An Investigation of Thermal Elasto-Plastic Stress Analysis of Laminated Thermoplastic Composites with a Circular Hole under Uniform Temperature Loading

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ABSTRACT

This investigation was performed to study the response of a thermal elasto-plastic stress analysis and residual stresses on symmetric cross ply [0/90]_s and angle-ply [30/-30]_s, [45/-45]_s, [60/-60]_s steel fiber reinforced thermoplastic laminated plate, which is subjected to a uniform temperature load with a circular hole. To evaluate the effects of uniform temperature value and stacking sequence of the laminated thermoplastic composite plates on the thermal stress, the study was carried out using the finite element solution. For this purpose the solution was calculated by ANSYS analysis code. The width to diameter (L/D) ratio in the plate was selected as 10. The composite materials were assumed to be linearly hardening. The Tsai-Hill theory was used as a yield criterion. Thermoplastic laminated composite plates were simply supported. All thermal stress values depending on applied temperature are shown in tables. It was found that increasing uniform temperature values and stacking sequence of composite plates considerably affected thermal stresses.

Key words: Thermal stress analysis, Laminated composites, Finite element method (FEM), Thermoplastic composites

1. INTRODUCTION

Availability of the new, low cost, high performance fiber reinforced plastic (FRP) structural composites for infrastructure construction and rehabilitation has given an impetus to the development of a new technology in the construction industry. In infrastructure the applications of the forms of FRP range from strengthening concrete columns, piles, piers, beams, decks, and other concrete structures to using high strength cables or tendons for prestressing or post-tensioning of concrete columns or beams and using the steel rebar profiled reinforcing FRP rods, gratings of different profiles and sections, continuous-fiber isogrids for imbedded concrete reinforcement, external wrappings of columns, or simple bonding of FRP plates to the concrete structure. Inherent lightness, placement flexibility, corrosion resistance, high strength to weight ratio, low relaxation/creep, high fatigue resistance, and magnetic transparency of FRP composites make them attractive as logical substitutes for steel structural elements /1/. Thermal stresses are important in engineering design since they can lead to premature failure or plastic yielding in structures /2/. Laminates, like many structures, must have holes to serve various purposes. An obvious purpose is to accommodate a bolt. Another purpose is to provide access from one side of

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the laminate to the other. The analysis of the stresses around holes is quite difficult /3/. Thermal stress analyses for bars, beams, plates and cylinders are more easily understandable /4/.

Sayman /2/ carried out a thermal elastic-plastic and residual stress analysis on symmetric cross-ply and angle ply thermoplastic laminated plates. In that study temperature changed linearly along the cross sections of the plates. Wang *et al.* /5/ offered an analytical method in order to determine the thermal residual stresses in an isotropic plate reinforced with a circular orthotropic patch. Akay and Ozden /6/ obtained thermal residual stresses in injection-moulded thermoplastics by removing thin layers from specimens. Sayman *et al.* /7/ carried out elastic-plastic stress analysis on a steel-fiber-reinforced high-density thermoplastic composite cantilever beam loaded by a bending moment at its free end. Sayman and Esendemir /8/ presented a low density polyethylene thermoplastic composite reinforced by woven steel fibers supported at the ends acted upon with a force at the midpoint. Jeronimidis and Parkyn /9/ studied residual stresses in carbon fiber-thermoplastic matrix laminates. Sayman and Kayrici /10/ studied elasto-plastic stress analysis on a thermoplastic composite cantilever beam subjected to a single transverse force applied to the free end of the beam. Senel *et al.* /11/ performed residual stress analysis of laminated composite plates under thermal loads using analytical methods. Chung /12/ studied the thermal analysis of carbon fiber polymer-matrix composites by electrical resistance measurement. Gigliotti *et al.* /13/, focused on the out-of-plane displacements, i.e., curvatures, induced by uniform thermal fields in [0/90] laminated square plates. In Sayman *et al.* /14/, an elastic-plastic thermal stress analysis was carried out on a steel fiber reinforced thermoplastic matrix composite beam. In that study temperature was chosen to vary linearly, that was zero and T_0 at the upper and lower surfaces, respectively. Shabana and Noda /15/ performed a thermo-elasto-plastic stress analysis in functionally graded materials subjected to thermal loading. They obtained elastic-plastic stresses in a rectangular plate under three kinds of temperature conditions by using finite element technique. Ji-Fan /16/ used a discrete-layer shear deformation laminated plate theory in order to analyze the steady-state thermal

stresses in laminated plates. Bektas and Sayman /17/ carried out an analytical elasto-plastic stress analysis on thermoplastic laminated plates. Apalak *et al.* /18/ carried out a transient thermal analysis of an adhesively bonded and laser-spot welded joint, based on a thermal model developed for the laser-spot welding of multi-layered sheets using a pulsed Nd:YAG laser /18/.

