ON THE ROLE OF STRAIN NON-UNIFORMITIES

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ABSTRACT

The main objective of this article is to explore the implications of the analytical structure of thermomechanical processes on the evolution of strain non-uniformities and instability phenomena in simple shearing. Using as model the inelastic material: thermoviscous flow, viscoplastic solids, thermoviscoplastic solids, we attempt to demonstrate how internal dissipation, initial deformation, external load parameters and material inhomogeneities affect the smoothness of processes. We verify that the formation of shear bands is attributed to strain-softening or thermal-softening response of the material and present some stability criteria. Finally, we present some stabilizing effects, such as the "double-gradient" approximation.

INTRODUCTION

In this article, we focus on a theoretical description of plastic stability, suggested for explaining the formation of instabilities in the deformation process of solids (shear bands) or in the velocity profile of fluids.

We consider a thermoviscoplastic material exhibiting strain hardening or softening, strain rate sensitivity and thermal softening. This material occupies the space between two parallel planes $x=0$ and $x=h$ and is sheared in the y -direction, uniformly in y and z . This simple problem modelizes the qualitative behaviour of some basic experiments, such as the torsion of thin-walled tubes or the thermoviscous Couette flow. In more general situations, the simple shearing modelizes the deformation zone in the localization region by using the generalized stress and strain.

The material is characterized by the (constant) density ρ , the specific heat coefficient c , the thermal softening function μ , which is a decreasing function of temperature, the strain-rate sensitivity s and the strain sensitivity ϕ . Under the assumption of adiabatic conditions, which is the most unstable situation in the context of dissipative thermomechanical processes, the system of partial differential equations describing the behaviour of the velocity $v(x,t)$, temperature $\theta(x,t)$ and plastic strain $\gamma(x,t)$ is the following

$$\rho v_t = \sigma_x \quad (1)$$

$$\rho c \theta_t = \sigma \gamma_t, \quad (2)$$

$$\gamma_t = v_x, \quad (3)$$

where $\sigma(x,t)$ is the shear stress according to the constitutive law

$$\sigma = G \mu(\theta) s(\gamma_t) \phi(\gamma), \quad (4)$$

where G is the shear modulus. In (1)-(4) the subscript indicates differentiation with respect to the variable indicated.

It was established by several authors [1-6] that the properties of the functions μ , s and ϕ and especially the amounts of the slopes of μ , s and ϕ and their relationship, determine the stability of the deformation process. In particular, the softening properties of the metal may outweigh the tendency of the material to harden, destabilizing the process in the following manner: non-uniformities in the strain-rate profile induce non-uniform heating, which, in turn, results in non-uniform distribution of the plastic flow (by enhancing the plastic strain only in hotter regions) and induces an autocatalytic mechanism. In the sequel, we illustrate this belief by choosing two simple models, the isothermal case and the so-called "gradient approach" [7].

Another important aspect is the asymptotic behaviour of solutions of (1)-(4) (under appropriate initial and boundary conditions) when the time becomes very large. Related works [2, 8] showed that the asymptotic form of v , θ and γ depends crucially on the boundary conditions of the problem (1)-(4), i.e. on the control parameters of shearing. More specifically, the boundary conditions seem to "attract" the asymptotic behaviour, with the influence of prescribed tractions being more powerful than the prescribed velocities. Moreover, among all the types of asymptotic stability, the "steady states" are of the greatest technological interest. These situations are equivalent to "quasi-static equilibrium" (when the velocity tends to a function of the position x only) [2, 8] or to "low-temperature equilibrium" [9] (where the stress and the strain rate tend to zero) or to "uniform strain and temperature distribution" (where strain and temperature are increasing functions of time). This last category provides a weaker stability, since it induces the development of high values for the strain and the temperature, which favour the material instabi-

