Identification of Equivalent Elastic and Damping Parameters for Roll Core Sandwich Panels of Railway Vehicles

Kin'ya Matsumoto*

Mie University, 1515, Kamihama-cho, Tsu, Mie 514-8507, Japan

Kenji Hosokawa

Chubu University, 1200, Matsumoto-cho, Kasugai, Aichi 487-8501, Japan

Toshiaki Makino, Takeshi Kawasaki

Hitachi Ltd., 794, Higashitoyoi, Kudamatsu, Yamaguchi 744-8601, Japan

and

Hiroaki Ohtsubo

Hokuyo Paper Ltd., 1-3-9, Kogaba, Iwaki, Saitama 339-0072, Japan

ABSTRACT

Since composite materials such as roll core sandwich panels have high specific strength, high structural efficiency and durability, in recent years they have been used in many structural applications of railway vehicles. It is therefore very important for the design and the structural analysis to make clear the dynamic properties of the roll core sandwich panels. Especially, elastic and damping parameters are essential for dynamic analyses. On the other hand, an inverse analysis method has already been proposed by one of the authors to identify equivalent parameters of composite materials using the FEM eigenvalue analysis. The purpose of this paper is to apply the proposed method to a roll core sandwich panel, and to identify the equivalent elastic and damping parameters of the panel. First, by applying the experimental modal analysis technique to a roll core sandwich square panel, natural frequencies and modal damping ratios of the panel are obtained. Next, from the natural frequencies and modal damping ratios, the equivalent elastic and damping parameters of the panel are identified numerically.

Key words: Vibration, Elastic and Damping Parameters, Identification, Composite Materials, FEM, Strain Energy Method, Eigenvalue Analysis, Sandwich Panels

1. INTRODUCTION

Aluminum roll core sandwich panels are adopted for railway vehicles as self-supported interior modules on account of their high stiffness and strength. The module is one of the elements for a new concept of the car body system integration to compose the high performance railway cars with aluminum double skin body shells. The roll core is made of meandered paper filled with phenol form as illustrated in Figure 1. This unique style of the core enables the panel to bend, unlike other kinds of sandwich panels.

* Corresponding author :
Associate Professor Kin'ya Matsumoto
Faculty of Education, Mie University
1515, Kamihama-cho, Tsu, Mie 514-8507, Japan.
Tel. : +81-59-231-9309 ; Fax : +81-59-231-9352 ;
e-mail : matumoto@edu.mie-u.ac.jp

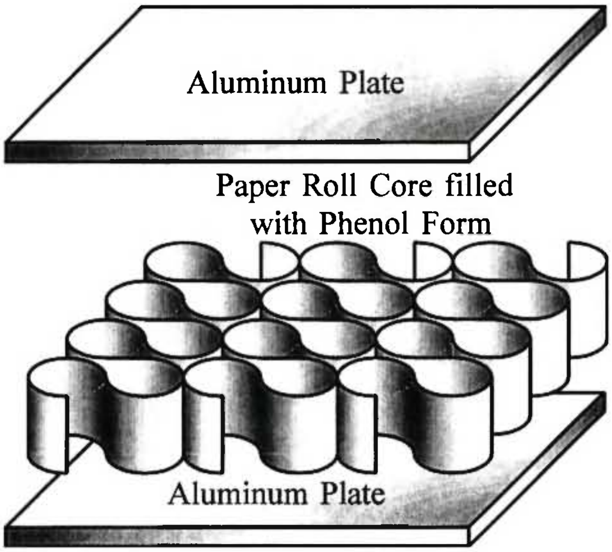


Fig. 1: Structure of roll core sandwich panel

Generally, sandwich panels have relatively soft cores between two hard surface plates, whose layered system confers high stiffness and strength. But the complex structure of sandwich panels also makes it difficult to measure the material properties by the usual experimental methods. Especially, it seems impossible to extract the transverse shear moduli, which are key parameters to fix the elastic behavior of panels. Furthermore, the damping properties are indispensable to obtain the vibrational characteristics of structures. Therefore, developing an evaluation technique of these elastic and damping parameters is required.

For this requirement, some identification methods of elastic and damping parameters from vibration data are proposed. Gladwell surveys literature relating to the inverse problems in vibration /1/. The identification methods of elastic parameters are presented /2-7/. Qian *et al.* propose an identification method of elastic and damping parameters of laminated composites /8/. A paper about parametric determination method of honeycomb sandwich panels has been published by Saito *et al.* /9/.

