Identification of Behaviour Laws and Failure Criteria Parameters of Composites Based on Off Axes Tests Application : Composite Polyester/Alfa

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ABSTRACT

The purpose of the present study is to propose a method for identifying the mechanical behaviour of composite materials based on off axes tests for various directions, in a more general framework including a large range of geometrical variables used in industry. We propose a mechanism based mode which strongly depends on the structural anisotropy of the material and on the orientation of loading. For failure we use a quadratic surface similar to Hill criterion, the coefficients of which depend on the type and the direction of loading.

The proposed identification method is applied to a composite made out of woven vegetal fibres and a polyester resin matrix. The fibres are extracted from Alfa leaves which are abundant in North Africa, and are commonly used in the manufacturing of high quality paper.

Key Words: Composite, vegetal fibres, Behaviour law, Directional effects, Failure.

1. INTRODUCTION

Composites are new materials that have appeared recently and are, generally, composed of two phases: The **fibres** that are dispersed into another material

called the **matrix**. The composites perform better mechanically than each phase on its own. The research of high technical performances has led to the development of composites which are heterogeneous structures, highly anisotropic due to the arrangement of the phases. This anisotropy has two principal consequences; one is favourable: the possibility of making the fibres in the direction of high stress. The second is unfavourable: the complexity of the modes of possible fracture. The orientation causing this anisotropy must be considered in the modelling of the mechanical behaviour.

During the last twenty years, composite materials have been the subject of a great deal of literature dealing with the analysis and the prediction of their mechanical behaviour. Homogenization in elasticity, which is a classical example, makes it possible to deal with the most complex fiber arrangements, like woven composites /8/ in the linear domain, and in the non linear domain /5/. In /6,7/ the damage is taken into account and computed, whereas the failure analysis by fatigue based criteria, has been developed for instance by Hashin and Rotem /9/. These different approaches include the directional effects, but the composite materials are equally sensitive to the loading nature and modes, as has been shown by Chevalier *et al.* /4/. It is in the nature of composites to exhibit sensitivity with respect to loading mode. This paper presents a three-dimensional model of mechanical behaviour and failure

in composite materials. The model relies on results of off-axes tests performed on samples for various orientations. Our theory takes into account the features of loading (type and direction). To interpret the experimental issues, we constructed a three dimensional model that deals with the composites in linear elasticity, non linear elasticity and plasticity at the macro and meso level referring to the loading characteristics and the mechanical behaviour. Recently we have been dealing with composites reinforced in several directions having one or multi directions, such as the case of woven composites as taffetas. This material belongs to the family of long fibres which has an important place in construction of high mechanical performance structures (aero spatial, aeronautic, automobile, biomechanics, etc.). The objective of this work is to present a modelling of the behaviour of composites in many forms and any mode of reinforcement met in the industry.

An identification procedure has been performed on a composite made out of woven vegetal fibres and a polyester resin matrix. The fibres are extracted from Alfa leaves which are abundant in North Africa. Alfa is an extremely abundant vegetation in Tunisia, especially in the western centre where it covers wide surfaces. It is, essentially, used in paper manufacturing (the Kasserine cellulose factory) which is an important industry in the country (3% of exported products). The idea of manufacturing this composite arose from the abundance and low cost of these natural vegetal fibres, which make them competitive with respect to glass or carbon fibres.

2. MODELLING OF ELASTIC BEHAVIOUR

In this part, we propose a three dimensional model for the elastic behaviour. We propose a theoretical study that considers the features of the loading (type and direction) and the non-linear elastic behaviour of composites. The incremental form of deformation and stress allows the step by step calculation for any loading that is also divided into increments.

