Pluralistic Efficiency-Equity Tradeoffs in Locating Public Services

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Abstract Aside from efficiency, equity is a particularly salient objective in siting public facilities. In this paper, we develop and test a bicriteria mixed integer program that trades off the efficiency and equity measures between an individual and the group. Individual efficiency is measured by data envelopment analysis (DEA) while individual equity is modeled by a center location model. On the other hand, group efficiency is measured by the median location model, while group equity is measured by the Gini coefficient. Available computational results suggest that the multicriteria model provides a promising approach to locate public services in a pluralistic society, when the individual rights are often weighed against common good.

Keywords efficiency; data envelopment analysis; equity; median; center

1 Introduction

Governments and public agencies are continually faced with the problem of locating public service facilities. The term public service, as used here, refers to any of the common, everyday services provided by federal, state, and local governments. Typical examples of these services are police and fire stations, emergency medical care, and social services. For such spatially distributed services, performance is determined to a significant degree by the location of public service facilities, which determines their responsiveness to demands in the study area.

Efficiency and equity are two key performance indices for public-service facility location. To date, the most widely used criterion in public-service location is efficiency. The majority of such public-service facility models minimize travel distance or time, or to maximize demand coverage^[1–4]. In police, fire and ambulance services, an efficiency measure that has been widely used is response time, or the elapsed time between the call for help and the arrival of the emergency unit. For stationary service centers such as a public library, a comparable measure to response time is travel time or travel distance experienced by service recipients to access the facility. For example, Cheol^[5] examined the spatial distribution of health facilities needed to minimize the travel distance per capita and the total distances patients traveled.

The main limitation of the traditional efficiency maximizing model is that facility location is an extremely complex endeavor that encompasses at least several other factors besides minimal

response time. As a departure, a number of authors, including Fisher and Rushton^[6], have focused on the multi-criteria nature of the public service location. Data envelopment analysis (DEA)^[7], which can consider multiple-cost and -benefit measures is a methodology that can incorporate such concerns.

Equity is another important measure of public service delivery. Essentially, equity is an abstract socio-political concept that implies fairness and justice^[8]. Equity maximization suggests that inequality is minimized, where inequity measures were primarily studied in economics^[9]. Recently, there has been an increasing concern for equity in the facility-location literature. Marsh and Schilling^[10] surveyed twenty different measures to gauge the level of equity in facility location. They refer to the fairness, impartiality, or equality of service.

The centdian problem seeks p points that minimize a convex combination of the median and center distance objectives. Median is often associated with efficiency and center with equity. In this paper, we implement the centdian concept using a more direct approach. Although the model shares similar characteristics as the centdian model, its efficiency measure and its equity measure are defined differently than the min-sum and min-max functions. Efficiency is modeled in a DEA framework, while equity is modeled as a Gini coefficient. In this way, it clearly distinguishes itself from the classic centdian model.

Since we are locating public facilities, we wish to tradeoff equity-efficiency between individuals and a group. In the public arena, individual rights are often weighed against common good. Our goal is to come out with a facility expansion strategy that is good for serving the individual citizen as well as the general public.

The rest of this paper is organized as follows. In section 2, basic public service facility location and DEA models are reviewed. Similar to Thomas et al.^[12], a model incorporating location and DEA models into one formulation will be produced. In section 3, we develop an efficiency-equity tradeoff model. In section 4, we discuss the relationship between the efficiency-equity tradeoff model and the classic centdian model. A public-service location example with multiple inputs and outputs is given in section 5. Conclusions and discussion are presented in the final section.

2 Literature review

2.1 Public service facility-location methods

Public services are typically provided and managed by governments in response to perceived and expressed need. The spatial distribution of public services is often determined by facility location decisions. In a typical location problem, a smaller delivery or access distance means a better service quality or customer satisfaction. One objective is to identify the furthest distance between a facility site and a demand and then select the sites that minimize this distance. In this way, even the most remote customer would receive quality service^[11].

