Effect of Particle Rotations on the Interface Behaviour between a Granular Layer and Parallel Rough Plates

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ABSTRACT

This paper presents the results of numerical studies of plane shearing of a thin layer of a granular material located between parallel rough walls under a constant pressure. Herein the essential mechanical properties of the granular material are described using a hypoplastic Cosserat continuum approach. Along the bounding wall in motion the slide and rotation resistance of particles in contact is mainly determined by the interaction between the wall roughness, the pressure level, and the size, shape and roughness of the particles. It is demonstrated in the paper that with the so-called Cosserat rotations the rotation resistance between the granular layer and the adjoining rough walls can be modelled in a physically natural manner. The numerical investigations show that for large shearing the deformation is localized within a narrow zone parallel to the interface. The location and thickness of the localized zone is strongly influenced by the interface behaviour.

Key words: Layered material, shear localization, granular materials, micro-polar continuum, hypoplasticity.

1 INTRODUCTION

The concentration of shear deformations in narrow zones called shear bands is a well known phenomenon in granular materials like sand (e.g. Mühlhaus and Vardoulakis, 1987, Vardoulakis and Sulem, 1995, Desrues *et al.*, 1996, Gudehus, 1997, Oda *et al.*, 1997, Uesugi *et al.*, 1998, Gudehus, 1999). Location,

orientation and thickness of a shear band mainly depends upon the grain size distribution and grain shape, the stress level, initial backing density of the grain assemble, the slide and rotation resistance of grains in contact with the bounding structure and the dilatancy resistance of the whole system. The aim of this paper is to study theoretically the shear behaviour of a sandwich structure composed of a layer of a cohesionless granular material with empty voids located between two parallel stiff plates with rough walls. The bottom plate is fixed while the top plate is moved under a constant pressure parallel to the granular layer as sketched out in Figure 1. The focus of the present investigation is on modelling the rotation resistance of particles in contact with a rough surface of the bounding structure. In order to take into account particle rotations a micro-polar continuum approach or so called Cosserat continuum is proposed within the framework of hypoplasticity.

Fig. 1: Layer of a granular material located between two parallel stiff plates under a constant normal pressure

Originally, the concept of hypoplasticity was proposed by Kolymbas (1991) for a nonpolar

continuum. He formulated the mechanical behavior of a nonlinear and anelastic material by using a single tensorial function of the rate-type. In this way, the hypoplasticity differs from the concept in elastoplastic theory because it neither requires a yield surface and flow rule nor the decomposition of the deformation into elastic and plastic parts. Based on this general concept a remarkable progress has been achieved in the work by Wu and Bauer (1992), Gudehus (1996) and Bauer (1996, 2000) where so-called critical states were integrated into the hypoplastic model to describe the dependence of the material behavior on the pressure level and on the density with a single set of constitutive constants. The proposed nonpolar hypoplastic model was extended by Tejchman (1997) with quantities which are relevant for a Cosserat continuum, i.e. micro-rotations, couple stresses and the mean grain diameter as the internal length were included in the model. In the present paper a slightly modified version proposed by Bauer and Huang (2001) is used. The hypoplastic Cosserat model takes into account the influence of the pressure level, the void ratio, the mean grain diameter and the slide and rotation resistance of grains. In the present paper it is demonstrated that with the Cosserat boundary conditions the rotation resistance of particles in contact with the rough surfaces of the adjoining plates under shearing can be modelled in a physically natural manner. Due to the presence of an internal length the boundary value problems are mathematically well-posed and the predicted thickness of the shear zone is not sensitive with respect to the element size provided that it is small enough (de Borst *et al.*, 1992).

The paper is organized as follows: The hypoplastic Cosserat model by Bauer and Huang (2001) is briefly outlined in Section 2. The modelling of the influence of the particle size and wall roughness of the bounding structure on the rotation resistance of particles in contact with a rough bounding structure under shear motion is described in the following section. The results obtained from the numerical study for plane shearing of an infinite sand layer located between two parallel plates is presented in Section 4. The evolution of the displacement field, the void ratio, the micro-rotation, the stress components and couple stress components is

shown for both a very rough wall and a medium rough wall of the bounding plates.

