An Elasto-Plastic Stress Analysis of Simply Supported Aluminum Metal Matrix Composite Beams Under a Transverse Linearly Distributed Load by Use of Anisotropic Elasticity Theory

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ABSTRACT

A beam is a structural element loaded transversely in general conditions. This loading causes shear forces and bending moment. In this study, an analytical elasto-plastic stress analysis is carried out on simply supported stainless steel fiber reinforced aluminum metal matrix composite beams under linearly distributed load using anisotropic elasticity theory. In the elasto-plastic solution, the material is assumed to be perfectly plastic. Sample problems are given for various angles analytically. The yielding begins at the upper and lower surfaces of the beam at the same distances from the ends. The intensity of residual stress component of σ_x is maximum at the upper and lower surfaces but residual stress component of τ_{xy} is maximum on the x axis of the beam. The composite beams can be strengthened by using the residual stresses.

Key Words: Residual stress, elasto-plastic stress analysis, metal-matrix composite, simply supported beam, linearly distributed load.

INTRODUCTION

Metal-matrix composites offer an increased service temperature and improved specific mechanical properties over existing metal alloys. They consist of a ductile, usually low density, matrix reinforced with elastic or ductile and strong fibers. Aluminum-matrix composites, which are particularly cited for their

superior performance-to-weight advantage, have many applications in the aerospace and other industries. Many different technologies for production of metal-matrix composites have emerged over the years.

Chou *et al.* /1/ reviewed the work on fiber-reinforced metal-matrix composites explaining fabrication methods, mechanical properties, secondary working techniques and interfaces. Kaczmar *et al.* /2/ discussed the production methods and properties of metal matrix composite materials reinforced with dispersion particles, platelets, non-continuous (short) and continuous (long) fibres. Khatibzadeh and Piggott /3/ investigated the effect of width on the mechanical properties of angle-ply laminates by using + 45 balanced angle-ply laminates. Bahei-El-Din and Dvorak /4/ studied the elastic-plastic behaviour of symmetric metal-matrix composite laminates for the case of in-plane mechanical loading. Kang and Kang /5/ reviewed the work on the short fiber-reinforced aluminum composite material, fabricated by stirring method and extruded at high temperatures at various extrusion ratios. Yeh and Krempl /6/ investigated the influence of cool-down temperature histories on the residual stress in fibrous metal-matrix composites. Sayman and Kayrıci /7/ studied a low density polyethylene thermoplastic composite beam, reinforced with woven steel fibers, which is loaded by a single force at its free end. Esendemir /8/ carried out an elasto-plastic stress analysis in a thermoplastic composite beam of arbitrary orientation subjected to transverse uniformly distributed load by using an analytical solution. Esendemir /9/ carried out an analytical elasto-plastic stress analysis for

a fiber-reinforced polymer matrix composite beam subjected to a linearly distributed load. Sayman /10/ performed an elastic-plastic stress analysis in metal-matrix composite beam under a transverse force. Karakuzu and Özcan /11/ determined residual stresses in an aluminum metal-matrix composite cantilever beam analytically. Küçük *et al.* /12/ carried out an elastic-plastic stress analysis in an aluminum metal matrix composite beam supported from two ends with a force acting at the mid point. Elastic-plastic and residual stresses in metal matrix or thermoplastic composite cantilever beams were studied by using analytical methods /13-16/.

In this study an analytical elastic-plastic stress analysis is carried out on metal matrix composite beams supported from two ends under transverse linearly distributed load in the x-y plane.