This class of location models is called the p -center models. The models have this distinctly nonlinear objective:

$$\min \max_{\substack{i=1,2,\dots,n \\ j=1,2,\dots,m}} d_{ij}x_{ij},$$

where d_{ij} is the distance between demand i and facility-site j , and x_{ij} is the binary decision-

variable to assign demands to facilities. There are all together m facility-sites and n demand-nodes. This objective function, because of its non-linearity, is inconvenient for standard computational procedures. Here we present a linearized formulation that is amenable to subsequent merging with the DEA model^[12]. Let us define d'_j as the maximum distance between a facility-site j and all demand-nodes. The objective is to minimize d'_j for the case when one public service facility j is to be sited. For a multi-facility case when p ($p > 1$) facilities are involved, we minimize the sum of the distances from each demand to the closest selected facility^[12]:

$$\min \sum_{j^* \in \sigma_p} D'_{j^*}.$$

Here σ_p is the set of p individually-selected location, each of which is defined by

$$D'_{j^*} = \min d'_j.$$

We wish to generate constraints that force d'_j to assume the desired values. First, a set of constraints needs to be included to ensure that proper arc between a demand i and a facility j is chosen:

$$\sum_{j=1}^m d_{ij} x_{ij} \leq d_{ij} s_j + (1 - s_j)M.$$

Here M is any arbitrarily-selected large-number and has to be larger than any d_{ij} in the minimum-distance matrix. Assisted by the binary variable s_j , the above equation activates the appropriate x_{ij} corresponding to the direct arc between i and j ^[12].

When the maximum distances d'_j are to be identified, the following companion constraint finds the appropriate maximum-distance. For each proposed site j , the following constraint set will determine the maximum-distance d'_j :

$$d'_j - d_{ij} x_{ij} + M s_j \geq M, \quad \forall i \neq j; i = 1, 2, \dots, n; j = 1, 2, \dots, m.$$

The next constraint specified the number of p facilities to be sited among m candidate locations, where $p < m$.

$$\sum_{j=1}^m s_j = p \tag{1}$$

Another constraint that must be met is that each demand must be served by a facility:

$$\sum_{j=1}^m x_{ij} = 1, \quad i = 1, 2, \dots, n.$$

Demand at a node i (among n demands) cannot be served by facility j unless the facility is built. The following constraint requires a facility to be in place to meet demands. This happens when a supply-demand assignment is necessitated by activation of the appropriate x_{ij} allocation-variable:

$$\sum_{i=1}^n x_{ij} - (n - p + 1) s_j \leq 0, \quad j = 1, 2, \dots, m.$$

If any x_{ij} are activated for a given facility j , this constraint will force the facility to be built at j ($s_j = 1$). The second term in this inequality ensures that the second facility chosen for construction serves at least a demand. Otherwise, the formulation could recommend the construction of a “white elephant” facility that serves none of the demands^[12].

2.2 DEA model

DEA evaluates the relative efficiencies of a homogeneous set of Decision Making Units (DMUs)^[7]. Since Charnes et al. proposed this model^[7], it has been widely used to evaluate the relative efficiency of DMUs such as healthcare, military, banks, university facilities. Fisher and Rushton^[6], Desai and Storbeck^[13], and Athanassopoulos and Storbeck^[14] applied DEA to measure the relative spatial efficiency of location decisions. As part of their DEA models, they used two measures of access as input variables: the total travel distance and the extent of non-coverage (i.e., population outside the service distance of a facility). Based on the concept of “locational benchmarking”, Shroff et al.’s^[15] sited long-term care facilities. They used DEA to measure the relative efficiencies of potential service regions. In DEA the efficiencies of DMUs are computed using a mathematical program involving weights attached to input/output measures. To gain productivity, each DMU selects a set of weights that maximize its individual efficiency. A more detailed exposition of the theory and models of DEA is available in Charnes and Cooper^[7], Seiford and Thrall^[16], Ali and Seiford^[17], and Charnes et al.^[18].

