho \cdot c \cdot \frac{\partial \theta}{\partial y} \Big|_\delta} \quad (1.11)$$

where $\dot{\gamma}_a = \int_0^\delta \dot{\gamma} dy \Big|_\delta$. Since $\frac{\partial \theta}{\partial y} \Big|_\delta \leq 0$, and

$$\int_0^\delta \frac{\partial^2 \theta}{\partial y^2} dy \Big|_\delta - \frac{\partial^2 \theta}{\partial y^2} \Big|_\delta < 0 \quad \text{for shear band, the implication}$$

of the formula for $\dot{\delta}$ becomes clear. As external source, plastic work tends to make the shear band narrower and heat diffusion tends to widen it. This constitutes a proper mechanism for the quasi-static dissipative localized structure.

In fact, quasi-stationary approximation of equations (1.1-1.2) for a general thermo-viscous material

$$\dot{\gamma} = G(\theta, \tau) \quad (1.12)$$

gives much simpler expression of shear band width^[6]

$$\delta \approx \int_{\theta_\delta}^{\theta_m} \frac{\sqrt{\lambda} d\zeta}{\sqrt{\int_{\zeta}^{\theta_m} \tau \cdot G(\eta, \tau) d\eta}} \sim \sqrt{\frac{\lambda(\theta_m - \theta_\delta)}{\tau_c \cdot \dot{\gamma}_c}} \quad (1.13)$$

The afore-mentioned mechanism of the internal structure of shear band is shown straightforwardly.

The whole process of shear banding is schematically shown in Fig.1. Various experimental justifications of the estimation of the shear band width have been reported^[7,8] for several metals, such as steel, titanium, etc. Therefore, the mechanism governing shear band has been qualitatively clarified.

However, if one intended to extrapolate the essential assumptions mentioned above to pattern of shear band, he would encounter difficulties. For example, if $\dot{\gamma} = 0$ outside shear band, then how could mere heat conduction form regular spacing of shear band patterns?

Fig. 1 Scheme of shear band formation.

Scaling of Shear Bands and Their Spacing

In some cases, a narrow shear band is not the unique feature of shear localization. It has been reported that a pattern or network of shear bands with regular spacing can occur even under simple and uniform loading. For example, a network of shear bands, with width of less than $1\mu\text{m}$ and spacing $10\mu\text{m}$ in aluminum^[9], similar network of shear bands in steel^[10] and in polymer^[11]. All were observed in samples subjected to uniaxial impact. Though it seemed to be clear in the last section that plastic work rate and heat diffusion can govern the internal structure of shear band, what kind of mechanism leads to the regular spacing of shear bands remains open. Anyhow, this unknown mechanism should be in force simultaneously with those affecting shear band.

A major difficulty involved in the analysis results from the large difference in variables within and outside shear band. The first impression of the difference is the very thin width of shear band and comparatively wide spacing of the bands. In order to put our analysis on a solid basis, establishment of well-posed model equations governing the pattern is of primary importance.

The general one-dimensional differential equations of momentum and energy conservations (1.1-1.2) should still be valid. However, the key point is to sort out the most significant factors controlling spatial structures.

The following table lists characteristic parameters appearing in the model and corresponding dimensions and range of values in metals.

Carefully examination of these parameters may provide a clue to the solution.

		Dimension				Range of value
		M	L	T	θ	
Material parameter	ρ Density	1	-3	0	0	$10^3 \text{ kg} \cdot \text{m}^{-3}$
	c Specific heat	0	2	-2	-1	$10^3 \text{ J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$
	λ Heat conductivity	1	1	-3	-1	$10^1 \text{ W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$
	$\dot{\gamma}_c$ Strain rate	0	0	-1	0	$10^{5 \sim 7} \text{ s}^{-1}$
	θ_c Temperature	0	0	0	1	10^2 K
	τ_c Stress	1	-1	-2	0	10^8 Pa
Boundary condition	v Velocity	0	1	-1	0	$10^{1 \sim 3} \text{ m} \cdot \text{s}^{-1}$
	b Width of specimen	0	1	0	0	$10^{-3 \sim -2} \text{ m}$

Subscript c denotes the characteristic parameters appearing in constitutive relation describing mechanical behavior of material within the shear band. Here adiabatic boundary condition is assumed. Clearly this assumption could not affect the main feature of the problem.