This paper is concerned with the prediction and examination of the thermal elasto-plastic stress analysis of steel fiber reinforced symmetric thermoplastic composite plates with a circular hole. Finite element method (FEM) was used to compute the thermal stresses under various uniform temperatures loading. In addition thermal residual stress components were calculated, therefore the solution is also carried out in two steps, as thermal elastic stress analysis and thermal elasto-plastic stress analysis.

2. PROBLEM STATEMENT

Consider a square composite plate of length $L=200$ mm, thickness $h=8$ mm with a hole of diameter $d=20$ mm is shown in Figure 1-a. The laminates with symmetric stacking sequences of cross ply [0°/90°]_s and angle ply [30°/-30°]_s, [45°/-45°]_s, [60°/-60°]_s were chosen to investigate and these arrangements are displayed as schematic in Figure 1-b, too. Thermoplastic laminated composite plates were simply supported at all edges.

The distributions of thermal stress components were calculated by FEM, so ANSYS, that is a general-purpose finite element code, was used for solving the problem. Element type was selected as SHELL 181, which is suitable for analyzing thin to moderately thick shell structures. It is a four-noded element with six degrees of freedom at each node: translations in the x, y, and z directions, and rotations about the x, y, and z-axes. The degenerate triangular option should only be used as filler elements in mesh generation. The geometry, node locations, and the coordinate system for this element are shown in Figure 2. Four nodes define the element: I, J, K, and L. Element formulation is based on logarithmic strain and true stress measures. Element kinematics allow for finite membrane strains (stretching) /19/. In addition, description of stress output for SHELL 181 element type is illustrated in Figure 3 /19/.

The schematic finite element model of the composite plate with circular hole is shown in Figure 4. As can be seen in this figure, a mapped mesh shape is chosen for the construction of the model since, compared to a free mesh; a mapped mesh is limited in terms of the element shape it contains and the pattern of the mesh. In addition, when we require this type of mesh, we must construct the geometry as a series of fairly regular volumes [19-20]. Meanwhile, 1453 elements and 1547 nodes modeled the laminated plates, after the mesh process.

Uniform temperature load is applied to the whole of the laminated plate. To investigate the effect of increasing temperature value on thermal stresses, each model was subjected to loading under 60, 70, 80 and 90°C uniform temperature. In addition, to examine residual stresses, elastic and elasto-plastic solutions were performed for these models.

2.1 Manufacturing of the Composite Plate

The composite material was prepared in Dokuz Eylul University, Mechanics Laboratory. The

thermoplastic laminated plate consists of high-density polyethylene (HDPE) as a thermoplastic matrix material and steel fibers, as shown in Figure 5 of a preceding study [17]. Therefore, it is named steel fiber reinforced thermoplastic laminated composite in the literature. First, polyethylene granules are put into the moulds and the temperature of the electrical heater is increased up to 190°C. Following this, the material is held for 5 minutes under 2.5 MPa at this temperature value. Then, the temperature of the electrical heater is decreased to 30°C under 15 MPa pressure value for 3 minutes and a polyethylene layer is produced. Finally, the steel fibers are put between two thermoplastic layers and processed as the former process. Each layer of fiber-reinforced thermoplastic is of 2 mm thickness. The final laminated plate is made from four layers so that it has an 8 mm thickness. The mechanical properties of the plate are shown in Table 1. These mechanical properties are found experimentally. The thermoplastic composite material is assumed to be linearly hardening. In addition, it is assumed that the mechanical properties do not change with increasing temperature.