A major “drawback” of the traditional DEA-formulation is that there must be a separate formulation for each individual DMU. Here we formulate a combined model that can assess the efficiency of several DMUs all at once. We call this the multi-alternative DEA (MADEA). To complete this re-formulation, we define a new variable d_j ($j = 1, 2, \dots, m$) as a measure of “inefficiency”. It carries a value bounded by $[0, 1)$. The smaller the value d_j , the less inefficient (and thus the more efficient) a facility-location is. It is known from the theory of DEA that a DMU_j is efficient if and only if $d_j = 0$. Similar to the public-service facility-location model above, we use s_j ($j = 1, 2, \dots, m$) to denote a binary variable, indicating the selection of an alternative in the output. If a siting alternative j will not be included into the output, $s_j = 0$; otherwise it is unitary-valued. Let b_{kj} denote the amount of k th benefit of the j th alternative and c_{lj} the l th cost for the j th alternative. Let u_{kj} denote the weight placed on the k th benefit of the j th alternative, and the weight v_{lj} for the l th cost for the j th alternative. To assess the efficiency of all DMUs simultaneously, the multi-alternative DEA model can be specified as follows^[12,19–20]:

$$\begin{aligned} & \min \sum_{j=1}^m d_j \\ \text{s.t.} \quad & \sum_{l=1}^L v_{lj} c_{lj} = s_j, \quad j = 1, 2, \dots, m \end{aligned} \quad (2)$$

$$\begin{aligned} & \sum_{k=1}^K u_{kj} b_{kj} + d_j = s_j, \quad j = 1, 2, \dots, m \\ & \sum_{k=1}^K u_{kr} b_{kj} - \sum_{l=1}^L v_{lr} c_{lj} \leq 0, \quad j = 1, 2, \dots, m; r = 1, 2, \dots, m \\ & u_{kj} - Ms_j \leq 0, \quad u_{kj} \geq \varepsilon s_j, \quad j = 1, 2, \dots, m; \quad k = 1, 2, \dots, K \end{aligned} \quad (3)$$

$$v_{lj} - Ms_j \leq 0, \quad v_{lj} \geq \varepsilon s_j, \quad j = 1, 2, \dots, m; \quad l = 1, 2, \dots, L \quad (4)$$

where ε is an infinitesimal value. Again, we specify p facilities via Equation (1).

The first set of constraints acts as scaling equations, which ensure that the weights are 0-1

ranged. The second set of constraints is used to compute the efficiency score of the facilities under consideration. Since costs are bigger than benefits, the third set of constraints assures that the efficiency score for any efficient facility cannot exceed a value of unity. Finally, the constraints that make the whole apparatus work are the weight-forcing constraints. The constraints are “either-or” in nature and assure that the weights for a facility cannot be greater than zero unless the facility is open ($s_j = 1$). In addition if the facility is open, it assures that each weight will be greater than a very small number ε . In short, the weight-forcing constraints effect the triggering mechanism of the binary siting variable s_j .

2.3 Combined DEA/location formulation

Here, we will combine the DEA and location models by mixed-binary-programming. The combined-model looks like^[12]:

$$\min \sum_{j=1}^m d_j \quad (5)$$

$$\text{s.t.} \quad \sum_{l=1}^{L-1} v_{lj} c_{lj} + \sum_{i=1}^n v_{lij} d_{ij} = s_j, \quad j = 1, 2, \dots, m \quad (6)$$

$$\sum_{k=1}^K u_{kr} b_{kj} - \sum_{l=1}^{L-1} v_{lr} c_{lj} - \sum_{i=1}^n v_{Lir} d_{ij} \leq 0 \quad (7)$$

Here, the second and third are the expanded constraints accounting for spatial and non-spatial factors.

Introducing the binary ‘marker’ variable $m_{ij} = \{0, 1\}$ to register the locational-cost of alternative j in serving demand i (if any)

$$m_{ij} - x_{ij} \leq 0 \quad (8)$$

$$v_{Lij} \leq M m_{ij} \quad (9)$$

$$v_{Lij} \geq \varepsilon m_{ij} \quad (10)$$

$$\sum_{j=1}^m x_{ij} = 1 \quad (11)$$

The next set of constraints suggests the demand cannot be served unless facility is in place:

$$\sum_{i=1}^n x_{ij} - (n - p + 1) s_j \leq 0 \quad (12)$$

The remaining facility-location constraints ensure the maximum distance is minimized:

$$\sum_{j=1}^m d_{ij} x_{ij} \leq d_{ij} s_j + (1 - s_j) M \quad (13)$$

$$d'_j - d_{ij} x_{ij} + M s_j \geq M \quad (14)$$

$$d_{ij} m_{ij} - d'_j + M(1 - m_{ij}) \geq 0 \quad (15)$$

In addition, we specify p facilities via Equation (1), and we add weight-scaling equation set (2). Also, weight-forcing constraints (3) and (4) are included. It is important to note that there are only three sets of binary variables, the x_{ij} , s_j and the m_{ij} . These binary variables are simply used to select a facility and to mark the $i - j$ pair concerned.