ELASTIC ANALYSIS

The simply supported composite beam is subjected to a transverse linearly distributed load q , as shown in Figure 1. The angle between the first principal axis of the composite fibers and the x axis is θ . The governing differential equation for an orthotropic beam in the plane stress case is given by Lekhnitskii /17/.

$$a_{22} \frac{\partial^4 F}{\partial x^4} - 2a_{26} \frac{\partial^4 F}{\partial x^3 \partial y} + (2a_{12} + a_{66}) \frac{\partial^4 F}{\partial x^2 \partial y^2} - 2a_{16} \frac{\partial^4 F}{\partial x \partial y^3} + a_{11} \frac{\partial^4 F}{\partial y^4} = 0 \quad (1)$$

Fig. 1: Simply supported composite cantilever beam subjected to a linearly distributed load.

where F is a stress function. Solutions of some anisotropic beams are given by Lekhnitskii /18/. The constants in Equation (1) are given by Jones /19/ as,

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{16} \\ a_{12} & a_{22} & a_{26} \\ a_{16} & a_{26} & a_{66} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} \quad (2)$$

where a_{ij} are the components of the compliance matrix. The elements of compliance matrix are given as

$$\begin{aligned} a_{11} &= S_{11}m^4 + (2S_{12} + S_{66})m^2n^2 + S_{22}n^4 \\ a_{12} &= S_{12}(n^4 + m^4) + (S_{11} + S_{22} - S_{66})n^2m^2 \\ a_{22} &= S_{11}n^4 + (2S_{12} + S_{66})n^2m^2 + S_{22}m^4 \\ a_{16} &= (2S_{11} - 2S_{12} - S_{66})nm^3 - (2S_{22} - 2S_{12} - S_{66})n^3m \\ a_{26} &= (2S_{11} - 2S_{12} - S_{66})n^3m - (2S_{22} - 2S_{12} - S_{66})m^3n \\ a_{66} &= 2(2S_{11} + 2S_{22} - 4S_{12} - S_{66})n^2m^2 + S_{66}(n^4 + m^4) \end{aligned} \quad (3)$$

$$S_{11} = \frac{1}{E_1}, S_{12} = -\frac{\nu_{12}}{E_1}, S_{22} = \frac{1}{E_2}, S_{66} = \frac{1}{G_{12}}$$

where $m = \cos \theta$, $n = \sin \theta$

Boundary conditions for this beam are given as,

$$y = c \Rightarrow \sigma_y = 0 \quad (4)$$

$$y = -c \Rightarrow \sigma_y = \frac{q_0 x}{tL} \quad (5)$$

$$y = \pm c \Rightarrow \tau_{xy} = 0 \quad (6)$$

$$x = 0 \Rightarrow \int_{-c}^c \tau_{xy} t dy = \frac{q_0 L}{6} \quad (7)$$

$$x = L \Rightarrow \int_{-c}^c \tau_{xy} t dy = \frac{q_0 L}{3} \quad (8)$$

$$x = 0 \Rightarrow \int_{-c}^c \sigma_x t dy = 0 \quad (9)$$

$$x = L \Rightarrow \int_{-c}^c \sigma_x t dy = 0 \quad (10)$$

$$x = 0 \Rightarrow \int_{-c}^c \sigma_x t y dy = 0 \quad (11)$$

$$x = L \Rightarrow \int_{-c}^c \sigma_x t y dy = 0 \quad (12)$$

where t is the thickness of the beam. For satisfying both the differential equation and boundary conditions, the stress function F_6 is chosen as

$$F_6 = \frac{xy^5}{20} f_6 + \frac{x^3 y^3}{6} d_6 + \frac{x^2 y^4}{12} e_6 \quad (13)$$

Substituting Equation (1) gives

$$-6a_{16} d_6 + 2a_{11} e_6 = 0$$

$$e_6 = 3 \frac{a_{16}}{a_{11}} d_6, \quad m = \frac{a_{16}}{a_{11}} \quad (14)$$

and,

$$(12a_{12} + 6a_{66}) d_6 - (8a_{16}) 3md_6 + 6a_{11} f_6 = 0$$

$$f_6 = -2n d_6, \quad n = \frac{1}{2} \frac{2a_{12} + a_{66} - 4a_{16} m}{a_{11}} \quad (15)$$