The main purpose of this study is to propose the inverse analysis method to identify equivalent elastic and damping parameters of sandwich panels regarding them as single layered shells. In this paper, the relations between equivalent parameters and such structural properties (natural frequencies or modal damping ratios) are considered as nonlinear systems. So the identification of equivalent parameters can be converted to a nonlinear optimization problem. The quasi-Newton method is employed for the optimization algorithm to solve these nonlinear systems. A shell element using the first-order shear deformation theory is formulated for the modeling of sandwich panels. Modal damping ratios of the sandwich panels are calculated by the strain energy method proposed by Adams *et al.* /10/.

Excitation tests of a square sandwich panel are carried out and the natural frequencies and modal damping ratios are measured. The equivalent elastic and damping parameters are identified using the measured properties. With these identified parameters natural frequencies and modal damping ratios are calculated and compared with the results of excitation tests.

2. THEORY

The stress and strain relation matrix of the r-th orthotropic lamina in the material principle direction LTV (see Figure 2) is given by

$$Q_r = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & Q_{66} \end{bmatrix}_r \quad (1)$$

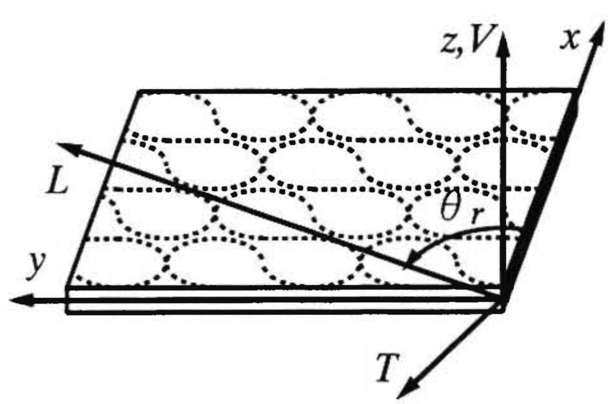


Fig. 2: Coordinate system of orthotropic plate

The stiffness matrix in the direction of the elemental coordinate axes xyz is given by transforming the stiffness matrix $\boldsymbol{Q}_r$ with the transfer matrix $\boldsymbol{T}_r$ calculated by the orientation angle.

$$\overline{\boldsymbol{Q}_r} = \boldsymbol{T}_r^T \boldsymbol{Q}_r \boldsymbol{T}_r \qquad (2)$$

For unsymmetrically laminated sandwich panels, we can obtain the generalized stress and strain relation matrix of a finite shell element model using first-order shear deformation theory.

$$\boldsymbol{D} = \begin{bmatrix} \boldsymbol{D}_p & \boldsymbol{D}_c & 0 \\ \boldsymbol{D}_c & \boldsymbol{D}_b & 0 \\ 0 & 0 & \boldsymbol{D}_s \end{bmatrix} \qquad (3)$$

where the component matrices are

$$\boldsymbol{D}_p = \sum_{r=1}^{N_{TL}} \int_{h_{r-1}}^{h_r} \overline{Q_{ijr}}\, dz \quad (i,j=1,2,6) \qquad (4)$$

$$\boldsymbol{D}_c = \sum_{r=1}^{N_{TL}} \int_{h_{r-1}}^{h_r} z\overline{Q_{ijr}}\, dz \quad (i,j=1,2,6) \qquad (5)$$

$$\boldsymbol{D}_b = \sum_{r=1}^{N_{TL}} \int_{h_{r-1}}^{h_r} z^2\overline{Q_{ijr}}\, dz \quad (i,j=1,2,6) \qquad (6)$$

$$\boldsymbol{D}_s = \sum_{r=1}^{N_{TL}} k \int_{h_{r-1}}^{h_r} \overline{Q_{ijr}}\, dz \ (i,j=4,5) \qquad (7)$$

where z is the distance from the neutral surface to each lamina and N_{TL} is the total number of laminae.

Adams *et al.* reported that the specific damping capacity (SDC) of the r-th lamina of unidirectional composite materials can be expressed as a diagonal matrix in the form with

$$\boldsymbol{\Psi}_r = \begin{bmatrix} \psi_L & 0 & 0 & 0 & 0 \\ 0 & \psi_T & 0 & 0 & 0 \\ 0 & 0 & \psi_{TV} & 0 & 0 \\ 0 & 0 & 0 & \psi_{VL} & 0 \\ 0 & 0 & 0 & 0 & \psi_{LT} \end{bmatrix}_r \qquad (8)$$

where the component ψ_{ij} is the specific damping capacity of each strain direction.

They also suggested that components of the SDC matrix can be measured by vibration tests for some kinds of unidirectional specimens with various fiber orientations.