3 A bicriteria efficiency-equity model

3.1 Equity measure in facility location

Key to a successful location decision is equity among the constituent groups. Correspondingly, the inclusion of equity in location models has been an area of recurring interest. Beginning with the work of Morrill et al.^[21], McAllister^[22] and Savas^[23], a plethora of models have been developed to incorporate equity in facility siting. Of all the summary measures of inequity, the Gini coefficient or Gini index (circa 1912) may be the most widely recognized in the economic and social welfare literature. Recently, the Gini coefficient has also found its way to the location literature^[24–25]. Typically it is defined in terms of the Lorenz curve^[26], essentially a cumulative frequency plot.

In this paper, we employ the Gini coefficient as the index of equity. The Gini coefficient satisfies the principle of transfers and results in models that are analytically tractable. When referenced against a Lorenz curve^[9], the Gini coefficient is defined as a ratio. It is the ratio of the area between the line of perfect equity (the 45-degree line) and the Lorenz curve to the total area below the line of perfect equity (the right triangle). Thus in Figure 1, the further below the diagonal the Lorenz curve is, the farther away from perfect equity is the Gini coefficient, and the greater the ratio is. Hence, a Gini coefficient of unity represents absolute inequity, and a value of zero is perfect equity.

For a facility-location problem, we would rank order all demand nodes by the total distance traveled by the customers to their nearest facilities. For each rank, we calculate the proportion of the customers that ranks below. Similarly, we compute the proportion of the total distance traveled by the customers at that node or below, or the cumulative share of miles of travel from that node. Let us define “travel impedance” from a node as the sum of the travel distances to the facilities, expressed in terms of the total miles traveled. The travel distance from node k (or accessibility) to all facility is $\sum_j d_{kj}x_{kj}$. The total demands in the network is simply $\sum_i A_i$. Total person-miles-of-travel in the network from all demand nodes i is

Adapted from Wikipedia, the free encyclopedia

Figure 1 Graphical representation of the Gini coefficient

$$\sum_i \sum_j A_i d_{ij} x_{ij} \quad (16)$$

Likewise, total person-miles-of-travel in the network from demand nodes k is $\sum_k \sum_j A_k d_{kj} x_{kj}$. Notice these two figures may not be the same, since the valuations on x are likely to be different for the node set $\{i\}$ and the set $\{k\}$. The total distance traveled per capita is $(\sum_i \sum_j A_i d_{ij} x_{ij}) / \sum_i A_i$.

The travel distance or accessibility for demand node h is $\sum_j d_{hj} x_{hj}$. If node i is higher in ranking than k in travel accessibility, the difference in travel accessibility from node i vis-à-vis node h is then $(\sum_j d_{ij} x_{ij} - \sum_j d_{hj} x_{hj})$. The difference in total travel distance or accessibility is $\sum_i \sum_h (\sum_j A_i d_{ij} x_{ij} - \sum_j A_h d_{hj} x_{hj})$. The absolute difference in distance traveled per capita is $\sum_i \sum_h |\sum_j A_i d_{ij} x_{ij} - \sum_j A_h d_{hj} x_{hj}| / (\sum_i A_i)^2$. Correspondingly, the Gini coefficient can be written as the ratio of the difference between nodes i and h and the total distance per capita:

$$G = \frac{\sum_i \sum_h \left| \sum_j A_i d_{ij} x_{ij} - \sum_j A_h d_{hj} x_{hj} \right|}{2 \left(\sum_i A_i \right)^2 \left(\sum_i \sum_j A_i d_{ij} x_{ij} / \sum_i A_i \right)} \quad (17)$$

3.2 Bicriteria model

The combination of the above DEA/location model with the Gini equity measurement results in the following multi-objective formulation. We wish to minimize both inefficiency and inequity through two criterion functions. On top of minimizing the inefficiency criterion (5), we introduce an equity criterion G , which is to be minimized:

$$\min G,$$

where G is defined by Equation (17).