For satisfying all the boundary conditions we also select the stress function as second and third orders:

$$F = \frac{d_6 x^3 y^3}{6} + \frac{f_6 x y^5}{20} + \frac{d_4 x y^3}{6}$$

$$+ \frac{t_4 x^3 y}{6} + \frac{a_2 x^2}{2} + b_2 xy \quad (16)$$

$$\sigma_x = \frac{\partial^2 F}{\partial y^2} = d_6 x^3 y + f_6 x y^3 + d_4 x y \quad (17)$$

$$\sigma_y = \frac{\partial^2 F}{\partial x^2} = d_6 x y^3 + b_4 x y + a_2 \quad (18)$$

$$\tau_{xy} = -\frac{\partial^2 F}{\partial x \partial y} = -\frac{3}{2} d_6 x^2 y^2 - \frac{1}{4} f_6 y^4$$

$$- \frac{1}{2} d_4 x^2 - \frac{1}{2} t_4 x^2 - t_2 \quad (19)$$

From boundary conditions Eqn.(4) and (5)

$$a_2 = -\frac{q_0 x}{2Lt}, \quad t_4 = \frac{q_0}{2tLc} - d_6 c^2 \quad (20)$$

are obtained. From Eqn (4)

$$b_2 = -\frac{1}{20} f_6 c^4 - \frac{1}{6} d_4 c^2 - \frac{q_0 L}{12tc} \quad (21)$$

is obtained. From Eqn (4)

$$d_6 = -\frac{q_0}{4tLc^3}, \quad d_4 = \frac{1}{4} \frac{q_0 L}{tc^3} - \frac{3}{5} f_6 c^2 \quad (22)$$

are obtained. Thus, by using the relationship between the equations, we obtain:

$$t_4 = \frac{3}{4} \frac{q_0}{tLc}, \quad f_6 = \frac{1}{2} \frac{q_0 n}{tLc^3}, \quad t_2 = \frac{1}{40} \frac{q_0 nc}{tL} - \frac{1}{8} \frac{q_0 L}{tc} \quad (23)$$

if we replace these coefficients into Equations (17), (18) and (19), we obtain,

$$\sigma_x = -\frac{1}{4} \frac{q_0}{tLc^3} x^3 y + \frac{1}{2} \frac{q_0 n}{tLc^3} x y^3 + \frac{1}{4} \frac{q_0 L}{tc^3} x y - \frac{3}{10} \frac{q_0 n}{tLc} x y$$

$$\sigma_y = -\frac{1}{4} \frac{q_0}{tLc^3} x y^3 + \frac{3}{4} \frac{q_0}{tLc} x y - \frac{1}{2} \frac{q_0}{tL} x \quad (24)$$

$$\tau_{xy} = -\frac{3}{8} \frac{q_0}{tLc^3} x^2 y^2 - \frac{1}{8} \frac{q_0 n}{tLc^3} y^4 - \frac{1}{8} \frac{q_0 L}{tc^3} y^2 + \frac{3}{20} \frac{q_0 n}{tLc} y^2$$

$$- \frac{3}{8} \frac{q_0}{tLc} x^2 - \frac{1}{40} \frac{q_0 nc}{tL} + \frac{1}{8} \frac{q_0 L}{tc}$$

ELASTO-PLASTIC SOLUTION

During the elastic-plastic solution, the material is

assumed to be purely plastic, because the analytical solution of the beam is almost impossible under the condition of work hardening. The equations of equilibrium for the plane-stress neglecting body force are given as,

$$\begin{aligned}\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} &= 0 \\ \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} &= 0\end{aligned}\quad (25)$$