According to the π theorem of dimensional analysis, the six material parameters can constitute only two independent dimensionless parameters, which can be chosen as

$$\text{Prandtle number } P_c = \frac{c \cdot \tau_c}{\lambda \cdot \dot{\gamma}_c} \sim \frac{10^{10}}{\dot{\gamma}_c} \gg 1$$

$$B = \frac{\tau_c}{\rho \cdot c \cdot \theta_c} \sim 10^0 \sim 1$$

As a matter of fact, the Prandtle number P_c can be rewritten as

$$P_c = \frac{c \cdot \tau_c}{\lambda \cdot \dot{\gamma}_c} = \frac{\tau_c / \rho \cdot \dot{\gamma}_c}{\lambda / \rho \cdot c} = \frac{\nu}{K} \quad (2.1)$$

P_c signifies the relative importance of viscosity and heat diffusion,

whilst, parameter B manifests the balance of impact plastic work and temperature change within shear band.

Now, let us see how to scale the governing equations and explore mechanisms underlying band formation and their spacing in a unified approach.

Suppose the variations of variables within a mature shear band be in the order of characteristic parameters in constitutive equation, then

$$\Delta y \sim \delta, \quad \Delta \theta \sim \theta_c, \quad \frac{\Delta \dot{\gamma}}{\dot{\gamma}_c} \sim \frac{\Delta \tau}{\tau_c}, \quad \Delta \tau \sim \tau_c$$

Where δ is the half-width of the shear band, which is undetermined for the time being. Therefore, dimensionless equations are expressed in the form⁽¹²⁾

$$\left\{ \begin{array}{l} \frac{\rho \cdot \dot{\gamma}_c \cdot \delta^2}{\tau_c \cdot t_k} \cdot \frac{\partial \tilde{\gamma}}{\partial \tilde{t}} = \frac{\partial^2 \tilde{\tau}}{\partial \tilde{y}^2} \end{array} \right. \quad (2.2)$$

$$\left\{ \begin{array}{l} \frac{\partial \tilde{\theta}}{\partial \tilde{t}} = \frac{\tau_c \cdot \dot{\gamma}_c \cdot t_k}{\rho \cdot c \cdot \theta_c} \tilde{\gamma} + \frac{\lambda}{\rho \cdot c} \frac{t_k}{\delta^2} \frac{\partial^2 \tilde{\theta}}{\partial \tilde{y}^2} \end{array} \right. \quad (2.3)$$

Where ' $\sim$ ' denotes dimensionless and well scaled variables and t_k is the time scale for shear band formation.

In accordance with the intuition of balance of dissipative plastic work and temperature rise within the shear band, a proper choice of the magnitude of the coefficients in the equations is

$$\frac{\tau_c \cdot \dot{\gamma}_c \cdot t_k}{\rho \cdot c \cdot \theta_c} \sim 1$$

This leads to

$$t_k \sim \frac{\rho \cdot c \cdot \theta_c}{\tau_c \cdot \dot{\gamma}_c} \sim B \cdot \frac{1}{\dot{\gamma}_c} \sim \frac{1}{\dot{\gamma}_c} \sim (10^{-5} \sim 10^{-7}) \cdot s \quad (2.4)$$

This choice of t_k leaves two selections of the expression of δ .

$$(1) \quad \frac{\lambda \cdot t_k}{\rho \cdot c \cdot \delta^2} \sim 1 \quad \text{giving}$$

$$\delta \sim \sqrt{\frac{\lambda}{\rho \cdot c} \cdot t_k} \sim \sqrt{\frac{\lambda \cdot \theta_c}{\tau_c \cdot \dot{\gamma}_c}} \sim (10^{-5} \sim 10^{-6}) \cdot m \quad (2.5)$$

This makes the coefficient of the inertia term

$$\frac{\rho \cdot \dot{\gamma}_c \cdot \delta^2}{\tau \cdot t} \sim \frac{1}{P_c} \ll 1 \quad (2.6)$$

Therefore, internal structure of shear band should be quasi-static and heat-dissipative, with plastic work as heat source. This is consistent with the conclusion cited in the previous section.

$$(2) \text{ Let } \frac{\rho \cdot \dot{\gamma}_c \cdot \delta^2}{\tau_c \cdot t_k} \sim 1 ,$$

then another δ , denoted by δ_1 , can be defined, such that

$$\delta_1 \sim \frac{\sqrt{c \cdot \theta}}{\dot{\gamma}_c} \sim \sqrt{\frac{P_c}{\dot{\gamma}_c}} \cdot \delta \sim 10^2 \cdot \delta \quad (2.7)$$

Obviously, δ_1 is much large than δ . In addition, this choice makes the term of heat diffusion negligible, because

$$\frac{\lambda}{\rho \cdot c} \cdot \frac{t_k}{\delta_1^2} \sim \left(\frac{\delta}{\delta_1} \right)^2 \sim \frac{1}{P_c} \ll 1 . \quad (2.8)$$

More importantly, δ_1 indicates another dissipative mechanism, i.e. momentum diffusion or viscosity, as shown by the definition of Prandtle number P_c . Therefore, does δ_1 give an indication of the spacing of shear bands? Nevertheless, since the discussion is based on the estimation of the internal behavior of a shear band, the conjecture should be justified.