lities. However, a feature of this deformation is that the strain localization (i.e. the "blow-up" of $\dot{\gamma}_x$) is excluded, as well as the related formation of shear bands [7]. Another category of weak stability corresponds to the development of very high values of strain gradient only in a very narrow material region, generally located near the boundaries where velocity or shear force are prescribed. This situation may destabilize the process by the combined action of the boundary conditions, which gives heat into the body via the mechanical work, and the adiabaticity of the process, which prevents the heat diffusion into all the body and so it limits it into a narrow region. In [10], the destabilizing source is the initial deformation state and the symmetry of the plastic flow, which may favour the development of the strain rate only in some regions and prevent it in others. In this case, the cause of the weak asymptotic stability is the non-uniform distribution of the strain rate and the reduction of the internal cohesion of the material. However, inspite of the fact that the strain rate approaches to very high values near the boundaries, the product $\mu \dot{\gamma}_t$ (i.e. the stress, in this case) remains bounded, stabilizing the evolution of the process.

In this context of stability analysis the shear banding appears as the "blow-up" of the strain gradient in a finite time, dependend on the initial strain, the thermomechanical functions μ , s and ϕ and the prescribed tractions or velocities at the boundaries. More elaborated concepts (as the " L_∞ -localization") can be found in [11]. In this paper, we present some results illustrating the appearance of material instabilities in a finite time, which are based on the non-linear analysis.

There is an extensive litterature on the non-linear and linearized analysis studies of shear banding and the reader is invited to see the articles of Tzavaras [12] and Aifantis [13] for an excellent survey.

MATERIALS AND THE MAIN RESULTS

In this section, we present some examples illustrating the destabilizing role of non-uniform strain or temperature distribution, when it is combined with material behaviour exhibiting strain or thermal softening. Moreover, we present a "regularization" based on the "double-gradient" approach.

Strain softening

To illustrate the effect of strain softening on the evolution of small non-uniformities, we choose two models, the (isothermal) "elliptic" model

$$\sigma = G\phi(\gamma) \quad (5)$$

where $\varphi'(\gamma) < 0$, and the (isothermal) "parabolic-hyperbolic" model

$$\sigma = G\varphi(\gamma) s(\dot{\gamma}_t) . \quad (6)$$

Both models are special cases of the constitutive law (4).

(i) "Elliptic" model.

It is generally established that strain softening has a destabilizing influence on the deformation regime by amplifying initial defects or strain non-uniformities. To see that, we write (1),(3)-(5) in the form

$$\rho \dot{\gamma}_t = G(\varphi(\gamma))_x \quad (7)$$

$$\dot{\gamma}_t = \dot{\gamma}_x , \quad (8)$$

from which, by differentiation with respect to x of (7) and using (8) we obtain the partial differential equation in γ

$$\gamma_{tt} - \frac{G}{\rho} \varphi'(\gamma) \gamma_{xx} = G \varphi(\gamma) \gamma_x^2 . \quad (9)$$

Since $\varphi'(\gamma) < 0$, the initial-value problem of (9) with initial conditions

$$\gamma(x,0) = \gamma_0(x) \quad (10)$$

is elliptic, therefore even small non-uniformities in $\gamma_0(x)$ tend to increase with time. We conclude that this model must be "regularized" by incorporating strain rate -dependence or strain-gradient-dependence effects [14,15,16]. The next model provides such a model.

(ii) "Hyperbolic-parabolic" model.

We choose a type of plastic material with a non-monotone yield function $\varphi(\gamma)$ and a strain-rate sensitivity function s given by [17]

$$s(\dot{\gamma}_t) = (\beta + \dot{\gamma}_t)^\eta , \quad 0 < \eta < 1 . \quad (11)$$

The yield function is taken to be of the form [15]

$$\varphi(\gamma) = \varphi_m - \alpha(\gamma - \gamma_m)^2 , \quad \varphi_m, \alpha, \gamma_m > 0 , \quad (12)$$

and is characterized by the strain γ_m after which the slope of $\varphi(\gamma)$ passes from positive to negative values (see Fig. 1).