The composite beam possesses nearly the same yield point in tension and compression due to reinforcing by the steel wires; therefore the Tsai Hill theory is used as a yield criterion in this study. The equivalent stress in a principal material direction is given as,

$$\sigma^2 = \sigma_1^2 - \sigma_1 \sigma_2 + \frac{X^2}{Y^2} \sigma_2^2 + \frac{X^2}{S^2} \tau_{12}^2 - Y^2 \quad (26)$$

where X, Y are the yield points in the 1st and 2nd principal material directions respectively and S is the shear yield strength in the 1-2 plane. The stress components in the principal material directions are written as,

$$\begin{aligned}\sigma_1 &= \sigma_x \cos^2 \theta + 2\tau_{xy} \sin \theta \cos \theta \\ \sigma_2 &= \sigma_x \sin^2 \theta - 2\tau_{xy} \sin \theta \cos \theta \\ \tau_{12} &= -\sigma_x \sin \theta \cos \theta + \tau_{xy} (\cos^2 \theta - \sin^2 \theta)\end{aligned}\quad (27)$$

Writing $\sigma_y = 0$ in the second differential equations of equilibrium gives that τ_{xy} is a function of y in the ordinary form or $\tau_{xy} = f(y)$ and $\frac{\partial \tau_{xy}}{\partial x} = 0$. And putting the stress components σ_1, σ_2 and τ_{12} in Eqn (25) and deriving it with respect to x gives $\frac{\partial \sigma_x}{\partial x} = 0$, integration of which gives $\sigma_x = f(y)$. The plastic region begins at the lower and upper surfaces of the beam when the stress component σ_x reaches,

$$X_1 = \frac{X}{\sqrt{\cos^4 \theta - \sin^2 \theta \cos^2 \theta + \frac{X^2 \sin^4 \theta}{Y^2} + \frac{X^2 \sin^2 \theta \cos^2 \theta}{S^2}}}\quad (28)$$

where θ is the orientation angle of the fibres. When the plastic region starts on these surfaces σ_x becomes X_1 and τ_{xy} equals 0. From this condition, $\sigma_x = X_1$ is found as a constant in the plastic region and shear stress is equal to zero.

The composite cantilever beam is loaded by linearly distributed load q for elasto-plastic solution as shown in Figure 2.

The stress function F was given in Eqn (13). The boundary conditions for the elasto-plastic solution are given as

$$y = -h_1 \Rightarrow \tau_{xy} = 0 \quad (29)$$

Fig. 2: Simply supported composite cantilever beam subjected to a linearly distributed load for elasto-plastic solution

$$y = h_2 \Rightarrow \tau_{xy} = 0 \quad (30)$$

$$\int_{-h_1}^{h_2} \tau_{xy} t dy = -\frac{q_0 x^2}{2L} \quad (31)$$

$$y = -h_1 \Rightarrow \sigma_x = -X_1 \quad (32)$$

$$y = h_2 \Rightarrow \sigma_x = X_1 \quad (33)$$

The resultant of σ_x at any section is equal to zero:

$$X_1 t (c - h_1) - X_1 t (c - h_2) + \int_{-h_1}^{h_2} \sigma_x t dy = 0 \quad (34)$$

The resultant moment of σ_x at any section (x) is equal to the bending moment:

$$X_1 t (c - h_1) \frac{c + h_1}{2} + X_1 t (c - h_2) \frac{c + h_2}{2} + \int_{-h_1}^{h_2} \sigma_x t y dy = \frac{q_0}{2} x \left(L - \frac{x^2}{3L} \right) \quad (35)$$

h_1 is equal to h_2 because of the symmetry of the material properties with respect to the x axis.

From the boundary conditions, the unknown parameters are found as,

$$d_4 = -d_6 x^2 + 2nh_1^2 d_6 + \frac{X_1}{xh_1} \quad (36)$$

$$\frac{1}{2} b_4 x^2 + b_2 = -x^2 h_1^2 d_6 - \frac{1}{2} h_1^4 n d_6 - \frac{1}{2} h_1 \frac{X_1}{x} \quad (37)$$

$$d_6 = \frac{\frac{q_0 x^2}{2Lt} - \frac{2}{3} h_1^2 \frac{X_1}{x}}{\frac{4}{3} x^2 h_1^3 + \frac{8}{15} n h_1^5} \quad (38)$$

$$X_1 c^2 - X_1 h_1^2 - \frac{8}{15} n x h_1^5 d_6 + \frac{2}{3} X_1 h_1^2 = \frac{1}{2} q_0 x \left(L - \frac{x^2}{3L} \right) \quad (39)$$

Solving Equation (39) by the Newton-Raphson method gives h_1 and then the other constants can be determined.