(a)

(b)

Fig. 1: a) Geometry of a laminated thermoplastic composite plate with a circular hole
b) Schematic view of symmetric stacking sequence of laminates

Table 1
Mechanical properties and yield strengths of the composite layer

E_1 (MPa)	E_2 (MPa)	G_{12} (MPa)	K (MPa)	ν_{12}	Axial Strength X (MPa)	Transverse Strength Y (MPa)	Shear Strength S (MPa)	Thickness of each layer (mm)	Thermal expansion coefficients ($1/^\circ\text{C}$)	
									α_1	α_2
41000	1200	420	76	0.25	39	5	14	2	13.1E-06	131E-06

x_{IJ} = Element x-axis if ESYS is not supplied.

x = Element x-axis if ESYS is supplied.

Fig. 2: The geometry, node locations, and the coordinate system for SHELL 181 element /19/

Fig. 3: Description of stress output for SHELL 181 element type /19/

Fig. 4: The schematic finite element model of the composite plate with a circular hole

Fig. 5: Manufacturing of the thermoplastic composite material

2.2 Mathematical Formulation

The three-dimensional thermoelastic anisotropic strain-stress relations are [3],

$$\varepsilon_i = S_{ij}\sigma_j + \alpha_i\Delta T \quad i, j = 1, 2, \dots, 6 \quad (1)$$

where the total strains, ε_i , are the sum of the mechanical strains, $S_{ij}\sigma_j$, and the six free thermal strains, $\alpha_i\Delta T$, for a temperature change ΔT . The three-dimensional stress-strain relations are obtained by inversion:

$$\sigma_i = C_{ij}(\varepsilon_j - \alpha_j\Delta T) \quad i, j = 1, 2, \dots, 6 \quad (2)$$

In both equations (1) and (2), the six α_i are the coefficients of thermal deformation (expansion or contraction and distortion, i.e., shear), and ΔT is the temperature difference. In equation (2), the terms $C_{ij}\alpha_j\Delta T$ are the thermal stresses if the total strain is zero [3].

For plane stress on an orthotropic lamina in principal material coordinates,

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 - \alpha_1\Delta T \\ \varepsilon_2 - \alpha_2\Delta T \\ \gamma_{12} \end{bmatrix} \quad (3)$$

The stresses in laminate coordinates for the k^{th} layer are obtained by transformation of coordinates.

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}_k = \begin{bmatrix} \overline{Q_{11}} & \overline{Q_{12}} & \overline{Q_{16}} \\ \overline{Q_{12}} & \overline{Q_{22}} & \overline{Q_{26}} \\ \overline{Q_{16}} & \overline{Q_{26}} & \overline{Q_{66}} \end{bmatrix} \begin{bmatrix} \varepsilon_x - \alpha_x \Delta T \\ \varepsilon_y - \alpha_y \Delta T \\ \gamma_{xy} - \alpha_{xy} \Delta T \end{bmatrix}_k \quad (4)$$

The layer stresses are integrated through the thickness; the force resultants are,

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} K_x \\ K_y \\ K_{xy} \end{bmatrix} - \begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} \quad (5)$$

in which the A_{ij} and B_{ij} are the usual extensional and bending-extension coupling stiffnesses and the thermal forces are

$$\begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} = \int \begin{bmatrix} \overline{Q_{11}} & \overline{Q_{12}} & \overline{Q_{16}} \\ \overline{Q_{12}} & \overline{Q_{22}} & \overline{Q_{26}} \\ \overline{Q_{16}} & \overline{Q_{26}} & \overline{Q_{66}} \end{bmatrix}_k \begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{bmatrix}_k \Delta T dz \quad (6)$$

In a similar manner, the moment resultants are obtained by integrating the moment of the stresses through the thickness $z/3$:

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} K_x \\ K_y \\ K_{xy} \end{bmatrix} - \begin{bmatrix} M_x^T \\ M_y^T \\ M_{xy}^T \end{bmatrix} \quad (7)$$

in which the D_{ij} are the usual bending stiffnesses and the thermal moments are,

$$\begin{bmatrix} M_x^T \\ M_y^T \\ M_{xy}^T \end{bmatrix} = \int \begin{bmatrix} \overline{Q_{11}} & \overline{Q_{12}} & \overline{Q_{16}} \\ \overline{Q_{12}} & \overline{Q_{22}} & \overline{Q_{26}} \\ \overline{Q_{16}} & \overline{Q_{26}} & \overline{Q_{66}} \end{bmatrix}_k \begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{bmatrix}_k \Delta T dz \quad (8)$$