We first check the validity of the constitutive equation (6) and (11). Suppose that the boundary conditions are prescribed velocities and zero strain gradient at the planes $x = 0$ and $x = h$,

$$v(0,t) = 0, \quad v(h,t) = \bar{v}, \quad (13)$$

$$\gamma_x(0,t) = 0, \quad \gamma_x(h,t) = 0. \quad (14)$$

Then, differentiating the stress with respect to t (see (6) and (11))

$$\sigma(x,t) = G \varphi(\gamma) (\beta + \gamma_t)^{1/\eta} \quad (15)$$

and using (1), we find that the stress obeys to the reaction-diffusion equation (G is taken equal to 1 for simplicity)

$$L[\sigma] \equiv \sigma_t - \frac{\varphi(\gamma)^{1/\eta}}{\rho \sigma^{(1-\eta)/\eta}} \sigma_{xx} + \left[\beta - \left(\frac{\sigma}{\varphi(\gamma)} \right)^{1/\eta} \right] \sigma \frac{\varphi'(\gamma)}{\varphi(\gamma)} = 0, \quad (16)$$

under the boundary (see (1) and (13))

$$\sigma_x(0,t) = \sigma_x(h,t) = 0 \quad (17)$$

and the initial conditions

$$\sigma(x,0) = \sigma_0(x). \quad (18)$$

We choose $\sigma_0(x)$ such that (see Fig. 1)

$$\sigma_0(x) > \beta^\eta \varphi_m, \quad 0 \leq x \leq h, \quad (19)$$

in order to ensure the positiveness of the initial strain rate (see (15)). Then, the plastic flow can start. We will show that, if $\gamma \leq \gamma_m$, then the plastic flow is valid, i.e.

$$v_t(x, t) > 0, \quad 0 \leq x \leq h, \quad v \leq v_m. \quad (20)$$

In other words, the solutions of (16)-(18) comply with the restriction that the strain rate is positive in order to justify the assumption that the elastic effects are negligible. In order to show (20), we first recall [5] that, if σ_1 and σ_2 are sufficiently smooth and the following inequalities hold for $0 \leq t \leq t_m$

$$L[\sigma_1] \geq L[\sigma_2], \quad 0 \leq x \leq h \quad (21)$$

$$\sigma_{10}(x) > \sigma_{20}(x), \quad 0 \leq x \leq h \quad (22)$$

$$\sigma_{1x}(1, t) \geq \sigma_{2x}(1, t), \quad (23)$$

$$\sigma_{1x}(0, t) \leq \sigma_{2x}(0, t), \quad (24)$$

then

$$\sigma_1(x, t) > \sigma_2(x, t), \quad (25)$$

for $0 \leq x \leq h$, $0 \leq t \leq t_m$. In order to prove (20), we use this observation with $\sigma_1(x, t) = \sigma(x, t)$ and $\sigma_2(x, t) = \beta^\eta \varphi_m$ (see Fig.1). We verify that

$$L[\sigma_2] = \beta \left[1 - \left(\frac{\varphi_m}{\varphi(v)} \right)^{1/\eta} \right] \beta^\eta \varphi_m \frac{\varphi'(v)}{\varphi(v)} \leq 0, \quad (26)$$

therefore (21) is valid, since $L[\sigma_1] = 0$. On the other hand, (22) is satisfied by (19), and (23), (24) are satisfied as equalities. We conclude that

$$\sigma(x, t) > \beta^\eta \varphi_m, \quad 0 \leq x \leq h, \quad t \leq t_m. \quad (27)$$

Combining (15) and (27) we find (20). We note that t_m is the value of t at which for the first time $v(\bar{x}, t_m) = v_m$ for some $0 \leq x \leq h$. Integration of (15) shows that v satisfies the equation

$$v(x, t) = v_0(x) - \beta t + \int_0^t \left(\frac{\sigma(x, \tau)}{\varphi(v(x, \tau))} \right)^{1/\eta} d\tau. \quad (28)$$