SAMPLE AND DISCUSSION

During the analytical solution steel fiber-reinforced aluminum metal-matrix composite beams are used as shown in Figure 3 /20/. Its mechanical properties and

Fig. 3: Final product of the beam

yield points are given in Table 1. The analytical solution is applied to simply supported composite beams subjected to the linearly distributed load. We neglected the stress component σ_y in comparison with σ_x and τ_{xy} , and chose q_0 as 5 N per mm thickness. Elastoplastic solutions were carried out for orientation angles of 0° , 30° , 45° , 60° and 90° . The thickness and height of the beam have been taken as 8 and 14 mm respectively.

Plastic yielding starts first at the upper and lower surfaces for the 0° , 30° , 45° , 60° and 90° orientation angles as seen in Table 2. The plastically collapsed points are determined for each orientation angle. It is maximum for the 0° orientation angle (Table 3). During the solution, the length of the beam, L , is chosen smaller than the distance of the collapsed points for each orientation angle. The expansion of the plastic region and the residual stress component of the σ_x on the upper and lower surfaces are given in Table 4. As seen in this table, σ_x residual stress component is tensile and compressive on the upper and lower surfaces, respectively. The intensity of the residual stress components of σ_x is maximum on those surfaces. The residual stress component of the τ_{xy} is also given at the x axis in this table, because that is maximum on the x axis. The intensity of the residual stress component of σ_x is maximum for $x = L/\sqrt{3}$.

Table 1
Mechanical properties and yield strengths of the composite beam.

E_1 (MPa)	E_2 (MPa)	G_{12} (MPa)	ν_{12}	Axial Strength X (MPa)	Transverse Strength Y (MPa)	Shear Strength S (MPa)
85000	74000	30000	0.3	230.00	24.00	48.90

Table 2
The lengths of the plastic region at the upper and lower surfaces and length of beams

Orientation Angle	0°		30°		45°		60°		90°	
At the upper and lower surface	154.0	418.0	87.3	231.0	68.0	178.0	58.0	149.1	50.4	133.9
Length of beam	510.000		285.000		220.000		185.000		165.000	

mm]

Table 3
Yield points, and length of the plastic collapse

Orientation Angles	0°	30°	45°	60°	90°
X_1	230.000	71.800	43.090	30.822	24.000
Length (mm)	530.000	296.227	229.487	194.000	171.400

The distribution of the residual stress components of σ_x is given for the 0°, 30°, 45°, 60° and 90° orientation angles as shown in Figure 4. The residual stress component of σ_x is represented along the cross section of the beam at some sections.

CONCLUSIONS

The results given below are concluded in this investigation.

- The magnitude of the residual stress component of σ_x is maximum at the upper and lower surfaces. It is maximum for 0° orientation angle.
- When the orientation angle is increased, the plastic region starts at smaller distances.
- The magnitude of the residual stress component of

σ_x at the upper and lower surfaces is the same for 0°, 30°, 45°, 60° and 90° orientation angles because of the symmetry of the material properties with respect to the x axis.

- The magnitude of the residual stress component of τ_{xy} is greatest on or around the x axis. It is much smaller than that of σ_x .
- The plastic collapse occurs first for 90° oriented beam. It is largest for 0° oriented beam.
- Plastic yielding occurs at the greatest distance from the end for the 0° orientation angle. It is smaller for 90° orientation angle.
- The magnitude The strength of the beam is increased by residual stresses. of the residual stress component of σ_x is maximum for $x = L/\sqrt{3}$
- The strength of the beam is increased by residual stresses.