The energy dissipated during a cycle of the n-th modal vibration for laminated composite shells is calculated as follows.

$$\Delta U_n = \frac{1}{2}\int_S \boldsymbol{\varepsilon}_n^T \boldsymbol{\Psi} \boldsymbol{\varepsilon}_n\, ds \qquad (9)$$

Since $\boldsymbol{\Psi}$ is the SDC matrix of a shell element written by

$$\boldsymbol{\Psi} = \begin{bmatrix} \boldsymbol{\Psi}_p & \boldsymbol{\Psi}_c & 0 \\ \boldsymbol{\Psi}_c & \boldsymbol{\Psi}_b & 0 \\ 0 & 0 & \boldsymbol{\Psi}_s \end{bmatrix} \qquad (10)$$

where the components of $\boldsymbol{\Psi}$ are

$$\boldsymbol{\Psi}_p = \sum_{r=1}^{N_{TL}} \int_{h_{r-1}}^{h_r} \tilde{\psi}_{ijr}\, dz \quad (i,j=1,2,6) \qquad (11)$$

$$\boldsymbol{\Psi}_c = \sum_{r=1}^{N_{TL}} \int_{h_{r-1}}^{h_r} z\tilde{\psi}_{ijr}\, dz \quad (i,j=1,2,6) \qquad (12)$$

$$\boldsymbol{\Psi}_b = \sum_{r=1}^{N_{TL}} \int_{h_{r-1}}^{h_r} z^2\tilde{\psi}_{ijr}\, dz \quad (i,j=1,2,6) \qquad (13)$$

$$\boldsymbol{\Psi}_s = \sum_{r=1}^{N_{TL}} k \int_{h_{r-1}}^{h_r} \tilde{\psi}_{ijr}\, dz \quad (i,j=4,5) \qquad (14)$$

Since $\tilde{\psi}_{ijr}$ are the components of the energy-dissipated matrix $\tilde{\boldsymbol{\Psi}}_r$, which is calculated by the SDC matrix in equation (8) transforming into the direction of the elemental coordinate axes.

$$\tilde{\boldsymbol{\Psi}}_r = \boldsymbol{T}_r^T \boldsymbol{\Psi}_r \boldsymbol{Q}_r \boldsymbol{T}_r \qquad (15)$$

The maximum strain energy and energy dissipated during a cycle of the n-th modal vibration is given by

$$U_n = \frac{1}{2}\boldsymbol{\delta}_n^T \boldsymbol{L}^T \int_S \boldsymbol{B}^T \boldsymbol{D}\boldsymbol{B}\, ds\, \boldsymbol{L}\boldsymbol{\delta}_n \tag{16}$$

$$\Delta U_n = \frac{1}{2}\boldsymbol{\delta}_n^T \boldsymbol{L}^T \int_S \boldsymbol{B}^T \boldsymbol{\Psi}\boldsymbol{B}\, ds\, \boldsymbol{L}\boldsymbol{\delta}_n \tag{17}$$

where $\boldsymbol{\delta}_n$ is the n-th modal vector, $\boldsymbol{B}$ is the displacement-strain relation matrix and $\boldsymbol{L}$ is the transfer matrix of elemental coordinates.

Finally, the n-th modal damping ratio is calculated as the ratio of the energy dissipated and the maximum strain energy,

$$\zeta_n = \frac{1}{4\pi}\cdot\frac{\Delta U_n}{U_n} \tag{18}$$

Regarding the relationship between natural frequencies and elastic parameters or between the SDC matrix and the modal damping ratios as a nonlinear system, the quasi-Newton method can be used for identifying the equivalent parameters of the material principle direction. We define the error function as the difference between the vibrational properties f_{En} measured by the excitation test and calculated ones $f_n(\boldsymbol{x})$ by the eigenvalue analysis, as follows /11/:

$$g_n(\boldsymbol{x}) = f_{En} - f_n(\boldsymbol{x}) \tag{19}$$

Then, the identification is considered as a nonlinear optimization problem to find a solution $\boldsymbol{x}^*$ that minimizes the error norm $\Phi(\boldsymbol{x})$.

$$\Phi(\boldsymbol{x}) = \frac{1}{2}\sum_{n=1}^{N_{TM}} g_n(\boldsymbol{x})^2 \tag{20}$$

where N_{TM} is the total number of referring modes.