We note that the above comparison result (27) is valid only for the time period $0 < t \leq t_m$, where $\gamma \leq \gamma_m$ or

$$\varphi'(\gamma) \geq 0. \quad (29)$$

By using standard techniques of non-linear analysis [18] we can show that, if $\gamma \leq \gamma_m$, the plastic flow is well behaved and the material instabilities are excluded. The question is whether the process remains well behaved in the region of negative slopes for the yield function $\varphi(\gamma)$. To see that, we first verify that the system (1), (3), (6), (11), (12), (under the boundary conditions (13) and (14), and suitable initial conditions) admits the homogeneous solution (which also satisfies the "plasticity" criterium (27))

$$\gamma_0(t) = t + \bar{\gamma}, \quad (30)$$

$$v_0(x) = x, \quad (31)$$

$$\sigma(t) = (\beta+1)^\eta \varphi(t+\bar{\gamma}), \quad (32)$$

which corresponds to the uniform shearing. We also observe that this solution is equivalent to a "quasi-static" situation. Our objective is to follow the evolution of a perturbation of this solution [12]

$$v(x,t) = x + \delta v(x,t), \quad (33)$$

$$\gamma(x,t) = (t+\bar{\gamma}) + \delta \gamma(x,t), \quad (34)$$

in order to obtain the answer to our question. We know already that the relationship between the strain softening and the strain-rate sensitivity factors will determine if the process will continue without developing singularities or not [1-6].

In our case, the strain-rate sensitivity factor $\eta < 1$ is too weak to affront the quadratic function $\varphi(\gamma)$. To verify that, we use (33) and (34) to obtain from (1), (3), (6), (11) and (12) the linearized system in $(\delta \gamma, \delta v)$

$$\delta v_t - \eta \varphi(t+\bar{\gamma}) \delta v_{xx} - \varphi'(t+\bar{\gamma})(1+\beta)^\eta \delta \gamma_x = 0, \quad (35)$$

$$\delta v_x = \delta \gamma_t. \quad (36)$$

This is a hyperbolic-parabolic system provided $\eta > 0$. The mathematical structure of these two equations, competing each other, reveal

the aforementioned stabilizing role of $\eta > 0$. The boundary conditions (13) and (14) give

$$\delta v(0,t) = 0, \quad \delta v(h,t) = 0, \quad (37)$$

$$\delta v_x(0,t) = 0, \quad \delta v_x(h,t) = 0. \quad (38)$$

Following [12], we put

$$\delta v = Y_t \quad (39)$$

to obtain the partial differential equation

$$Y_{tt}(x,t) - \eta \varphi(t+\bar{v}) Y_{txx}(x,t) - (1+\beta)^\eta \varphi'(t+\bar{v}) Y_{xx}(x,t) = 0, \quad (40)$$

under the boundary conditions

$$Y(0,t) = Y(1,t) = 0. \quad (41)$$

This is parabolic in Y_t . The general solution can be written as [12]

$$Y(x,t) = \sum_{k=1}^{\infty} c_k(t) \sin k\pi x, \quad (42)$$

$$c_k(t) = 2 \int_0^h Y(x,t) \sin k\pi x \, dx. \quad (43)$$

By putting (42) in (40) we obtain the ordinary differential equation in $c_k(t)$ ($k = 1, 2, \dots$)

$$\begin{aligned} \mathcal{L}[c_k] &= \ddot{c}_k(t) + k^2 \pi^2 \eta (\varphi_m - a(t-t_m)^2) \dot{c}_k(t) - \\ &- 2a(1+\beta)^\eta k^2 \pi^2 (t-t_m) c_k(t) = 0, \end{aligned} \quad (44)$$

where

$$t_m = v_m - \bar{v}. \quad (45)$$

It is clear that this equation is stable if $t \leq t_m$.

On the contrary, for $t > t_m$ the solution of (44) becomes unstable, i.e. perturbations can grow faster than the uniform shear flow and the uniform shearing solutions are linearly unstable. We give a sketch of the proof [18]: We first verify that ($t_M = \sqrt{\phi_m/\alpha} + t_m$):

$$\mathcal{L}[\phi(t+\bar{V})] < 0, \quad t_m < t < t_M \quad (46)$$

independently of η . Therefore we find the first estimate

$$c_k(t) > \frac{1}{K} \phi(t+\bar{V}), \quad t_m < t < t_M, \quad (47)$$

which is used in (44) in order to obtain the aforementioned estimate

$$c_k(t) > \frac{1}{K}(t+\bar{V}), \quad t_m < t < t_M, \quad (48)$$

where K depends on the initial perturbation.