To take into consideration the main effects in a composite material, we introduce the following 6-dimensional vectorial spaces (Chafra *et al.*, 2002):

- Stress space (Σ) spanned by elements of the basis of vectors defined by :

$$\{\Sigma_{\alpha}\} = \left\{ \begin{array}{l} \Sigma_1 = \sigma_{11}, \Sigma_2 = \sigma_{22}, \Sigma_3 = \sqrt{2}\sigma_{12}, \\ \Sigma_4 = \sigma_{33}, \Sigma_5 = \sqrt{2}\sigma_{13}, \Sigma_6 = \sqrt{2}\sigma_{23} \end{array} \right\}^T$$

where σ_{ij} ($i, j = 1, 2, 3$) are the components of the stress tensor

- Strain space (E) spanned by elements of the basis of vectors defined by :

$$\{E_{\alpha}\} = \left\{ \begin{array}{l} E_1 = \varepsilon_{11}, E_2 = \varepsilon_{22}, E_3 = \sqrt{2}\varepsilon_{12}, \\ E_4 = \varepsilon_{33}, E_5 = \sqrt{2}\varepsilon_{13}, E_6 = \sqrt{2}\varepsilon_{23} \end{array} \right\}^T$$

where ε_{ij} ($i, j = 1, 2, 3$) are the components of the strain tensor.

We introduce two tensorial invariants, the stress intensity $\Sigma_0 = \sqrt{\Sigma_{\alpha}\Sigma_{\alpha}}$ and the strain intensity $E_0 = \sqrt{E_{\alpha}E_{\alpha}}$, deduced from the second order stress and strain tensor.

To study the unilateral effects, we propose to characterize the loading in direction and type:

The loading direction θ is defined by a unit vector $\{\sigma_{\alpha}^{\theta}\} \in \{\Sigma\}$, with $\sqrt{\sigma_{\alpha}^{\theta}\sigma_{\alpha}^{\theta}} = 1$ and $\Sigma_{\alpha} = \Sigma_0\sigma_{\alpha}^{\theta}$.

The type of loading is defined as a function of the sign configuration of the stress tensor, hence the stress space is decomposed into zones characterized by the sign attributed to each component of stress. For that purpose we define a parameter $(\cdot)$ which corresponds to the number of rows in the sign matrix $[I_{\chi\alpha}]$ and is equal to the line:

$$\{\text{sign}(\sigma_1), \text{sign}(\sigma_2), \dots, \text{sign}(\sigma_6)\}$$

$$\text{with } \text{sign}(\sigma_{\alpha}) = \begin{cases} 1 & \text{if } \sigma_{\alpha} > 0 \\ -1 & \text{if } \sigma_{\alpha} < 0 \end{cases}$$

We consider all the combinations of signs in the stress tensor, therefore the sign matrix is composed of 2^6

rows : ($\alpha=1,2,\dots,6$; $\chi=1,2,\dots,2^6$)

$$[1 \chi \alpha] = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & -1 \\ 1 & 1 & 1 & 1 & -1 & -1 \\ - & - & - & - & - & - \\ - & - & - & - & - & - \end{bmatrix}$$

In the one dimensional case, the parameter (χ) is equal to 1 or 2. We have two spaces with differences in behaviour, and recognize the theory of Ladeveze /7/. These differences in behaviour are due to cases of closing (in compression) and opening (in traction) of micro-cracks. When $\alpha, (\alpha \leq 6)$ is the dimension of the subspace of stresses ($\chi=1,2,\dots,2^6$) we have 2^6 zones of loading with different behaviours.

A sign convention is adopted according to the following rules: a positive sign is taken for the tensile stress and a negative sign is taken for the compressive stress. For the shearing stress, we consider the stress vector $\mathbf{T}$ belonging to the shear plane and corresponding to the surface element $dS(\mathbf{T}=[\sigma]\mathbf{n})$. Sign (shear stress) = $\text{sign}(\mathbf{N} \cdot \mathbf{T} \cdot \mathbf{tg}\varphi)$, where $\mathbf{N}$ is the projection of the normal vector $\mathbf{n}$ on the shear plane and φ the polar angle of $\mathbf{N}$ (see also Chevalier *et al.* 1998 /4/).