For this spacing, all variations are expressed as

$$\Delta y \sim l, \quad \Delta \theta \sim \theta_1, \quad \Delta \dot{\gamma} \sim \dot{\gamma}_1, \quad \Delta \tau \sim \tau_1, \quad t_k \sim \frac{\rho \cdot c \cdot \theta}{\tau_c \cdot \dot{\gamma}_c}$$

where subscript 1 denotes half spacing. Noticeably, in the discussion of shear band spacing, the adopted time scale t_k is the same as that used previously in investigation of shear band. This implies that formation of shear bands and their spacing is in the same range of time.

We have now the following scaled dimensionless equations

$$\left\{ \begin{array}{l} \frac{\rho \cdot \dot{\gamma}_1 \cdot l^2}{\tau_1 \cdot t_k} \frac{\partial \bar{\gamma}}{\partial \bar{t}} = \frac{\partial^2 \bar{\tau}}{\partial \bar{y}^2} \end{array} \right. \quad (2.9)$$

$$\left\{ \begin{array}{l} \frac{\partial \bar{\theta}}{\partial \bar{t}} = \frac{\tau_1 \cdot \dot{\gamma}_1 \cdot t_k}{\rho \cdot c \cdot \theta_1} \cdot \bar{\tau} \cdot \bar{\gamma} + \frac{\lambda}{\rho \cdot c} \frac{t_k}{l^2} \frac{\partial^2 \bar{\theta}}{\partial \bar{y}^2} \end{array} \right. \quad (2.10)$$

Where '-' denotes dimensionless variables but scaled by corresponding variations in band spacing.

The same consideration of balance of dissipative work and temperature rise leads to

$$\frac{\tau_1 \cdot \dot{\gamma}_1 \cdot t_k}{\rho \cdot c \cdot \theta_1} \sim 1$$

or more clearly

$$\frac{\dot{\gamma}_1}{\dot{\gamma}_c} \sim \frac{\theta_1}{\theta_c} \cdot \frac{\tau_c}{\tau_1} \quad (2.11)$$

Obviously, this represents an essential estimation of correlation of states within and outside a shear band.

If the term of heat conduction were in $O(1)$, then

$$\frac{\lambda}{\rho \cdot c} \cdot \frac{t_k}{l^2} \sim 1$$

$$\text{or } l^2 \sim K \cdot t_k^2 \sim \delta^2.$$

Doubtlessly, this is invalid. Now, the unique choice is

$$\frac{\rho \cdot \dot{\gamma}_1 \cdot l^2}{\tau_1 \cdot t_k} \sim 1$$

i.e.

$$l^2 \sim \frac{\tau_1}{\rho \cdot \dot{\gamma}_1} \cdot t_k \sim \nu \cdot t_k \sim \frac{\tau_1 \cdot c \cdot \theta_c}{\tau_c \cdot \dot{\gamma}_c \cdot \dot{\gamma}_1} \sim \frac{\dot{\gamma}_c}{\dot{\gamma}_1} \cdot \delta_1^2 \quad (2.12)$$

if $\tau_c \sim \tau_1$.

Moreover

$$\frac{l^2}{\delta^2} \sim P_1 = \frac{\tau_c \cdot c}{\dot{\gamma}_1 \cdot \lambda} = \frac{\dot{\gamma}_c}{\dot{\gamma}_1} \cdot P_c. \quad (2.13)$$

The estimation of spacing l qualitatively justifies the mechanism of momentum diffusion, which governing the spacing, but indicates an additional factor $\frac{\dot{\gamma}_c}{\dot{\gamma}_1}$ attached to the previous estimation of spacing δ_1 .

Finally the equations governing spacing become

$$\left\{ \begin{array}{l} \frac{\partial \bar{\gamma}}{\partial \bar{t}} = \frac{\partial^2 \bar{\tau}}{\partial \bar{y}^2} \end{array} \right. \quad (2.14)$$

$$\left\{ \begin{array}{l} \frac{\partial \bar{\theta}}{\partial \bar{t}} = \bar{\tau} \cdot \bar{\gamma} + \frac{1}{P_1} \cdot \frac{\partial^2 \bar{\theta}}{\partial \bar{y}^2} \end{array} \right. \quad (2.15)$$

Provided $P_1 \gg 1$, the deformation in band spacing is adiabatic and heat-assisted, but the structure of spacing is controlled by momentum diffusion.

In summary, the pattern of shear bands is due to simultaneously existing diffusions — momentum and heat diffusions. The quasi-steady structure of shear band plays an essential role in the eventual feature of the pattern. The scheme of these features is shown in Fig.2.

Discussion

In the literature available now, there are very few quantitative description and observation of pattern of shear bands. Perhaps, the followings are the rare exceptions.