Double-gradient regularization

We now present an example of the "smoothing" role of double-strain gradient included in the constitutive law [13]. We will show that this inclusion, which reflects the microstructural non-homogeneities, implies the "smoothing out" of possible non-uniformities in strain. Moreover, in the onset of instability, it has a retarding effect on the appearance of instability. The model is very similar to the thermoviscoplastic material considered in [7] and the results presented here follow the same lines. We consider the isothermal model

$$\sigma(\gamma, \gamma_t) = \phi(\gamma) \left(\frac{\partial \gamma}{\partial t} \right)^\eta \quad (49)$$

where $\phi(\gamma) > 0$ and

$$\phi'(\gamma) < 0, \quad (50)$$

i.e. a material exhibiting strain softening. A typical example is a "power-law" softening function

$$\varphi(\gamma) = \gamma^m, \quad -1 < m < 0. \quad (51)$$

Previous analyses [5, 6] give that the non-linear dynamic problem (1), (3), (49), (51) is stable provided the strain-rate sensitivity is greater than the strain softening, i.e. if

$$\eta > |m|. \quad (52)$$

However, the non-uniformities of strain increase much more rapidly at material regions with initial non-uniformities, localizing the plastic flow in this region. Thus, the formal "blow-up" of strain gradient is excluded by the stability criterium (52), but the persistence of non-uniformities may act as precursor of instabilities. To see that, we write (49) in the form

$$\sigma^{1/\eta} = \frac{\eta}{m+\eta} \frac{\partial}{\partial t} (\gamma^{m+\eta/\eta}), \quad (53)$$

from which we deduce by integration

$$\gamma^{m+\eta/\eta}(x,t) = \frac{m+\eta}{\eta} \int_0^t \sigma^{1/\eta}(x,\tau) d\tau + \gamma_0^{m+\eta/\eta}(x). \quad (54)$$

We suppose, for simplicity, that the boundary condition at the plane $x=h$ is a shear load q ,

$$\sigma(h,t) = q. \quad (55)$$

Then, (54) gives

$$\gamma^{m+\eta/\eta}(h,t) = \frac{m+\eta}{\eta} q^{1/\eta} t + \gamma_0^{m+\eta/\eta}(h), \quad (56)$$

from which we conclude that, if $\eta < |m|$, there exists a critical time

t_c , given by

$$t_c = \frac{\eta}{|m|-\eta} q^{-1/\eta} \gamma_0^{m+\eta/\eta}(h), \quad (57)$$

such that the strain at the boundary $x=h$ becomes infinite. This is a typical case of strong instability.

We close this parenthesis and return to our problem of gradient approach, which consists to write (49) in the form

$$\sigma(v, v_t, v_{xx}) = \varphi(v) v_t^\eta - c v_{xx} \quad (58)$$

Here, $c > 0$ denotes the phenomenological coefficient specifying the influence of the gradient effect [16]. Then, assuming quasi-static situation as in [7] and by differentiation with respect to time of (58) we deduce that the strain rate $\psi = v_t$ satisfies the following parabolic partial differential equation

$$\eta \varphi(v) \psi^{\eta-1} \psi_t - c \psi_{xx} + \varphi'(v) \psi^\eta = 0, \quad (59)$$

under initial

$$\psi_0(x) > 0 \quad (60)$$

and boundary conditions

$$\psi_x(0, t) = \psi_x(h, t) = 0. \quad (61)$$

The last condition is a direct consequence of (14). We give below the results of the analysis of the problem. For more details this point, the reader may consult [18]. The first observation concerns the positiveness of the strain rate for each $t > 0$ and x , i.e. the validity of the assumption that the elastic effects are negligible. Indeed, application of maximum principle for (59), (60) and (61) gives that

$$\psi(x, t) > 0, \quad 0 < t < \infty, \quad 0 \leq x \leq h. \quad (62)$$

Next, we present the behaviour of solutions of (59)-(61). For simplicity, we assume that $\varphi(v)$ is given by (51). Then, (59) takes the form

$$\psi_t - \frac{c}{\eta} v^{-m} \psi^{1-\eta} \psi_{xx} + \frac{m}{\eta} v^{-1} \psi = 0. \quad (63)$$