During a proportional loading, $\{\sigma_\alpha = \text{cst}\}$, we define a generalized modulus which relates a stress intensity to a strain intensity

$$\Sigma_0 = E^{(\chi)} \varepsilon_0. \quad (1)$$

We associate to each type of loading, a Hookean operator $\mathbf{C}$ and its inverse $\mathbf{S}$ such that

$$\Sigma_\alpha = C_{\alpha\beta}^{(\chi)} \varepsilon_\beta \text{ or } \varepsilon_\alpha = S_{\alpha\beta}^{(\chi)} \Sigma_\beta \quad (2)$$

In the case of non proportional loading, the constitutive law is defined as an incremental linear elastic law :

$$\Delta \Sigma_\alpha(i) = C_{\alpha\beta}^{(\chi)} \Delta \varepsilon_\beta(i)$$

or

$$\Delta \varepsilon_\alpha(i) = S_{\alpha\beta}^{(\chi)} \Delta \Sigma_\beta(i). \quad (3)$$

To describe the variation of elastic compliance in the linear domain which depends on the type and the direction of loading, we construct an iso-strain quadratic surface in the space of stress:

$$\phi(\varepsilon) = \bar{\mathbf{R}}_{\alpha\beta}^{(\chi)} \varepsilon_\alpha \varepsilon_\beta - \frac{2}{\varepsilon_0} = 0 \quad (4)$$

($\varepsilon_0 = \text{constant}, \alpha, \beta = 1, 2, 3, \dots, 6$)

Where $\mathbf{R}_{\alpha\beta}^{(\chi)}$ is related to the components of the compliance matrix :

$$\mathbf{R}_{\alpha\beta}^{(\chi)} = \mathbf{S}_{\alpha\gamma}^{(\chi)} \mathbf{S}_{\gamma\beta}^{(\chi)} \quad (5)$$

and is related to the generalized modulus:

$$E^{(\chi)} = \frac{1}{\sqrt{\mathbf{R}_{\alpha\beta}^{(\chi)} \sigma_\alpha \sigma_\beta}} \quad (\{\sigma_\alpha\} = \text{constant}) \quad (6)$$

Chafra 1994 /2/ has given details on the identification of this process.

It can be shown (Chafra *et al.*, 1996 /3/), that the matrix $[\mathbf{R}_{\alpha\beta}]$ is non negative and the generalized modulus $E^{(\chi)}$ exists.

a- Case of biaxial loading

In the of biaxial loading the matrix $[I_{\chi\alpha}]$ is defined by

$$[I_{\chi\alpha}] = \begin{bmatrix} 1 & 1 \\ -1 & 1 \\ -1 & -1 \\ 1 & -1 \end{bmatrix}$$

($\alpha=1,2$) we have four spaces with differences in behaviour ($\chi=1,2,3,4$).

The constitutive law :

$$\begin{cases} \Delta E_1 = S_{11}^{(\chi)} \Delta \Sigma_1 + S_{12}^{(\chi)} \Delta \Sigma_2 \\ \Delta E_2 = S_{21}^{(\chi)} \Delta \Sigma_1 + S_{22}^{(\chi)} \Delta \Sigma_2 \end{cases} \quad (7)$$

with $S_{21}^{(\chi)} = S_{12}^{(\chi)}$

And the isostrain criteria is written as follows:

$$\Re_{11}^{(\chi)} \Sigma_1^2 + 2\Re_{12}^{(\chi)} \Sigma_1 \Sigma_2 + \Re_{22}^{(\chi)} \Sigma_2^2 - E_0^2 = 0 \quad (8)$$

The three parameters that define the quadratic equations (8) are determined uniquely from three experimental points or from more points using a least square method.

Identification procedure:

- From 3 angles of cutting (0°, 45° and 90°)

In the case of biaxial loading we obtain the following relation between Σ_1 and Σ_2

$$\begin{cases} \Sigma_2 = K_a \Sigma_1, & K_a = \tan \alpha \\ \Sigma_1 = K_b \Sigma_2, & K_b = \tan \alpha \end{cases}$$

$\Rightarrow$

$$\sigma_1 = \frac{1}{\sqrt{1+K_a^2}} = \frac{K_b^2}{\sqrt{1+K_b^2}}$$

$$\sigma_2 = \frac{1}{\sqrt{1+K_b^2}} = \frac{K_a^2}{\sqrt{1+K_a^2}}$$

The isostrain criteria can be written as follows:

$$\begin{aligned} \Re_{11}^{(\chi)} \sigma_1^2 + 2\Re_{12}^{(\chi)} \sigma_1 \sigma_2 + \Re_{22}^{(\chi)} \sigma_2^2 &= E_0^2 \\ \Rightarrow \Re_{11}^{(\chi)} \frac{1}{1+K_a^2} + 2\Re_{12}^{(\chi)} \frac{K_a}{1+K_a^2} + \Re_{22}^{(\chi)} \frac{K_a^2}{1+K_a^2} &= \\ &= \frac{E_0^2}{\bar{\Sigma}_0^2(\alpha)} = \left[S'(\chi)(\alpha) \right]^2 \end{aligned}$$