Actually, only in the restricted case of perfect constraint are the N^T and M^T thermal forces and moments, respectively. However, the force and moment resultants can be rearranged to read,

$$\begin{bmatrix} \overline{N_x} \\ \overline{N_y} \\ \overline{N_{xy}} \end{bmatrix} = \begin{bmatrix} N_x + N_x^T \\ N_y + N_y^T \\ N_{xy} + N_{xy}^T \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \quad (9)$$

$$\begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} K_x \\ K_y \\ K_{xy} \end{bmatrix}$$

$$\begin{bmatrix} \overline{M_x} \\ \overline{M_y} \\ \overline{M_{xy}} \end{bmatrix} = \begin{bmatrix} M_x + M_x^T \\ M_y + M_y^T \\ M_{xy} + M_{xy}^T \end{bmatrix} =$$

$$\begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \quad (10)$$

$$\begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} K_x \\ K_y \\ K_{xy} \end{bmatrix}$$

In the form of equations (9) and (10), the thermal portion of thermal and mechanical stress problems can be treated as equivalent mechanical loads defined by N^T and M^T in Equations (6) and (8), respectively in addition to the mechanical loads, N and $M/3$.

The temperature causing yielding of the layers can be found by using the stress components. The stress components in the principal material directions are

written as,

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} \cos^2 & \sin^2 & 2\sin\cos \\ \sin^2 & \cos^2 & -2\sin\cos \\ -\sin\cos & \sin\cos & \cos^2 - \sin^2 \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} \quad (11)$$

These principal stresses are substituted into the Tsai-Hill criterion in order to find the temperature, which is causing yielding of a lamina [3].

The elasto-plastic solution is performed for small plastic deformations. The Tsai-Hill theory is used as a yield criterion because of the same yield points in tension and compression for fiber reinforced thermoplastic composite layers. The criterion can be written below as [3],

$$\frac{\sigma_1^2}{X^2} + \frac{\sigma_2^2}{Y^2} - \left(\frac{1}{X^2} + \frac{1}{Y^2} - \frac{1}{Z^2} \right) \sigma_1 \sigma_2 + \frac{\tau_{12}^2}{S^2} = 1 \quad (12)$$

Multiplying Eq.(12) by X^2 gives us in the first principal material direction as,

$$\bar{\sigma}_{eq} = \sqrt{\sigma_1^2 - X^2 \left(\frac{1}{X^2} + \frac{1}{Y^2} - \frac{1}{Z^2} \right) \sigma_1 \sigma_2 + \frac{\sigma_2^2 X^2}{Y^2} + \frac{\tau_{12}^2 X^2}{S^2}} \quad (13)$$

The yield strengths of the lamina in the second and third principal material directions are the same, owing to the same alignment of the fibers in these directions, so that Eq. 13 can be written as,

$$\bar{\sigma}^2 = \sigma_1^2 - \sigma_1 \sigma_2 + \frac{\sigma_2^2 X^2}{Y^2} + \frac{\tau_{12}^2 X^2}{S^2} \quad (14)$$

where $\bar{\sigma}$ (equivalent stress) is given by Ludwik's equation as,

$$\bar{\sigma} = \sigma_0 + K \epsilon_p \quad (15)$$

To calculate the residual stresses it is necessary to superpose on the thermal stress system. The superposition of the elastic stresses and plastic stresses

provides the residual stress values as,

$$\begin{aligned} (\sigma_x)_r &= (\sigma_x)_p - (\sigma_x)_e \\ (\sigma_y)_r &= (\sigma_y)_p - (\sigma_y)_e \\ (\tau_{xy})_r &= (\tau_{xy})_p - (\tau_{xy})_e \end{aligned} \quad (16)$$

For this purpose, the solution was performed in two phases using the finite element method by ANSYS program. Firstly, elastic solution was applied on the thermoplastic composite plate. Then elasto-plastic solution was applied on it. Finally, residual stresses were obtained using Eq. 16.

3. RESULTS AND DISCUSSION

In this investigation, a thermal elastic, elasto-plastic and residual stress analysis of symmetric steel fiber reinforced thermoplastic laminated composite plates with a central circular hole, under different uniform temperature loadings, have been calculated using finite element analysis. The width plate-to-the hole diameter, L/D, ratio was selected as 10. The laminated composite plates are assumed to be simple supported. Temperature values were chosen as 60, 70, 80 and 90°C and temperature loadings were uniform. The symmetric laminated composite plates with a circular hole diameters were composed of four orthotropic layers oriented with cross-ply, $[0^\circ/90^\circ]_s$, and angle ply, $[30^\circ/-30^\circ]_s$, $[45^\circ/-45^\circ]_s$, $[60^\circ/-60^\circ]_s$ in a symmetrical manner.