Backman et al.^[13] found that in their tests of penetration, no significant change in the width of adiabatic shear bands for three fold increase in the linear size.

Grady et al.^[14] reported TEM studies of recovered samples of shock loaded aluminum. It was observed that intense localized shear bands are 0.1~0.3 μm in width and 2~10 μm in spacing. Hence, the ration of band width to spacing is ranging over 10^{-1} ~ 10^{-2} .

Fig. 2 Scheme of width and spacing of shear band.

Now let us examine the obtained theoretical estimation. For aluminum under planar shock loading, the relevant parameters can be evaluated as^[14]

$$c \sim 10^3 \text{ J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$$

$$\lambda \sim 200 \text{ W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$$

$$\tau_c \sim 500 \text{ MPa}$$

$$\theta_c \sim 1000 \text{ K}$$

Assuming $\dot{\gamma}_c \sim 10^8 \text{ s}^{-1}$, then

$$P_c = \frac{c \cdot \tau_c}{\lambda \cdot \dot{\gamma}_c} \sim \frac{10^3 \cdot 5 \cdot 10^8}{200 \cdot 10^8} \sim 2 \times 10^1$$

$$\text{and } \delta \sim \sqrt{\frac{\lambda \cdot \theta_c}{\tau_c \cdot \dot{\gamma}_c}} \sim \sqrt{\frac{200 \times 10^3}{5 \times 10^8 \times 10^8}} \sim 2 \mu\text{m}$$

This estimation of shear band width is one order higher than the observed. Whereas

$$\delta_1 \sim \sqrt{P_c} \cdot \delta \sim 10 \mu\text{m}$$

is roughly equal to the upper bound of observed spacing. Furthermore, the estimation 1 would be even larger

$$1 \sim \sqrt{\frac{\dot{\gamma}_c}{\dot{\gamma}_1} \cdot P_c} \cdot \delta \sim 10^2 \mu\text{m}$$

one order higher than observed value too, if $\dot{\gamma}_1$ is taken to be 10^6 s^{-1} as evaluated in most planar impact tests. The reason of the overestimation is still not very clear. It is also noticeable that another approach proposed by Grady and Kipp^[14] also lead to greater prediction of 1 μm of band width and 9 μm of band spacing.

Another feature worthy discussing is the number of shear bands. The constant velocity boundary condition confines the shear bands as follows

$$\begin{cases} 2 \cdot (1 + \delta) \cdot N \sim b & (3.1) \end{cases}$$

$$\begin{cases} 2 \cdot (1 \cdot \dot{\gamma}_1 + \delta \cdot \dot{\gamma}_c) \cdot N \sim V & (3.2) \end{cases}$$

Elimination of number of shear bands N, gives

$$\left(\frac{1 \cdot \dot{\gamma}_1}{\delta \cdot \dot{\gamma}_c} + 1 \right) / \left(\frac{1}{\delta} + 1 \right) \sim \frac{V}{b \cdot \dot{\gamma}_c} = v_b \quad (3.3)$$

Recalling

$$\frac{1}{\delta} \sim \left(\frac{\dot{\gamma}_c}{\dot{\gamma}_1} \right)^{1/2} \cdot P_c^{1/2} \sim x^{1/2} \cdot P_c^{1/2}, \quad x = \frac{\dot{\gamma}_c}{\dot{\gamma}_1} \quad (3.4)$$

We obtain the equation to determine $x = \frac{\dot{\gamma}_c}{\dot{\gamma}_1}$

$$\left(\sqrt{P_c/x} + 1 \right) \sim V_b \cdot \left(\sqrt{P_c \cdot x} + 1 \right) \quad (3.5)$$

Then

$$\sqrt{x} \sim \frac{\left(\frac{1}{V_b} - 1 \right)}{2 \cdot \sqrt{P_c}} \cdot \left\{ 1 + \sqrt{1 + \frac{4 \cdot P_c \cdot V_b}{(V_b - 1)^2}} \right\} \quad (3.6)$$

$$\frac{N}{b/2 \cdot \delta} \sim \frac{1}{1 + \sqrt{P_c \cdot x}} \quad (3.7)$$

Illustrative results are shown in Figs. 3-4. Noticeably, with

increasing boundary velocity, $\frac{\dot{\gamma}_1}{\dot{\gamma}_c}$ increases but $\frac{1}{\delta}$ decreases, hence the number of shear bands N increases.

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Fig. 3 Schematic diagram showing relation between ratio of spacing and width of shear band, l/δ , and dimensionless boundary velocity $v_b = \frac{V}{b \cdot \dot{\gamma}_c}$.

Fig. 4 Schematic diagram showing relation between normalized number of shear bands, $N/(b/2\delta)$, and dimensionless boundary velocity, $v_b = \frac{V}{b \cdot \dot{\gamma}_c}$.

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