$$\text{with: } \bar{\Sigma}_0(\varphi) = \sqrt{\Sigma_1^2 + \Sigma_2^2} \text{ and } \left[S'(\chi)(\alpha) \right] = \frac{E_0}{\bar{\Sigma}_0(\alpha)}$$

$$\text{if } \alpha = 0^\circ \Rightarrow K_a = 0 \text{ and } \Re_{11}^{(\chi)} = \left[S'(\chi)(0^\circ) \right]^2$$

$$\text{if } \alpha = 90^\circ \Rightarrow K_a \rightarrow 0 \text{ and } \Re_{22}^{(\chi)} = \left[S'(\chi)(90^\circ) \right]^2$$

if $\alpha = 45^\circ \Rightarrow K_a = 1$ and

$$\Re_{12}^{(\chi)} = \left[S'(\chi)(45^\circ) \right]^2 - \frac{1}{2} \left(\left[S'(\chi)(0^\circ) \right]^2 + \left[S'(\chi)(90^\circ) \right]^2 \right) \quad (9)$$

- From the least square method (0°, 30°, 45°, 60° and 90°)

$$\{\Re\} = - \left[A^T \cdot A \right]^{-1} \cdot A^T \cdot d_0 \quad (10)$$

where

$$\Re = \left[\Re_{11}^{(\chi)} \quad \Re_{12}^{(\chi)} \quad \Re_{22}^{(\chi)} \right]^T$$

$$[A] = \begin{bmatrix} P_1 & P_2 & P_3 & P_4 & P_5 \end{bmatrix}^T$$

$$\{P_i\} = \left\{ \Sigma_1^2, \Sigma_1 \Sigma_2, \Sigma_2^2, \Sigma_1 \Sigma_2, \Sigma_2^2 \right\}^T$$

$$\{d_0\} = E_0^2 \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix}^T$$

Σ_{ji} stress value for the case i of cutting (0°, 30°, 45°, 60°, 90°)

b- Use of experimental results

We often use an experimental method using samples out off axes for various directions (Figure 1). The results provided by a test are the longitudinal modulus, while the results used in the models are the generalized modulus.

By means of the transformation method, we establish the following relation between Σ_1 , Σ_2 and σ_L .

$$\begin{cases} \Sigma_1 = \sigma_L \cos^2 \theta \\ \Sigma_2 = \sigma_L \sin^2 \theta \\ \frac{1}{\sqrt{2}} \Sigma_3 = -\sigma_L \cos \theta \sin \theta \end{cases} \Rightarrow \tan \alpha = \tan^2 \theta$$

We define the longitudinal modulus and Poisson ratio, and we calculate Σ_0 and E_0

$$\Sigma_0 = \sqrt{\Sigma_1^2 + \Sigma_2^2 + \Sigma_3^2} = \sigma_L$$

$$\text{and } E_L^{(\chi)}(\theta) = \frac{\sigma_L}{\varepsilon_L}$$

$$E_0 = \sqrt{\varepsilon_L^2 + \varepsilon_T^2} = \varepsilon_L \sqrt{1 + \nu_{LT}^2}$$

$$\nu_{LT}^{(\chi)}(\theta) = -\frac{\varepsilon_T}{\varepsilon_L}$$

$$\text{Then } \Sigma_0 = \bar{\Sigma}_0 \sqrt{\cos^4(\theta) + \sin^4(\theta)}$$

$$\text{with } \bar{\Sigma}_0 = \sqrt{\Sigma_1^2 + \Sigma_2^2}$$

and

$$E^{(\chi)}(\theta) = \frac{\Sigma_0}{\varepsilon_0} = \frac{\sigma_L}{\varepsilon_L \sqrt{1 + \nu_{LT}^2(\theta)}} = \frac{E_L^{(\chi)}(\theta)}{\sqrt{1 + \nu_{LT}^2(\theta)}}$$