The magnitude of thermal stress components at the upper and lower surfaces of symmetric plates were the same for all uniform temperature loading, therefore they are presented in the tables only for upper surfaces.

The thermal elastic, elasto-plastic, residual stress components, and equivalent plastic strain depending on applied uniform temperature for x and y direction (σ_x , σ_y and τ_{xy}) for symmetric cross-ply, $[0^\circ/90^\circ]_s$ and angle ply, $[30^\circ/-30^\circ]_s$, $[45^\circ/-45^\circ]_s$, $[60^\circ/-60^\circ]_s$ laminated plate (Nodes A, B, C and D) with a circular hole are shown in Tables 2 and 3, respectively. It can be seen from these tables that the magnitude of the thermal elastic, elasto-plastic stresses and equivalent plastic strain increases related to rise of uniform temperature values, so the

highest values of thermal stress components and equivalent plastic strain are obtained for 90 °C uniform temperature loading.

Meanwhile, for all uniform temperatures; the absolute values of stress components σ_x and σ_y are the same for cross ply $[0^\circ/90^\circ]_s$ laminated plates in Table 2. This situation similar for angle ply $[45^\circ/-45^\circ]_s$ laminated plates in Table 3. However, the absolute values of thermal stress components for x and y directions show completely different values from each other for angle ply $[30^\circ/-30^\circ]_s$, and $[60^\circ/-60^\circ]_s$ laminated plates.

Maximum values of tensile elastic and elasto-plastic stresses depending on applied uniform temperature for symmetric cross-ply and angle-ply laminated plate with a circular hole are shown in Table 4, and maximum

values of compressive elastic and elasto-plastic stresses depending on applied uniform temperature for symmetric cross-ply and angle-ply laminated plate with a circular hole are illustrated in Table 5. As seen in these tables, absolute thermal stress values increase depending on increasing of applied uniform temperature loadings. Moreover, maximum tensile stress and maximum compressive stress values are calculated on angle-ply $[60^\circ/-60^\circ]_s$ laminated plate while minimum tensile stress and minimum compressive stress values are obtained on angle-ply $[45^\circ/-45^\circ]_s$ laminated plate. It is known that a composite structure can be damaged influence of uniform temperature any point or zone, for this reason Table 4 and 5 are given a thought related to failure of laminated thermoplastic composite structure.

Table 2

Elastic, elasto-plastic, residual stress components, and equivalent plastic strain for symmetric cross/ply $[0^\circ/90^\circ]_s$ laminated plate with a circular hole

T (°C)	Node	$(\sigma_x)_p$ (MPa)	$(\sigma_y)_p$ (MPa)	$(\tau_{xy})_p$ (MPa)	$(\sigma_x)_e$ (MPa)	$(\sigma_y)_e$ (MPa)	$(\tau_{xy})_e$ (MPa)	$(\sigma_x)_r$ (MPa)	$(\sigma_y)_r$ (MPa)	$(\tau_{xy})_r$ (MPa)	ϵ_p (10^{-6})
60	A	4.670	-4.670	0.896	7.694	-7.694	1.477	-3.024	3.024	-0.581	349
	B	4.915	-4.915	-0.168	8.097	-8.097	-0.480	-3.182	3.182	0.312	349
	C	-4.542	4.542	-1.504	-7.483	7.483	-2.478	2.941	-2.941	0.974	349
	D	-4.915	4.915	0.261	-8.097	8.097	-0.556	3.182	-3.182	0.817	349
70	A	4.672	-4.672	0.897	8.976	-8.976	1.723	-4.304	4.304	-0.826	496
	B	4.916	-4.916	0.952	9.446	-9.446	-0.539	-4.53	4.53	1.491	496
	C	-4.543	4.543	-1.505	-8.730	8.730	-2.891	4.187	-4.187	1.386	496
	D	-4.916	4.916	-0.521	-9.446	9.446	-0.685	4.53	-4.53	0.164	496
80	A	4.673	-4.673	0.897	10.259	-10.259	1.969	-5.586	5.586	-1.072	644
	B	4.918	-4.918	-0.157	10.796	-10.796	-0.681	-5.878	5.878	0.524	644
	C	-5.545	4.545	-1.505	-9.997	9.997	-3.304	4.452	-5.452	1.799	644
	D	-4.918	4.918	0.186	-10.796	10.796	-0.117	5.878	-5.878	0.303	644
90	A	4.674	-4.674	0.897	11.541	-11.541	2.216	-6.867	6.867	-1.319	792
	B	4.919	-4.919	-0.919	12.146	-12.146	-0.746	-7.227	7.227	-0.173	792
	C	-4.546	4.546	1.468	-11.225	11.225	-3.717	6.679	-6.679	5.185	792
	D	-4.919	4.919	0.624	-12.146	12.146	-0.123	7.227	-7.227	0.747	792