The quasi-Newton method takes an initial approximation x_0, and attempts to improve x_0 by the iteration formula using a search direction vector d and a step size parameter λ.

$$\boldsymbol{x}_{k+1} = \boldsymbol{x}_k + \lambda_k \boldsymbol{d}_k \tag{21}$$

A step size parameter λ is chosen by the line searcher algorithm, and a search direction vector $\boldsymbol{d}$ can be given as a solution of the equation followed by

$$\boldsymbol{H}_k \boldsymbol{d}_k = -\nabla\Phi(\boldsymbol{x}_k) = -\boldsymbol{J}^T(\boldsymbol{x}_k)\boldsymbol{g}(\boldsymbol{x}_k) \tag{22}$$

where $\boldsymbol{H}$ and $\boldsymbol{J}$ are the Hessian and Jacobian matrices of error function $\boldsymbol{g}(x_k)$, respectively. Figure 3 indicates the flowchart of the identification program.

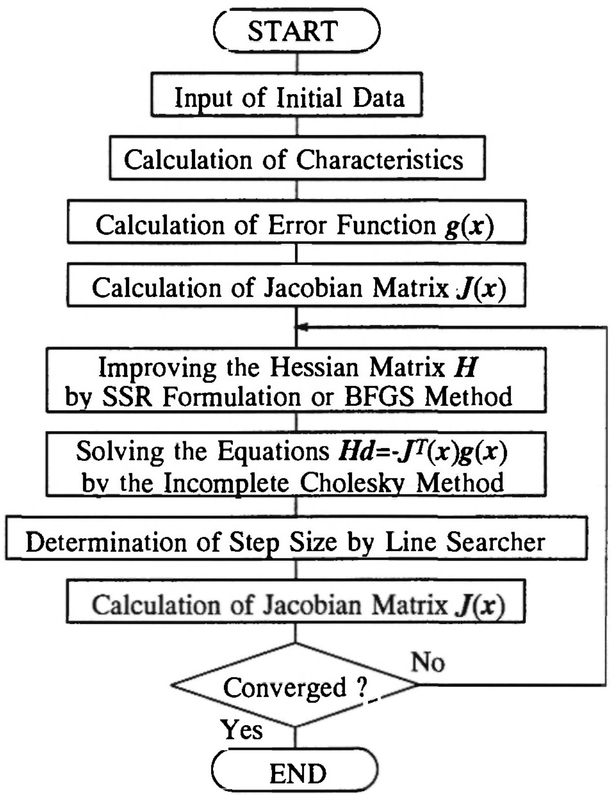


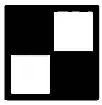
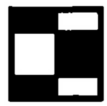
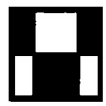
Fig. 3: Flowchart of the identification program

3. PARAMETER IDENTIFICATION OF ROLL CORE SANDWICH PANEL

To measure natural frequencies and modal damping ratios of a roll core sandwich panel impact tests are carried out. In addition, the elastic and damping parameters of the panel are identified by the developed program.

The specimen for impact tests is a roll core sandwich square panel of size 0.3×0.3×0.04m. The aluminum surface plates have a thickness of 0.5mm and the roll the calculated characteristics and measured ones with errors of less than 1% in natural frequencies and of no more than ±10% in damping ratios.

Table 1
Comparison between experiment and analysis
with identified parameters of the roll core sandwich square panel

Modal number		1st	2nd	3rd	4th	5th	6th
Modal shape							
Natural frequency	Experiment, Hz	904	1300	1520	1580	1720	2040
	Analysis, Hz	900	1292	1518	1576	1714	2022
	Error, %	-0.4	-0.6	-0.1	-0.3	-0.3	-0.9
Damping ratio	Experiment, %	0.478	0.690	0.832	0.789	0.867	0.835
	Analysis, %	0.516	0.730	0.870	0.721	0.797	0.796
	Error, %	7.9	5.8	4.6	-8.6	-8.1	-4.7

core cells made of aluminum hydroxide paper have a diameter of 14mm. The sandwich panel suspended by fine threads is excited by an impact hammer and the response acceleration is measured by a tiny pickup sensor. The natural frequencies and the modal damping ratios of the square sandwich panel obtained by the excitation tests are shown in Table 1.

The equivalent elastic and damping parameters of the roll core sandwich panel are identified in order for the analyzed results to agree with experimental ones. The identified parameters are shown in Table 2.

The comparison between analyzed properties and measured ones of the square sandwich panel is obtained in Table 1. These results show good agreement between

Table 2
Identified elastic and damping parameters of the roll core sandwich square panel

Elastic parameter [GPa]	E_L	E_T	G_{TV}	G_{VL}	G_{LT}
	6.84	5.14	0.034	0.047	1.98
Damping parameter SDC	ψ_L	ψ_T	ψ_{TV}	ψ_{VL}	ψ_{LT}
	0.159	0.048	0.110	0.091	0.0001

Using these identified elastic and damping parameters of the roll core sandwich square panel (shown in Table 2), natural frequencies and modal damping ratios of a roll core curved shell are calculated, and the calculated characteristics are compared with the results of experiments. Figure 4 shows the test specimen of a roll core sandwich curved shell, which is a part of a railway car's interior modules with the same structure as the sandwich square panel.