We establish the elastic modulus

$$E^{(\chi)}(\alpha) = \frac{\bar{\Sigma}_0}{\varepsilon_0} = \frac{E_L^{(\chi)}(\theta) \sqrt{\cos^4(\theta) + \sin^4(\theta)}}{\sqrt{1 + \nu_{LT}^2(\theta)}} \quad (11)$$

$$\text{with } S^{(\chi)}(\alpha) = (E^{(\chi)}(\alpha))^{-1}$$

Fig. 1: Off axes tests and loading direction

With three values of $S^{(\chi)}(\alpha)$, for $\alpha = 0^\circ, 45^\circ$ and 90° , we identify the parameters of surface $R_{11}^{(\chi)}$, $R_{22}^{(\chi)}$ and $R_{12}^{(\chi)}$ by the relation (9), and following these parameters we express for each the mechanical characteristic for the constitutive law (7).

$$\left\{ \begin{aligned} S_{11}^{(\chi)} &= \sqrt{\frac{R_{11}^{(\chi)}}{1 + (\nu_{LT}(90^\circ))^2}} \\ S_{22}^{(\chi)} &= \sqrt{\frac{R_{22}^{(\chi)}}{1 + (\nu_{LT}(0^\circ))^2}} \\ S_{12}^{(\chi)} &= \sqrt{\frac{R_{11}^{(\chi)} + 2R_{12}^{(\chi)} + R_{22}^{(\chi)}}{1 + (\nu_{LT}(45^\circ))^2}} - \sqrt{\frac{R_{11}^{(\chi)}}{1 + (\nu_{LT}(90^\circ))^2}} \end{aligned} \right. \quad (12)$$

3. FAILURE CRITERIA

For failure we have introduced a quadratic surface similar to the Hill criterion

$$F_{\alpha\beta}^{(\chi)} \Sigma_\alpha \Sigma_\beta = 1 \quad (\alpha, \beta = 1, 2, \dots, 6) \quad (13)$$

Coefficients $F_{\alpha\beta}^{(\chi)}$ depend on the type and direction of loading, and are related to the failure limit: $\Sigma_F(\alpha) = \frac{1}{\sqrt{F_{\alpha\beta}^{(\chi)} \sigma_\alpha \sigma_\beta}}$

In the case of biaxial loading (Σ_1, Σ_2) the constitutive law is defined by 3 compliances $S_{11}^{(\chi)}$, $S_{22}^{(\chi)}$, $S_{12}^{(\chi)}$, and the failure criteria is defined by three parameters $F_{11}^{(\chi)}$, $F_{22}^{(\chi)}$, $F_{12}^{(\chi)}$:

At the failure point, the model is given by the expression:

$$f_F^{(\chi)} \equiv F_{11}^{(\chi)} \Sigma_1^2 + 2 F_{12}^{(\chi)} \Sigma_1 \Sigma_2 + F_{22}^{(\chi)} \Sigma_2^2 - 1 = 0 \quad (15)$$

where,

$$\left\{ \begin{aligned} F_{11}^{(\chi)} &= \frac{1}{(\Sigma_F(0^\circ))^2} \\ F_{22}^{(\chi)} &= \frac{1}{(\Sigma_F(90^\circ))^2} \\ F_{12}^{(\chi)} &= \frac{1}{(\Sigma_F(45^\circ))^2} - \frac{1}{2} \left[\frac{1}{(\Sigma_F(0^\circ))^2} + \frac{1}{(\Sigma_F(90^\circ))^2} \right] \end{aligned} \right. \quad (16)$$

$\Sigma_F(0^\circ)$, $\Sigma_F(45^\circ)$ and $\Sigma_F(90^\circ)$ represent respectively the stresses of failure corresponding to $\alpha = 0^\circ, 45^\circ$ and 90° .

By means of the transformation method, we establish the following relation (see Figure 1 and use of experimental results):

$$\sigma_F(\theta) = \Sigma_F(\alpha) \sqrt{\cos^4(\theta) + \sin^4(\theta)} \quad (17)$$

$\sigma_F(\theta)$ is the value of the stress, measured directly from the diagram of stress-strain.

θ is the angle of cutting.