It is important to note that criterion (5) measures efficiency and equity from an individual standpoint, namely, from the point of view of individual demands i and individual facilities j — that each facility is efficient in serving demands, and that each demand is taken care of equitably. In the former case, the efficiency of each facility is maximized through the DEA model. In the latter case, each demand can access a facility within reasonable distance, including far away demands.

Aside from an individual vantage point, it is useful to examine a group of demands. The most efficient way to serve group $\{i\}$ is measured in terms of Equation (16), total miles of travel for the group. Using the aggregate measure of accessibility, one can examine which group is closer to service facilities vs. which group is further away. We like to take care of the group that has less access to service. Criterion (17) ensures that the service gap between those who are privileged and those who are under-served is narrowed.

The constraints of the bicriteria model are simply the combination of the ones we have discussed in some length earlier:

- p facilities are to be located in accordance with Equation (1).

- the weight-scaling equation set (2)
- weight-forcing constraints (3) and (4)
- the expanded constraints (6) and (7), accounting for spatial and non-spatial factors
- Equations (8), (9), (10) and (11) to register the locational-cost of alternative j in serving demand i (if any)
- precedence constraint (12)
- constraints (13) through (15) that ensure the maximum distance is minimized:

Only two new constraints are added in this bicriteria model, linking the site selection variable s_j and the benefit/cost weights u_{kj}/v_{lj} .

$$u_{kj} \geq \varepsilon s_j,$$

$$v_{lj} \geq \varepsilon s_j.$$

It is interesting that the binary variables s_j (and x_{ij}) are here to effect selection between the efficiency and equity sub-models, with s_j serving as the key variable that links the two sub-models. These binary variables provide a base from which issues of efficiency and equity can be simultaneously included in facility-location models.

4 Discussion

There are two traditional criteria in locating facilities. One is the minimax criterion and the other is the minisum criterion. The minimax (or min-max) criterion results in a ‘center’, wherein the farthest demand is to be brought as close to the service facility as possible. The minisum (or min-sum) criterion, on the other hand, results in a ‘median’, a facility that is as close to the demands as possible on the average.

As in a centdian model, the Bicriteria Efficiency-Equity Model considers both figures-of-merit: min-max and min-sum. However, more thoughts are given here to a spatial interpretation of center and median in terms of economic principles. The combined DEA/location model sites public facilities following a center proximity measure. The Gini coefficient — after proper transformation — can be shown to locate facilities according to a median model. Some interesting economic and mathematical properties can be gleaned viewing the multi-criteria model from this angle.

In the DEA/location model, we wish to have the cost-benefit analysis of DEA include explicitly the spatial dimension. In so doing, the distance function is now truly a composite of both physical distance-separation and benefit measures. Conversely, embedded in the inherent min-max proximity measures are other cost/benefit metrics consistent with economic principles governing a production process^[12]. Unlike other DEA models, the convexity of input-output combinations is relaxed here. The combined location/DEA model assumes only disposability of input/output's, representing a more flexible formulation. Among the challenges of the combined model is its nonlinearity, which arises from the min-max location model. The non-linearity

difficulty is ameliorated by an innovative model formulation, which lends itself to integration with the DEA portion.

It is also to be noted that the center location component of the Location/DEA model has given due equity consideration to individual demands, ensuring that the furthest demand is to be taken care of by a proximal facility. This is performed within a DEA, which is inherently a prescriptive model for maximal efficiency. As will be shown below, the Gini coefficient, as defined in Equation (16) is really a variation of the median. By having the cost-benefit analysis of DEA include explicitly the spatial dimension, as shown in Equations (6) and (7), facilities are located most efficiently. Locating facilities by minimizing the sum of the distances to demands effect an efficient operation (for the “common good”). What is interesting is that it is done with due consideration to equity among geographic population groups, as can be seen immediately below.