Table 3

Elastic, elasto-plastic, residual stress components, and equivalent plastic strain for symmetric angle/ply $[30^\circ/-30^\circ]_s$, $[45^\circ/-45^\circ]_s$, $[60^\circ/-60^\circ]_s$, laminated plate with a circular hole

θ	T (°C)	Node	$(\sigma_x)_p$ (MPa)	$(\sigma_y)_p$ (MPa)	$(\tau_{xy})_p$ (MPa)	$(\sigma_x)_e$ (MPa)	$(\sigma_y)_e$ (MPa)	$(\tau_{xy})_e$ (MPa)	$(\sigma_x)_r$ (MPa)	$(\sigma_y)_r$ (MPa)	$(\tau_{xy})_r$ (MPa)	ε_p (10^{-6})
$[30^\circ/-30^\circ]_s$	60	A	-0.326	4.681	4.107	-0.177	4.842	5.030	-0.149	-0.161	-0.923	0
		B	0.099	-2.115	3.881	0.104	-3.538	4.891	-0.005	1.423	-1.010	184
		C	5.921	-1.289	-4.804	5.865	-0.618	-4.824	0.056	-0.671	0.020	0
		D	-2.094	-0.219	-3.762	-3.706	-0.434	-4.559	1.612	0.215	0.797	170
	70	A	-0.331	4.689	4.222	-0.206	5.649	5.868	-0.125	-0.96	-1.646	0
		B	0.102	-2.117	3.884	0.121	-4.127	5.706	-0.019	2.010	-1.822	325
		C	5.927	-0.948	-5.555	6.843	-0.721	-5.628	-0.916	-0.227	0.073	0
		D	-2.116	-0.225	-3.781	-4.323	-0.506	-5.319	2.207	0.281	1.538	305
	80	A	-0.373	4.777	4.765	-0.236	6.456	6.706	-0.137	-1.679	-1.941	0
		B	0.105	-2.118	3.886	0.138	-4.717	6.521	-0.033	2.599	-2.635	468
		C	5.977	-1.290	-6.296	7.820	-0.824	-6.433	-1.843	-0.466	0.137	0
		D	-2.126	-0.227	-3.795	-4.941	-0.579	-6.079	2.815	0.352	2.284	445
	90	A	-0.376	4.881	5.107	-0.266	7.263	7.545	-0.110	-2.382	-2.438	84
		B	0.199	-2.225	3.889	0.156	-5.306	7.337	0.043	3.081	-3.448	611
		C	5.981	-1.298	-6.804	8.798	-0.927	-7.237	-2.817	-0.371	0.433	66
		D	-2.194	-0.229	-3.802	-5.559	-0.651	-6.839	3.365	0.422	3.037	587
$[45^\circ/-45^\circ]_s$	60	A	-0.896	0.896	4.670	-1.477	1.477	7.694	0.581	-0.581	-3.024	349
		B	0.664	-0.664	4.915	0.169	0.452	8.097	0.495	-1.116	-3.182	349
		C	1.504	-1.504	-4.542	2.478	-2.478	-7.483	-0.974	0.974	2.941	349
		D	0.664	0.664	-4.915	-0.413	0.454	-8.097	1.077	0.21	3.182	349
	70	A	-0.897	0.897	4.672	-1.723	1.723	8.976	0.826	-0.826	-4.304	496
		B	0.804	-0.804	4.916	0.612	-0.861	9.446	0.192	0.057	-4.530	496
		C	1.505	-1.505	-4.543	2.891	-2.891	-8.730	-1.386	1.386	4.187	496
		D	0.804	0.804	-4.916	-0.490	-0.595	-9.446	1.294	1.399	4.530	496
	80	A	-0.898	0.898	4.674	-1.969	1.969	10.259	1.071	-1.071	-5.585	644
		B	0.855	-0.855	4.918	0.639	0.864	10.796	0.216	-1.719	-5.878	644
		C	1.506	-1.506	-4.544	3.304	-3.304	-9.977	-1.798	1.798	5.433	644
		D	0.805	0.805	-4.917	-0.639	-0.821	-10.796	1.444	1.626	5.879	644
	90	A	-0.899	0.899	4.675	-2.216	2.216	11.541	1.317	-1.317	-6.866	792
		B	0.906	-0.906	4.932	0.666	0.985	12.146	0.240	-1.891	-7.214	792
		C	1.507	-1.507	-4.535	3.717	-3.717	-11.225	-2.210	2.210	6.690	792
		D	0.806	0.806	-4.918	-0.706	-0.829	-12.146	1.512	1.635	7.228	792
$[60^\circ/-60^\circ]_s$	60	A	-1.275	-9.966	1.656	-1.540	-15.622	-1.837	0.265	5.656	3.493	0
		B	-0.019	8.156	7.667	-0.200	12.966	11.922	0.181	-4.810	-4.255	312
		C	-14.289	-2.011	-0.410	-23.215	-2.047	3.667	8.926	0.036	-4.077	0
		D	8.193	0.025	-7.660	12.679	0.035	-11.779	-4.486	-0.010	4.119	298