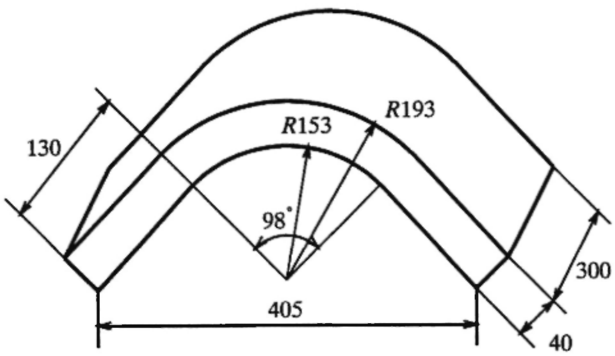


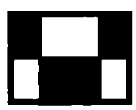
Fig. 4: Test specimen of roll core sandwich curved shell

A comparison between the experiment and the calculations is shown in Table 3. In the second mode of the natural frequencies and the third mode of the modal damping ratios, there are great differences between estimated values and measured ones. This suggests that the sandwich curved shells have different characteristics from the sandwich plane panels.

4. CONCLUSIONS

An method of identifying elastic and damping parameters for sandwich panels is proposed by using the nonlinear optimization method. Comparing the identified results with measured ones, it is shown that the

Table 3
Comparison between experiment and analysis with identified parameters of the roll core sandwich curved shell

Modal number		1st	2nd	3rd	4th	5th
Modal shape						
Natural frequency	Experiment, Hz	449	486	716	897	1130
	Analysis, Hz	447	257	687	888	1180
	Error, %	-0.4	-47.1	-4.1	-1.0	4.4
Damping ratio	Experiment, %	0.603	0.521	0.600	0.715	0.681
	Analysis, %	0.630	0.595	1.523	0.721	0.757
	Error, %	4.5	14.2	154	0.8	11.2

proposed method is capable of determining the elastic and damping parameters of sandwich panels. However, the results of the excitation tests for roll core sandwich curved shell suggest that the sandwich curved shell has different characteristics from the sandwich plane panel.

5. REFERENCES

1. G.M.L. Gladwell, Inverse problems in vibration, *ASME AMR*, 39 (7), 1013-1018 (1986).
2. L.R. Debald and R.F. Gibson, Determination of elastic constants of orthotropic plates by a modal analysis/Rayleigh-Ritz technique, *Journal of Sound and Vibration*, **124** (2), 269-283 (1988).
3. D.C. Kammer, Optimum approximation for residual stiffness in linear system identification, *AIAA Journal*, **26** (1), 104-112 (1988).
4. M. Grediac and P.A. Paris, Direct identification of elastic constants of anisotropic plates by modal analysis: Theoretical and numerical aspects, *Journal of Sound and Vibration*, **195** (3), 401-415 (1996).
5. E.O. Ayorinde and L. Yu, Towards an improved general vibration-based method for elastic identification of materials, *ASME NCA*, **24**, 259-267 (1997).
6. K. Hosokawa, K. Matsumoto and M. Zako, Identification of elastic parameters for laminated composites using FEM eigenvalue analysis, *ASME PVP*, **374**, 325-329 (1998).
7. K. Matsumoto, K. Hosokawa and M. Zako, Identification of vibrational damping parameters for laminated composite material by using FEM eigenvalue analysis, *ASME PVP*, **374**, 319-324 (1998).
8. G.L. Qian, S.V. Hoa and X. Xiao, Identification of elastic and damping properties of composite laminates from vibration test data, *Proceedings of International Conference of Composite Materials*, **10** (4), IV25-32 (1995).
9. T. Saito, R.D. Parbery, S. Okuno and S. Kawano, Parameter identification for aluminum honeycomb sandwich panels based on orthotropic Timoshenko beam theory, *Journal of Sound and Vibration*, **208** (2), 271-287 (1997).
10. D.X. Lin, R.G. Ni and R.D. Adams, Prediction and measurement of the vibrational damping parameters of carbon and glass fiber-reinforced plastics plates, *Journal of Composite Materials*, **18**, 133-152 (1984).
11. J.E. Dennis, Jr. and J.J. More, Quasi-Newton methods, motivation and theory, *SIAM Review*, **19** (1), 46-89 (1977).