2 HYPOPLASTIC COSSERAT MODEL

The kinematics of a Cosserat continuum is defined by the macro-displacement field u_i and micro-rotations or so-called Cosserat rotations ω^i ($i = 1; 2; 3$). The rate of deformation and the rate of curvatures are defined as:

$$\dot{\epsilon}_{ij} = \frac{\partial \dot{u}_i}{\partial x_j} + \pi_{kij} \dot{\omega}_k^i \quad (1)$$

and

$$\dot{\kappa}_{ij} = \frac{\partial \dot{\omega}_i^j}{\partial x_j} \quad (2)$$

respectively, where $\pi_{i,j,k}$ denotes the permutation tensor.

The proposed hypoplastic Cosserat model includes three state variables, i.e. the stress tensor σ , the couple stress tensor μ and the void ratio e , which is defined as the ratio of the volume of the voids to the volume of the solid grains. The evolutions of the state variables are described by the following objective rate type equations (Bauer and Tejchman, 1995, Tejchman, 1977, Tejchman and Gudehus, 2001, Bauer and Huang, 2001):

$$\sigma_{ij} = f_s [\hat{a}^{\frac{2}{3}} \dot{\epsilon}_{ij} + (\hat{\sigma}_{kl} \dot{\epsilon}_{kl} + \hat{\mu}_{kl} \dot{\kappa}_{kl}) \hat{\sigma}_{ij} + f_d \hat{a} (\dot{\sigma}_{ij} + \hat{\sigma}_{ij}^*) \sqrt{\dot{\epsilon}_{kl} \dot{\epsilon}_{kl} + \dot{\kappa}_{kl} \dot{\kappa}_{kl}}], \quad (3)$$

$$\mu_{ij} = f_s d_{s0} [a_c^{\frac{2}{3}} \dot{\kappa}_{ij} + \hat{\mu}_{ij} (\hat{\sigma}_{kl} \dot{\epsilon}_{kl} + \hat{\mu}_{kl} \dot{\kappa}_{kl} + 2 f_d a_c \sqrt{\dot{\epsilon}_{kl} \dot{\epsilon}_{kl} + \dot{\kappa}_{kl} \dot{\kappa}_{kl}})], \quad (4)$$

$$\dot{e} = (1 + e) \dot{\epsilon}_{kk}, \quad (5)$$

with the normalised quantities:

$$\begin{aligned} \hat{\sigma}_{ij} &= \sigma_{ij} / \sigma_{kk}, \\ \hat{\sigma}_{ij}^* &= \hat{\sigma}_{ij} - \delta_{ij} / 3, \\ \hat{\mu}_{ij} &= \mu_{ij} / (d_{s0} \sigma_{kk}), \\ \hat{\kappa}_{ij} &= d_{s0} \dot{\kappa}_{ij}. \end{aligned}$$

Herein δ_{ij} is the Kronecker delta and d_{50} denotes the mean grain diameter, which enters the constitutive model as an internal length. Eq. (5) can be derived from the balance equation of mass based on the assumption that the grain compressibility is negligible. Thus, the rate of the void ratio is proportional to the volume strain rate $\dot{\epsilon}_{kk}$ and the granular material can be modelled as a single component continuum. Function $\hat{a}$ in Eq.(3) and factor a_c in Eq. (4) are related to stationary stress states which can be reached asymptotically under large shearing (Huang, 2000). Factor a_c is assumed to be a constant (Tejchman and Gudehus, 2001, Bauer and Huang, 2001) and $\hat{a}$ depends on the so-called angle of internal friction φ_c (Bauer, 2000) and on the symmetric part of the normalised stress deviator $\hat{\sigma}_{kl}^{*s}$, i.e. $\hat{\sigma}_{kl}^{*s} = (\hat{\sigma}_{kl}^* + \hat{\sigma}_{lk}^*)/2$. The proposed representation for $\hat{a}$ reads:

$$\hat{a} = \frac{\sin \varphi_c}{3 + \sin \varphi_c} [\sqrt{b} + \sqrt{\hat{\sigma}_{kl}^{*s} \hat{\sigma}_{kl}^{*s}}] \quad (6)$$

with

$$b = \frac{8/3 - 3(\hat{\sigma}_{kl}^{*s} \hat{\sigma}_{kl}^{*s}) + g\sqrt{3/2} (\hat{\sigma}_{kl}^{*s} \hat{\sigma}_{kl}^{*s})^{3/2}}{1 + g\sqrt{3/2} (\hat{\sigma}_{kl}^{*s} \hat{\sigma}_{kl}^{*s})^{1/2}},$$

and

$$g = -\sqrt{6} \frac{\hat{\sigma}_{kl}^{*s} \hat{\sigma}_{lm}^{*s} \hat{\sigma}_{mk}^{*s}}{(\hat{\sigma}_{kl}^{*s} \hat{\sigma}_{kl}^{*s})^{3/2}}$$

The influence of the mean pressure and the current void ratio on the response of the constitutive equations (3) and (4) is taken into account with the stiffness factor f_s and the density factor f_d . The dilatancy behaviour, the peak stress ratio and strain softening depends on the density factor f_d which represents a relation between the current void ratio e , the critical void ratio e_c and the minimum void ratio e_d , i.e.

$$f_d = \left(\frac{e - e_d}{e_c - e_d} \right)^\alpha \quad (7)$$

where $\alpha < 0.5$ is a constitutive constant. The stiffness factor f_s is proportional to the granular hardness h_s and depends on the stress level σ_{kk} , i.e.

$$f_s = \left(\frac{e_i}{e} \right)^\beta \frac{h_s (1 + e_i)}{nh_s (\hat{\sigma}_{kl} \hat{\sigma}_{kl}) e_i} \left(-\frac{\sigma_{kk}}{h_s} \right)^{1-n}, \quad (8)$$

with

$$h_s = \frac{8 \sin^2 \varphi_c}{(3 + \sin \varphi_c)^2} + 1 - \frac{2\sqrt{2} \sin \varphi}{3 + \sin \varphi} \left(\frac{e_{i0} - e_{d0}}{e_{c0} - e_{d0}} \right)^\alpha.$$

Herein $\beta > 1$ is a constitutive constant and e_i denotes the maximum void ratio which is assumed to decrease with the mean pressure σ_{kk} according to the compression relation (Bauer, 1995):

$$e_i = e_{i0} \exp\{-(\sigma_{kk}/h_s)^n\}. \quad (9)$$

The constants e_{i0} , h_s and n in relation (9) can be adapted to an isotropic compression test starting from the loosest state of the grain material. It was postulated by Gudehus (1996) that e_c and e_d in Eq. (7) decreases with the mean pressure as e_i (Figure 2), i.e.

$$\frac{e_c}{e_{c0}} = \frac{e_d}{e_{d0}} = \frac{e_i}{e_{i0}}. \quad (10)$$

Herein e_{i0} , e_{c0} and e_{d0} denote the values of e_i , e_c and e_d for $\sigma_{kk} \approx 0$.

Fig. 2: Decrease of e_i , e_c and e_d with σ_{kk}

For the present numerical investigations the following material constants for a medium sand are used (Herle and Gudehus, 1999, Huang and Bauer, 2001):

$$\begin{aligned} e_{i0} &= 1.2, \quad e_{d0} = 0.51, \quad e_{c0} = 0.82, \quad \varphi_c = 30^\circ, \\ h_s &= 190 \text{ MPa}, \quad \alpha = 0.11, \quad \beta = 1.05, \quad n = 0.4, \\ d_{50} &= 0.5 \text{ mm}, \quad a_c = 1.0. \end{aligned}$$