Between the two criterion functions, the equity Gini-coefficient can be folded into a constraint by way of the “constraint-reduced feasible region” method — setting aside for the time being the integrality considerations. Following this method, Steuer^[27] suggested parametrically setting a ceiling for G and optimize using the remaining “efficiency” criterion, turning the model into a single-objective model in the process. Such a procedure, when executed parametrically, will trace out the non-inferior frontier, representing the tradeoff between efficiency and equity.

Following this line of thought, the equity criterion can now be transformed into a constraint equivalent to the min-sum (median) function, when the Gini-coefficient G is set to different ceilings parametrically.

$$\sum_i \sum_h A_i A_h \left| \sum_j d_{ij} x_{ij} - \sum_j d_{hj} x_{hj} \right| \leq K \sum_i A_i \sum_j d_{ij} x_{ij},$$

where K is a parametric constant. To appreciate the absolute value specified in this expression, let $y^* = \sum_i \sum_h A_i A_h \sum_j d_{ij} x_{ij}$ and $y = \sum_i \sum_h A_i A_h \sum_j d_{hj} x_{hj}$. Furthermore, let us define these deviational variables:

$$d^+ = \begin{cases} y - y^*, & \text{if } y > y^*, \\ 0, & \text{otherwise.} \end{cases}$$

$$d^- = \begin{cases} y^* - y, & \text{if } y < y^*, \\ 0, & \text{otherwise.} \end{cases}$$

Then $y^* - y = d^- - d^+$, and $|y^* - y|d^- + d^+$. Correspondingly, we can write the equity expression (with its absolute value sign) as:

$$d^- + d^+ \leq K \sum_i A_i \sum_j d_{ij} x_{ij}.$$

This constraint, complete with the travel control total of $\sum_i A_i \sum_j d_{ij} x_{ij} = \sum_i \sum_j A_i d_{ij} x_{ij}$, is a classic min-sum distance measure used in median location models. Another insight is that we are clearly minimizing the deviational variables d^- and d^+ , which represent the inequities between the accessibilities from disparate parts of the study area.

This model generalizes the classic median and center solutions, which minimize only simple scalar characteristics: the maximal distance and the average distance, respectively. The metrics

are extended here to include individual and group considerations. The respective metric — d_j and G — measures both efficiency and equity. In the following example, we will show how the bicriteria model trades off individual against group considerations.

5 Example

In this section, we illustrate the use of the proposed model. The aim is to demonstrate the advanced feature of our approach and its practical relevance. For this purpose, we will consider a network of fire stations and site several fire stations from seven candidate sites. There is a total of 10 demand zones from which fire service are generated. Three inputs variables, including the distances used in the min-max criterion. Two output variables are used to evaluate the relative efficiency of candidate locations. Table 1 lists the population (“customers”) at each of the 10 demand nodes. Table 2 contains the input-output vectors for each of the candidate facilities. Table 3 gives the distances between facilities and demands^[19].

Table 1 Population at each demand node

Demand Node	1	2	3	4	5	6	7	8	9	10
Population	8	18	15	20	32	24	48	39	18	27

Table 2 Input/output vectors for each facility

Candidate sites	Input 1	Input 2	Output 1	Output 2
F1	76	69	38	33
F2	38	49	75	88
F3	43	48	59	53
F4	28	36	99	82
F5	66	43	31	34
F6	50	65	29	39
F7	60	76	34	38

Table 3 Distance matrix for facilities to demands

Sites	Demands									
	1	2	3	4	5	6	7	8	9	10
F1	56	84	82	50	72	70	51	65	64	45
F2	36	88	98	100	65	57	92	88	44	77
F3	92	59	50	48	61	108	43	42	94	86
F4	72	39	37	83	22	103	66	34	81	88
F5	52	79	72	66	55	68	56	61	69	55
F6	43	122	115	118	106	51	109	108	45	69
F7	67	127	104	109	116	50	85	102	57	75

The facility-location model was run with the data discussed above and yielded the following results, as shown in Table 4. In Table 4 we solved for the minimum total sum of inefficiency (pure efficiency) and minimum inequity (pure equity) of different number of open facilities. For

example, suppose the number of open facilities is four and the relative weight on efficiency is 1 (with no regard for equity). Sites F1, F2, F3, F4 are chosen to serve the demands. The total sum of inefficiency scores is 0.2367 and the Gini coefficient score is 0.1274. Now suppose the number of open facilities is four and the relative weight on the Gini coefficient is 1 (with no regard to the efficiency score $\sum_j d_j$). Then sites F1, F3, F5, F6 are chosen to serve the demands. The total sum of inefficiency scores is 1.9101 and the Gini coefficient score is 0.0627.