Table 3 (continued)

70	A	-1.412	-9.977	1.670	-1.797	-17.225	-2.143	0.385	7.248	3.813	0
	B	-0.021	8.210	7.679	-0.023	15.127	13.909	0.002	-6.917	-6.23	458
	C	-14.913	-2.380	-0.455	-27.085	-2.388	4.278	12.172	0.008	-4.733	0
	D	8.212	0.026	-7.675	14.792	0.039	-13.742	-6.58	-0.013	6.067	443
80	A	-1.597	-10.190	1.679	-2.054	-20.829	-2.450	0.457	10.639	4.129	336
	B	-0.026	8.266	7.691	-0.028	17.288	15.896	0.002	-9.022	-8.205	604
	C	-14.675	-2.534	0.465	-30.954	-2.729	4.889	16.279	0.195	-4.424	399
	D	8.236	0.027	-7.692	16.905	0.044	-15.705	-8.669	-0.017	8.013	587
90	A	-1.620	-10.583	0.213	-2.311	-23.433	-2.756	0.691	12.850	2.969	382
	B	-0.028	8.324	7.705	-0.030	19.448	17.883	0.002	-11.124	-10.178	750
	C	-14.747	-2.784	0.952	-34.823	-3.070	5.501	20.076	0.286	-4.549	441
	D	8.241	0.028	-7.711	19.018	0.050	-17.668	-10.777	-0.022	9.957	732

Table 4

Maximum values of tensile elastic and elasto-plastic stresses for symmetric cross/ply $[0^\circ/90^\circ]_s$ and angle/ply $[30^\circ/-30^\circ]_s$, $[45^\circ/-45^\circ]_s$ and $[60^\circ/-60^\circ]_s$ laminated plates with a circular hole

θ	T (°C)	$(\sigma_x)_p$ (MPa)	$(\sigma_y)_p$ (MPa)	$(\tau_{xy})_p$ (MPa)	$(\sigma_x)_e$ (MPa)	$(\sigma_y)_e$ (MPa)	$(\tau_{xy})_e$ (MPa)
$[0^\circ/90^\circ]_s$	60	4.916	4.916	4.782	8.097	8.097	7.878
	70	4.917	4.917	4.784	9.447	9.447	9.191
	80	4.918	4.918	4.785	10.796	10.796	10.504
	90	4.920	4.920	4.787	12.146	12.146	11.817
$[30^\circ/-30^\circ]_s$	60	16.323	23.667	12.153	29.603	44.106	20.597
	70	16.635	23.952	12.291	34.536	51.457	24.030
	80	16.875	24.195	12.475	39.470	58.808	27.462
	90	16.889	24.370	12.590	44.404	66.159	30.895
$[45^\circ/-45^\circ]_s$	60	4.809	4.782	4.916	7.922	7.878	8.097
	70	4.811	4.784	4.917	9.243	9.191	9.447
	80	4.812	4.785	4.918	10.563	10.504	10.796
	90	4.814	4.787	4.920	11.884	11.817	12.146
$[60^\circ/-60^\circ]_s$	60	22.454	9.289	9.191	40.418	20.324	15.270
	70	22.908	9.682	9.273	47.154	23.711	17.815
	80	23.293	10.296	9.366	53.890	27.099	20.360
	90	23.699	10.479	9.805	60.627	30.486	30.486