α is the angle of loading.

$$\tan \alpha = \tan^2(\theta)$$

4. MODE OF MANUFACTURING OF THE ALFA/ POLYESTER COMPOSITE

The tested composite material is made manually, out of long Alfa fibres, woven in taffeta pattern, and a polyester resin.

The manufacturing steps are as follows:

1st stage: Elimination of lignin and hemicelluloses

These two elements reduce the fibre resistance. They may be eliminated by heating in a solution of sodium hydroxide (NaOH) as follows:

- Introduce, into a glass balloon containing 1 litre of water, 120g of NaOH, stir to dissolve the grains of sodium hydroxide, and then add about 60g to 70g of Alfa.

- Heat the balloon above 100°C for 2 hours.
- Cool the content of the balloon by washing it several times with distilled water.
- Pour into the balloon 1 litre of purified water and 40 ml of 12% sodium chloride, then leave for one hour to dissolve the Na⁺ ions.
- Wash several times the contents of the balloon with distilled water.
- Dry the content on a paper and then put it into a steamer for 24 hours at a temperature about 40°C.

2nd stage: Refinement of fibres:

The quantity of Alfa removed from the steam is brushed with a hair brush then with a tooth brush to make the fibres as fine as possible.

3rd stage: Weaving of the textures of Alfa fibres:

Textures are woven by hand in taffeta pattern in two perpendicular and crossed directions.

The parameters of texture are illustrated in the following table:

Parameter	Denomination	Definition	Value
n_t	parameter of texture	number of threads in a stitch	2
$\xi = a/l$	factor of constriction	fraction of the width of a wick to the width of a stitch	0.33
V_f	volume fraction of fibres	fraction of fibres volume to total volume	0.233
$\eta = \frac{L_c - L_t}{L_t}$	ambulate	Relative shortening of chain threads. (L_c length of the chain thread, L_t length of texture).	4.5%
$R = \xi (2 - \xi)$	factor of recovery	Fraction of reinforcement to that cross section area of the texture	0.55
$\theta = \frac{d}{e} = \frac{V_f}{\xi (2 - \xi)}$	factor of stacking	fraction of thickness of fibre to thickness of stitch (e)	0.42
$Gr = R d \rho_f$	Surface weight	weight of texture by square meter. ρ_f is the specific mass of the fibre.	118,31 gr/m ²

4th stage: Milling of the composite material:

The milling of the material is done by an aluminium mold (270 x 270 x 30 mm³ and 270 x 270 x 20 mm³). The manufacturing process is summarized in the following steps:

1. Apply and sweep milling off wax over the surface of the mould.
2. Spread of 1st layer of resin in the mould
3. Put the texture in place.
4. Spread 2nd layer of resin
5. Roll tightly to eliminate air
6. Regularize surface with a trowel

7. Dry at room temperature

8. Mill off plate.

9. Cut plate into test samples.

Physical -features of used materials:

- Average diameter of the fibres: 0,75 mms.
- Average spacing of fibres: 1,5 mms
- Volume percentage of fibres: 24 %
- Specific weight of the fibres: 250 Kg/m³
- Specific weight of the polyester resin: 1100 Kg/m³
- Specific weight of the composite: 1150Kg/m³

Speed of traction test:

It is the speed of separation of two clamps of the traction machine. It is equal to 2 mm/mins.

Results of the traction tests:

The graphic results of traction tests are illustrated by the stress-strain diagrams.

The results of traction tests to the 0°'s direction are illustrated in the following table:

N° of plate	Stress at failure rupture [MPa]	Stress at 0,05 % of strain [MPa]	Stress at 0,25 % of strain [MPa]	Strain at failure [%]
P1	132	6,25	20	2
P2	105	3,4	16,2	1,8
P3	108	3,2	15,2	1,8
P4	95	4,2	20,5	1,6
P5	92	3,2	16,4	2

The results of traction tests to the 45° 's direction, are illustrated in the following table:

N° of plate	Stress at failure [MPa]	Stress at 0,05 % of strain [MPa]	Stress at 0,25 % of strain [MPa]	Strain at failure [%]
P6	87	4,1	14,2	1,8
P7	70	4,5	11,5	1,2
P8	75	4,5	13,8	1,4

The results of traction tests to the 30° 's direction, are illustrated in the following table:

N° of plate	Stress at failure [MPa]	Stress at 0,05 % of strain [MPa]	Stress at 0,25 % of strain [MPa]	Strain at failure [%]
P9	88	4,1	15,4	1,4
P10	80	3,7	14,6	1,4
P11	77	3,5	14,86	1,35

Principal mechanical characteristics of the composite

$$E_L^{(\chi)}(0^\circ) = E_L^{(\chi)}(90^\circ) = 6385 \text{ MPa} ;$$

$$E_L^{(\chi)}(45^\circ) = 4710 \text{ MPa} ;$$

$$E_L^{(\chi)}(30^\circ) = E_L^{(\chi)}(60^\circ) = 5594 \text{ MPa} .$$

$$G_{12} = 1600 \text{ MPa} ;$$

$$G_{13} = 2900 \text{ MPa} ;$$

$$G_{23} = 1400 \text{ MPa}$$

$$\nu_{1,T}(0^\circ) = \nu_{1,T}(90^\circ) = 0,27 ;$$

$$\nu_{1,T}(45^\circ) = 0,48 ;$$

$$\nu_{1,T}(30^\circ) = \nu_{1,T}(60^\circ) = 0,46$$

5- VALIDATION OF THE MODEL:

In this part, we will try to validate the model by using the results of tractions tests on the composite.

a) Elastic domain

An identification procedure has been performed on woven Alfa/Polyester composite.

Criteria of iso strain's expression :

- From three angles of cutting (0°, 45° and 90°)

$$26,31 \Sigma_1^2 + 164,40 \Sigma_1 \Sigma_2 + 26,31 \Sigma_2^2 - 10^9 F_0^2 = 0$$

- From the least square method (0° , 30° , 45° , 60° and 90°)

$$33,18 \Sigma_1^2 + 149,84 \Sigma_1 \Sigma_2 + 33,18 \Sigma_2^2 - 10^9 E_0^2 = 0$$

Experimental results concerning this formulation are given in Figures 2, 3 and 4.

Fig. 2: Isostrain criteria for a woven Alfa/Polyester composite in the linear range ($\chi = 1$). For $E_0 = 0,5\%$

Fig. 3: Isostrain criteria for a woven Alfa/Polyester composite in the linear range ($\chi = 1$). For $E_0 = 0,4\%$

Fig. 4: Isostrain criteria for a woven Alfa/Polyester composite in the linear range ($\chi = 1$). For $E_0 = 0,2\%$

Following the iso strain criteria we express the mechanical characteristic for the constitutive law (7):

$$\begin{Bmatrix} \Sigma_1 \\ \Sigma_2 \end{Bmatrix} = \begin{bmatrix} 6386,1 & 1754,5 \\ 1754,5 & 6386,1 \end{bmatrix} \begin{Bmatrix} E_1 \\ E_2 \end{Bmatrix}$$

b) Case of failure:

An identification procedure has been performed on woven Alfa/Polyester composite. (Figure 5 and Table 1).

Table 1

Stresses at failure corresponding to $\varphi=0^\circ$, 45° and 90° ..:

θ	$\sigma_F [MPa]$	$\Sigma_F [MPa]$
0° et 90°	106,4	106,4
30° et 60°	81,67	64,56
45°	77,3	54,66

Failure criteria expression :

- From three angles of cutting (0° , 45° and 90°)

$$88,33 \Sigma_1^2 + 495,20 \Sigma_1 \Sigma_2 + 88,33 \Sigma_2^2 = 10^6$$

- From the minimum square method (0° , 30° , 45° , 60° and 90°)

$$88,4\Sigma_1^2 + 499\Sigma_1\Sigma_2 + 88,4\Sigma_2^2 = 10^6$$

Graphic representation :

Fig. 5: Failure criteria (zone of loading ($\chi = 1$)).

6. CONCLUSION

In this work, we propose a method for identifying the mechanical behaviour of composite materials based on off-axes tests for various directions. The Hill criterion is adopted with parameters that depend on the type and direction of loading. An identification procedure has been performed on woven Alfa/Polyester composite. The mechanical characteristics of the composite Alfa/Polyester, in any direction, are easily identified.

The use of Alfa fibres is justified by the abundance and low cost of these natural vegetal fibres, which make them competitive with respect to glass or carbon fibres.

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