Table 4The residual stress component of σ_x at the upper and lower surfaces and τ_{xy} on the x axis

θ	x (mm)	h_1 (mm)	σ_x (MPa) at upper surface	σ_x (MPa) at upper surface	σ_{x_r} (MPa) at upper surface	σ_{x_p} (MPa) at lower surface	σ_{x_c} (MPa) at lower surface	σ_{x_r} (MPa) at lower surface	τ_{xyr} (MPa) on the x axis
0°	204.44	5.30	-279.106	-230.000	49.106	279.106	230.000	-49.106	0.676
	234.44	4.34	-300.762	-230.000	70.762	300.762	230.000	-70.762	3.737
	264.44	3.60	-314.504	-230.000	84.504	314.504	230.000	-84.504	7.826
	294.44	3.31	-319.316	-230.000	89.316	319.316	230.000	-89.316	12.03
	324.44	3.62	-314.188	-230.000	84.188	314.188	230.000	-84.188	14.58
	354.44	4.47	-298.105	-230.000	68.105	298.105	230.000	-68.105	15.47
	384.44	5.65	-270.056	-230.000	40.056	270.056	230.000	-40.056	16.03
30°	104.54	5.89	-82.269	-71.800	10.469	82.269	71.800	-10.469	-0.371
	124.54	4.69	-91.624	-71.800	19.824	91.624	71.800	-19.824	1.362
	144.54	3.71	-97.635	-71.800	25.835	97.635	71.800	-25.835	3.905
	164.54	3.30	99.765	-71.800	27.965	99.765	71.800	-27.965	6.747
	184.54	3.75	-97.476	-71.800	25.676	97.476	71.800	-25.676	8.289
	204.54	4.89	-90.232	-71.800	18.432	90.232	71.800	-18.432	8.771
	224.54	6.43	-77.495	-71.800	5.695	77.495	71.800	-5.695	9.191
45°	87.010	5.46	-51.539	-43.090	8.449	51.539	43.090	-8.449	0.175
	97.010	4.72	-54.875	-43.090	11.785	54.875	43.090	-11.785	1.102
	107.010	4.08	-57.367	-43.090	14.277	57.367	43.090	-14.277	2.278
	127.010	3.44	-59.473	-43.090	16.383	59.473	43.090	-16.383	4.996
	147.010	4.14	-57.160	-43.090	14.070	57.160	43.090	-14.070	6.396
	157.010	4.90	-54.130	-43.090	11.040	54.130	43.090	-11.040	6.657
	167.010	5.84	-49.734	-43.090	6.624	49.734	43.090	-6.624	6.879
60°	76.809	5.27	-37.550	-30.822	6.728	37.550	30.822	-6.728	0.422
	86.809	4.47	-39.950	-30.822	9.168	39.950	30.822	-9.168	1.436
	96.809	3.89	-41.533	-30.822	10.711	41.533	30.822	-10.711	2.685
	106.809	3.66	-42.075	-30.822	11.253	42.075	30.822	-11.253	3.950
	116.809	3.90	-41.512	-30.822	10.690	41.512	30.822	-10.690	4.838
	126.809	4.57	-39.740	-30.822	8.918	39.740	30.822	-8.918	5.304
	136.809	5.54	-36.658	-30.822	5.836	36.658	30.822	-5.836	5.804
90°	65.260	5.36	-29.013	-24.000	5.013	29.013	24.000	-5.013	0.152
	75.260	4.35	-31.415	-24.000	7.415	31.415	24.000	-7.415	1.158
	85.260	3.56	-32.944	-24.000	8.944	32.944	24.000	-8.944	2.535
	95.260	3.24	-33.485	-24.000	9.485	33.485	24.000	-9.485	3.979
	105.260	3.59	-32.921	-24.000	8.921	32.921	24.000	-8.921	4.792
	115.260	4.49	-31.137	-24.000	7.137	31.137	24.000	-7.137	5.058
	125.260	5.74	-28.016	-24.000	4.016	28.016	24.000	-4.016	5.228

Fig. 4: Distribution of the residual stress component of σ_x along the sections for 0° , 30° , 45° , 60° and 90° orientation angles, respectively

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