We can show [18] that, if $m+\eta > 0$ and $c > 0$, the process is stable and the strain evolves "almost uniformly" without strain localization (shear banding). Namely, we obtain

$$\frac{1}{K} t^{m/(m+\eta)} \leq v(x, t) \leq K t^{m/(m+\eta)}, \quad (64)$$

$$|v_x(x,t)| \leq K. \quad (65)$$

Here, K are constants independent of time, which can be estimated by the initial data and the load parameter of the problem. The most interesting feature of this result, is that, by (64) and (65), the strong non-uniformities are excluded, while the strain becomes large with the time. On the contrary, if $c=0$, then from (54) we deduce that the strain gradient is given by

$$v_x(x,t) = \left[v_0(x)^{(m+\eta)/\eta} + \frac{m+\eta}{\eta} q t \right]^{-m/\eta} v_0(x)^{m/\eta} \frac{dv_0}{dx}(x), \quad (66)$$

from which we obtain that the strain gradient increases much more rapidly at material regions with initial non-uniformities in the strain. We conclude that, as in [7], the inclusion of the microstructural parameter $c > 0$ implies the "smoothing out" of non-uniformities.

An easy extension of the model (49) to the anisothermal case can be obtained by considering a flow stress of the form (4) where the temperature acts as intermediate variable expressed via the strain by using the energy equation. Indeed, we observe that, under quasistatic equilibrium, the energy equation (2) gives

$$\rho c \theta_t = q v_t \quad (67)$$

from which we obtain a linear relation between strain and temperature, due to the adiabatic conditions,

$$\rho c [\theta(x,t) - \theta_0(x)] = q [v(x,t) - v_0(x)]. \quad (68)$$

Then, (4) leads to

$$\sigma(v, v_t, \theta) = \varphi(v) \mu(v) s(v_t) = \sigma'(v, v_t). \quad (69)$$

For instance, for a material exhibiting thermal softening according to the power law, we have

$$\sigma'(v, v_t) = \varphi(v) \left(\frac{q}{\rho c} v - \frac{q}{\rho c} v_0 - \theta_0 \right)^v s(v_t),$$

where $-1 < v < 0$.

Therefore, the non-linear analysis of the anisothermal model follows parallel lines with the aforementioned isothermal case. The

double gradient approach gives

$$\sigma = \tau(\gamma) \left(\frac{\partial \gamma}{\partial t} \right)^\eta - c \gamma_{xx} \quad , \quad (70)$$

where

$$\tau(\gamma) = \left(\frac{q}{\rho c} \right)^\nu \gamma^m (\gamma - \gamma_0 - \frac{\rho c}{q} \theta_0)^\nu \quad (71)$$

and $-1 < m < 0$. This material is interesting for the reason that exhibits both thermal and strain softening and the only stabilizing counterbalance is expected to be the strain-rate sensitivity η . More specifically, if $c=0$ and

$$|m + \nu| > \eta \quad , \quad (72)$$

there exists again a critical time t_c at which the strain "blows-up". To see that, we deduce from (70), (71) by separation of variables, as previously,

$$\int \sigma^{1/\eta} d\tau = \left(\frac{q}{\rho c} \right)^\nu \int \gamma^{m/\eta} (\gamma - \gamma_0 - \frac{\rho c}{q} \theta_0)^{\nu/\eta} d\gamma \quad , \quad (73)$$

from which (72) can be, eventually, obtained in a similar manner as for the model (49).

This criterium of instability agrees with the non-linear results of [12] as well as the linearized results of [11]. In a similar way, we can also verify that, if $c > 0$, the critical time is larger, so the retarding effect of double-gradient approximation is again present. Moreover, the instability mode is again different than for the case $c=0$ exhibiting a uniform loss of stability [18].