Table 5

Maximum values of compressive elastic and elasto-plastic stresses for symmetric cross-ply $[0^\circ/90^\circ]_s$ and angle/ply $[30^\circ/-30^\circ]_s$, $[45^\circ/-45^\circ]_s$ and $[60^\circ/-60^\circ]_s$ laminated plates with a circular hole

θ	T (°C)	$(\sigma_x)_p$ (MPa)	$(\sigma_y)_p$ (MPa)	$(\tau_{xy})_p$ (MPa)	$(\sigma_x)_e$ (MPa)	$(\sigma_y)_e$ (MPa)	$(\tau_{xy})_e$ (MPa)
$[0^\circ/90^\circ]_s$	60	-4.916	-4.916	-4.809	-8.097	-8.097	-7.922
	70	-4.917	-4.917	-4.811	-9.447	-9.447	-9.243
	80	-4.918	-4.918	-4.812	-10.796	-10.796	-10.563
	90	-4.920	-4.920	-4.814	-12.146	-12.146	-11.884
$[30^\circ/-30^\circ]_s$	60	-8.307	-6.526	-11.329	-14.126	-10.696	-17.318
	70	-8.414	-6.620	-11.432	-16.480	-12.479	-20.204
	80	-8.521	-6.724	-11.537	-18.834	-14.261	-23.091
	90	-8.605	-6.824	-11.559	-21.189	-16.044	-25.977
$[45^\circ/-45^\circ]_s$	60	-4.782	-4.809	-4.916	-7.878	-7.922	-8.097
	70	-4.784	-4.811	-4.917	-9.191	-9.243	-9.447
	80	-4.785	-4.812	-4.918	-10.504	-10.563	-10.796
	90	-4.787	-4.814	-4.920	-11.817	-11.884	-12.146
$[60^\circ/-60^\circ]_s$	60	-15.472	-13.523	-14.266	-25.902	-23.582	-26.435
	70	-15.726	-13.996	-14.474	-30.219	-27.513	-30.841
	80	-16.561	-15.206	-14.706	-34.536	-31.443	-35.247
	90	-17.369	-16.460	-14.915	-38.853	-35.773	-35.373

CONCLUSIONS

In the present investigation; the thermal elastic, elasto-plastic and residual stresses in symmetric steel fiber reinforced thermoplastic laminated composite plates with a central circular hole under uniform temperature effect were performed by using finite elements method (FEM). Thermoplastic laminated composite plates were assumed to be simply supported and the Tsai-Hill theory was used as a yield criterion. For this purpose a general-purpose software code ANSYS was used during the solution. From the study results presented it can be concluded that:

1. The magnitudes of the elastic, elasto-plastic, residual stress and equivalent plastic strain are increased depending on increasing of uniform temperature value.
2. Stacking sequence and orientation angles of laminated plates clearly affect thermal stress components.

3. The uniform temperature causes plastic yielding in the symmetric cross-ply $[0^\circ/90^\circ]_s$, and angle ply $[30^\circ/-30^\circ]_s$, $[45^\circ/-45^\circ]_s$, $[60^\circ/-60^\circ]_s$ laminated plates.
4. The magnitudes of the thermal stress components at the upper and lower layers of symmetric laminates are the same.
5. The selected uniform temperature values produce residual stress all stacking sequence of plates.
6. The magnitude of the equivalent plastic strain for the cross-ply $[0^\circ/90^\circ]_s$, and angle ply $[45^\circ/-45^\circ]_s$ laminated plates is higher than that of angle ply $[30^\circ/-30^\circ]_s$, $[60^\circ/-60^\circ]_s$.

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