3 MODELLING THE INTERFACE BEHAVIOUR

Apart from stress and displacement boundary conditions of a non-polar continuum, additional non-standard boundary conditions, i.e. couple stresses and Cosserat rotation boundary conditions, must also be defined for the present model. For rough and medium rough boundaries and pure translatory motion of the bounding structure the following assumptions are made to model the interface behaviour in a simplified manner (Figure 3):

(i) Boundary particles of the granular body are permanently in contact with the adjoining structure, thus the relative velocity of boundary particles perpendicular to the adjoining structure surface is zero, i.e.

$$\dot{v}_h - \dot{v}_p = 0 \quad (11)$$

Fig. 3: Components of the velocity of the bounding structure, i.e. $\dot{u}_h$, $\dot{v}_h$ and of particles in contact with the bounding structure, i.e. $\dot{u}_p$, $\dot{v}_p$, $\dot{\omega}_p$.

(ii) The component u_p of the particle velocity parallel to the surface of the adjoining structure is equal to or less than the velocity component u_h of the adjoining structure, i.e.

$$\dot{u}_p = f_u \dot{u}_h \quad (12)$$

Herein the dimensionless factor $0 < f_u < 1$ denotes the fraction of $\dot{u}_h$ which is transmitted to the boundary of the granular body. With respect to (12) the relative velocity $\dot{u}_r$ then reads:

$$\dot{u}_r = \dot{u}_h - \dot{u}_p = (1 - f_u) \dot{u}_h,$$

which can also be represented as the sum of the part $\dot{u}_{rr} = f_r \dot{u}_h$ due to grain rotation and the part $\dot{u}_{rs} = f_s \dot{u}_h$ due to sliding, i.e.

$$\dot{u}_r = \dot{u}_{rr} + \dot{u}_{rs} = (f_r + f_s) \dot{u}_h. \quad (13)$$

f_r and f_s in Eq. (13) denote the fractions of $\dot{u}_h$ which are transmitted as rotation and sliding respectively. Herein the condition $f_u + f_r + f_s = 1$ holds for consistency.

(iii) Between the rate of the Cosserat rotation $\dot{\omega}_p^c$ at the boundary of the granular body, the mean grain diameter d_{50} , the factor f_r and the velocity component $\dot{u}_h$ the following relation is assumed:

$$\dot{\omega}_p^c = f_r \frac{\dot{u}_h}{d_{50}/2}. \quad (14)$$

With respect to Eq. (12) relation (14) for $\dot{\omega}_p^c$ can alternatively be represented as a function of the particle velocity $\dot{u}_r$ at the bounding structure, i.e.

$$\dot{\omega}_p^c = f_\omega \frac{\dot{u}_p}{d_{50}}, \quad (15)$$

with $f_\omega = 2f_r / f_u$. In general f_u varies with the evolution of the state quantities in the granular layer. For the sake of the simplicity factor f_u is assumed to be constant for the present investigation. Then the rate type equation (15) can be integrated and leads by monotonic shearing to the relation:

$$\omega_p^c = f_\omega \frac{u_p}{d_{50}}, \quad (16)$$

which was originally proposed by Tejchman (1997). It should be noted that for the present continuum approach the quantities $\dot{u}_p$, $\dot{\omega}_p^c$ and u_p , ω_p^c are the mean values of the corresponding discrete grain assembly.

4 PLANE SHEARING OF AN INFINITE GRANULAR LAYER LOCATED BETWEEN TWO PARALLEL ROUGH WALLS

For numerical simulations of plane shearing the present hypoplastic Cosserat model was implemented into the finite element programme ABAQUS. A four-

node element with bilinear shape functions was used to describe the displacements and Cosserat rotations within the element (Huang, 2000). For the case of plane strain only three degrees of freedom remain for each node, i.e. u , v and ω^c , and the non-zero static quantities are σ_{11} , σ_{22} , σ_{33} , σ_{12} , σ_{21} , μ_{31} and μ_{32} as shown in Figure (4).

Fig. 4: Modelling of a plane granular layer with a Cosserat continuum.