Table 4 Sample solution for the bicriteria efficiency-equity model

Number of open facilities	$p = 3$		$p = 4$		$p = 5$	
Relative weight on DEA	1	0	1	0	1	0
Relative weight on Equity	0	1	0	1	0	1
Open facilities	2, 3, 4	3, 5, 6	1, 2, 3, 4	1, 3, 5, 6	1, 2, 3, 4, 7	1, 3, 5, 6, 7
Total sum of inefficiency scores for open facilities	0	1.4591	0.2367	1.9101	0.7202	2.4556
Inefficiency scores per station	0	0.4864	0.0592	0.4775	0.1440	0.4911
Gini coefficient score for open facilities	0.1820	0.0674	0.1274	0.0627	0.1141	0.0618

A series of multicriteria solutions for different number of open facilities was obtained by varying the relative weight on the DEA efficiency measure and the Gini coefficient. Figure 2 shows the tradeoff between a total sum of inefficiency scores and the Gini coefficient. It appears that increasing the number of fire stations improve both efficiency and equity for the individuals and the groups. This is an important result, since it identifies non-inferior solutions for the good of individuals and for the group.

While most tradeoffs are conducted between two criteria for the same target audience, the current analysis reveals the tradeoff between individuals and groups. Within the “cone” shown in Figure 2 contain the trajectories of facility expansion paths that will be appealing to both

Figure 2 Tradeoff of total sum of inefficiency scores and Gini coefficient

Figure 3 Tradeoff of total sum of inefficiency scores per fire station and Gini coefficient

the individuals and the public in general — the win-win solutions. In this case, the lower border of the cone is defined by the following equation:

$$-0.7548d + 0.2452G = 0,$$

while the upper border is defined by this equation:

$$-0.1161d + 0.8839G = 0.$$

Available computer runs suggest that the cone is obtuse, meaning that it is robust enough to take many different weighting between the DEA efficiency measure and the Gini coefficient. This is a desirable feature in a pluralistic society, in which there are many opinions regarding the importance of individual rights vs. common good. More important, tradeoffs between individuals and groups become more gentle as the number of fire stations increases in Figure 2. In other words, as we follow the trajectory of facility expansion, the individual does not need to sacrifice as much for the common good and vice versa^[26].

6 Conclusion

A variety of facility-location models have been developed to site public facilities. The various location criteria include cost, time, and coverage. In this paper, we develop a bicriteria model that simultaneously optimizes overall efficiency and equity for the individuals and for the groups. For the individual and for the group, the model incorporates the combined measures of efficiency and equity through the spatial version of DEA and through the Gini coefficient respectively. In this way we have been able to identify the tradeoffs of overall efficiency and equity between individuals and groups. The tradeoffs can be related to the familiar center vs. median locational philosophies, with the former equated with equity and the latter efficiency. The more important result is the explicit tradeoffs between individuals vs. groups.

Computational results to date suggest that such tradeoffs between individuals and groups may be substantial when there is only a limited number of facilities. The larger the number of facilities, the less sensitive is the tradeoff between the two target audiences. The ability to

use such a multi-criteria model in pluralistic decision-making provides a promising approach for public service location analysis.

There are two issues with our formulation that require more study. While measuring efficiency is more straightforward, not all users will view the Center Location model and Gini coefficient as appropriate measures of equity. An important area for future inquiry, is the development of models that satisfy the principle of transfers. Second, the model is constructed under the assumption of the “all-or nothing” rule, under which customers always go to the nearest public service facility. This does not necessarily reflect the real world, when the second closest facility is often frequented. A more refined approach should be established if the location model is seriously considering more realistic usage patterns.

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