Non-homogeneous Thermoplastic Flow

We consider a stratified multicomponent shearing flow of materials exhibiting thermal softening according to the law

$$\sigma_i = \theta_i^{-\nu_i} \frac{\partial \gamma_i}{\partial t} \quad , \quad \nu_i > 0 \quad ,$$

for the i - material ($i=1,2,\dots,N$). We write the N equations of momentum and energy balance laws (1), (2), for the N materials and consider the boundary conditions (1.2). We now assume that at the intersurfaces the materials exhibit the same velocity and shear stress. As it can be easily verified [19] these compatibility conditions imply that

the strain rate and the temperature exhibit discontinuities at the interfaces. Therefore, an important question is if these strong non-uniformities of the strain rate distribution will persist, destabilizing the system. The answer is no, provided the maximum thermal softening factor of the materials is inferior than 1. Namely, in [19] it was shown that the stabilization of dissipation occurs through the thermal softening, in spite of the fact that the strenght of such a dissipative mechanism weakens in the course of motion. Then, if $\max V_1 < 1$, the stabilizing effect of dissipation induces the convergence of the flow to the uniform shearing. It is worth noticing that, in the present situation, dissipation is sufficiently strong to smoothen out discontinuities of strain rate. Moreover, the temperature becomes a uniform (in x) time-increasing function. A more elaborated analysis of the non-homogeneous thermoplastic flow was presentend in [20] and [21]. A homogenization result was established there for the problem of studying the mechanical properties of the "homogenized" slab consisting of numerous layers of different materials, with non-homogeneous specific heat $c^e(x)$ and strain-rate sensitivity $\mu^e(x, \theta)$, each of those with small thickness e . In such a context, it is natural to look for the relation between the asymptotic limits of the velocity, stress and temperature, as the number of layers tends to infinity while their thickness e tends to zero. It turns out that these limits satisfy an initial-boundary value problem very close to the type (1)-(3), for certain "homogenized" specific heat $c^0(x, \theta)$ and strain-rate sensitivity function $\mu_0(x, \theta)$. The result indicates that in the homogenization process the stress converges strongly, while the velocity and temperature exhibit oscillations, due to the heterogeneities of the material and to the heating caused by the thermal softening. It should be mentionned that the limit specific heat function $c^0(x, \theta)$ do depend also on θ implying that the limit problem is not identical to the initial one.

Numerical Results

In this section we illustrate the role of strain non-uniformities, on the mechanical behaviour of a viscoplastic material with constitutive law given by (57). We consider a viscoplastic slab of width $h=10^{-2}m$ and infinite extend, consisting of a softening material exhibiting strain rate sensitivity. We consider two cases : region of stability ($m+\eta > 0$) and region of instability ($m+\eta < 0$) to study the numerical behaviour of strain, strain-rate and strain gradient. For the numerical solution of the initial-boundary value problem we employ a method on lines based on a finite element semi-discretization procedure [4]. The software implementation was realised by the appropriate selection of parameters of a general finite element collocation program PDECOL [22]. In Fig. 2, we see the influence of the double-gradient coefficient to the evolution in time of the localization measure (defined in [6]), for $c=0$ and $c=10$ N. We verify that, when the material is unstable ($m+\eta < 0$) and $c=10$, the instability occurs at a critical time without large non-uniformities, i.e. without shear banding. On the

other hand, when the material is stable, the double gradient dependence enforces the localization measure to decay to zero very fast, i.e. the process tends to be uniform. The next Fig. 3 shows the evolution of strain localization for various values of c , where again the regularizing role of c is evident. In Fig. 4 the retarding effect of $c > 0$ to the emergence of strain-rate instabilities is illustrated. We observe that the retardation is almost 18%. Finally, the "smoothing" effect of $c > 0$ is shown in Fig. 5, where the evolution of the relationship between the maximum and the minimum value of strain is illustrated.

Conclusion

The main results of this paper show that the behaviour of the plastic strain can be defined by the analytical structure of the system describing the problem, where the form of constitutive law is dominant. In this context, the stable or unstable, uniform or non-uniform character of the process may depend not only on the well-known thermomechanical parameters (strain hardening or softening, strain rate sensitivity and thermal softening) but also on the initial non-uniformities in strain or temperature, the boundary conditions, the material heterogeneities and microstructural parameters. This point of view can eventually help the linearized techniques, based on the existence of uniform "ground" deformation, to be more efficient.

fig. 1

fig. 2

fig. 3

fig. 4

fig. 5

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