The calculation was performed for a plane granular layer with a height of $h = 4\text{cm}$ starting from a homogeneous and isotropic initial state, i.e. $\sigma_n = 100\text{ kPa}$; $\mu_n = 0$; $e_n = 0.6$. For an infinite shear layer the field quantities are independent of the co-ordinate in the direction of shearing, i.e. $\partial(\cdot)/\partial x_1 = 0$ with respect to the co-ordinate system in Figure (4). For plane strain conditions the field quantities are also independent of the co-ordinate x_1 . At the top surface a constant vertical pressure of $\sigma_{22} = \sigma_n$ is applied and a shear deformation is initiated by prescribed horizontal node displacements while the vertical displacement was obtained as a result of the dilatancy behaviour within the shear layer. For the case of very rough surfaces of the bottom and top plates the assumption is made that the rough wall can capture the adjoining grains so that neither sliding nor rotation can occur. In this case f_w in Eq. (16) is set to zero, which is equivalent to locked Cosserat rotations, i.e. $\omega^c = 0$ at the boundary of the granular layer. For shear displacements of 0.4, 1, 2 and 3 cm at the top of the layer the distribution of the horizontal displacements u across the height of the layer is shown in Figure (5a). It is obvious that the shear displacement field obtained from the hypoplastic Cosserat model is no longer linear

as it is for a classical continuum model. The deformations are inhomogeneous from the beginning of shearing as shown by Bauer and Huang (2001). For large shearing the deformation is localized in a narrow zone in the middle of the layer. Within the localized zone the void ratio e (Figure 5b) and the Cosserat rotation ω^c (Figure 5c) increases while outside this zone these quantities remain almost unchanged. The distribution of the stress components σ_{11} , σ_{21} and μ_{32} is influenced by the micro-polar effect while σ_{22} and σ_{12} are constant as required for the equilibrium (Figure 5d, 5e, 5f). Since $\sigma_{12} \neq \sigma_{21}$, the stress tensor in a Cosserat continuum is non-symmetric with the exception of states with $\partial\mu_{32}/\partial x_2 = 0$. The latter condition occurs at four points within the shear layer as can be detected by comparing Figure (5e) with Figure (5f), i.e. the points with $\sigma_{12} = \sigma_{21}$ correspond to the relative extrema of μ_{32} . Independent of the amount of shearing the couple stress and the rate of curvature are zero in the middle of the shear layer. In order to model a smoother top surface a certain coupling between the horizontal displacement and the Cosserat rotation is assumed according the relation (16) at the top of the granular layer. For the particular case of $\omega_T^c = -0.025 u_{\text{TOP}} / d_{50}$ the results are shown in Figure (6). It is obvious that the localized zone is located closer to the top boundary. Furthermore a comparison of Figure (5) with Figure (6) shows that the thickness of the localized zone is smaller for the smoother top surface and the distribution of the state quantities is different to the results obtained for locked Cosserat boundary conditions. Independent of the assumed Cosserat boundary condition the normal stresses and the void ratio in the middle of the localized zone tend towards the same stationary value that is also obtained from the corresponding non-polar hypoplastic model under plane shearing (Bauer, 2000), i.e. the state quantities tend towards $\sigma_{11} = \sigma_{22} = \sigma_{33}$ and $e = e_c$.

4 CONCLUSIONS

The effect of micro-rotations and couple stresses in a granular shear layer located between two parallel rough plates has been studied based on a hypoplastic Cosserat continuum approach. It is demonstrated in the paper that

Fig. 5: Plane shearing between two very rough walls

the influence of the wall roughness on the rotation resistance of particles in contact with the rough bounding wall can be modelled with the Cosserat boundary conditions in a simple manner. Finite element calculations show that for large shearing the deformation is localized within a narrow zone.

The thickness of the localized zone depends on the Cosserat boundary conditions, the mean grain diameter, the void ratio and the pressure level. For locked Cosserat rotations at the top and at the bottom of the shear layer the localized zone occurs in the middle of the shear layer, otherwise it is located close to the boundary with higher Cosserat rotations.

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Fig. 6: Plane shearing between a very rough bottom and a medium rough top wall

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