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Axioms and Structures of Conventional Accounting Measurement

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Abstract: This paper probes into the foundations of conventional accounting measurements in order to construct a relatively simple axiom system on which a purely mathematical measurement system can be erected and thus provide a consistent basis for examining pertinent aspects of conventional accounting practices. [...] We take conventional accounting measurement as given rather than, *ab initio*, seeking to prescribe what we think accounting measurement should be. [...] Our attempt has been directed toward *approximating* conventional accounting by a relatively simple set of axioms and measurement rules, in the same manner that scientists in physics or chemistry have tried to develop a relatively simple set of concepts and theories in order to explain, in satisfactory degrees of approximation, complicated phenomena in this world.

Keywords: accounting and economics, measurement theory, historical cost accounting

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Memorial Symposium for Professor Yuji Ijiri

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I Introduction

1. This paper probes into the foundations of conventional accounting measurements in order to construct a relatively simple axiom system on which a purely mathematical measurement system can be erected and thus provide a consistent basis for examining pertinent aspects of conventional accounting practices.
2. The objective of this paper -as well as its mode of development- needs to be distinguished from other, prior, attempts to axiomatize accounting.¹
 - i. Unlike the attempt by Moonitz and Sprouse² the object of our analysis is to develop and elucidate a uniform approach to *conventional* accounting

¹ The first such attempt appears to be due to E. L. Kohler. See the definition of "axiom" and related terms in (Kohler, 1952). See also (American Accounting Association 1936, 1941; 1948; 1951; 1957; American Institute of Certified Public Accountants, 1952; Canning, 1929; Chambers, 1955; Gilman, 1939), (Littleton, 1953; Mattessich, 1957; 1964; Moonitz, 1961; Paton & Littleton, 1940), (Sanders, Hatfield & Moore, 1938; Sprouse & Moonitz, 1962; Vatter, 1947) for a more general approach to systematizing accounting principles and practices: In particular (Mattessich, 1957) and (Moonitz, 1961) discuss specifically axiomatic approaches to accounting.

² See Moonitz (Moonitz, 1961) and Sprouse and Moonitz (Sprouse & Moonitz, 1962).

measurement as such. That is, we take conventional accounting measurement as given rather than, *ab initio*, seeking to prescribe what we think accounting measurement should be.

- ii. Conventional accounting is analyzed in this paper from a purely mathematical viewpoint of measurement. Hence, we are not interested in factors which do not affect the measure. We consider any two accounting systems as being equal if the measures derived from the systems are equal for any set of inputs to the systems.³ Therefore, unlike the approach by Mattessich adopted in (Mattessich, 1957) in which an emphasis is placed on a double classification scheme in accounting⁴ distinctions are not made in our analysis between an accounting system based upon double-entry bookkeeping and an accounting system based upon single-entry bookkeeping as long as methods of recording do not affect the measure which is of our primary interest.
- iii. The measurement system of conventional accounting is here viewed as though it consists of a set of axioms on the one hand and a set of measurement rules on the other hand. These are to be extracted from conventional accounting in such a way that the set of axioms and the set of measurement rules thus developed are not only *necessary* but also *sufficient* to explain the majority of the principles and practices in conventional accounting. Unfortunately, conventional accounting is a collection of many different accounting principles and practices. Moreover, in some cases, they are mutually inconsistent and hence no systematic theories can describe all of them. Therefore, numerous axioms and measurement rules may be needed to cover *all* of the principles and practices in conventional accounting, whereas there may be no axioms and measurement rules that can be used commonly to explain *every* principle and procedure in conventional accounting. Thus, our attempt has been directed toward *approximating* conventional accounting by a relatively simple set of axioms and measurement rules, in the same manner that scientists in physics or chemistry

³ Two functions f and g are said to be equal if both are defined on the same domain X and

$$f(x) = g(x) \quad (1.1)$$

for every x in X . The same definition of equality applies to the equality of accounting systems in our discussions. In order to avoid any possibility of misunderstanding, we would like to postpone stating our view on what the things to be measured in conventional accounting are until we have presented all axioms and measurement rules.

⁴ See, however, his recent work in (Mattessich, 1964) for a different approach.

have tried to develop a relatively simple set of concepts and theories in order to explain, in satisfactory degrees of approximation, complicated phenomena in this world.

- iv. The set of axioms and the set of measurement rules developed in this paper correspond to the set of axioms and the set of theorems (including lemmas and corollaries), respectively, in Euclidean geometry in the sense that if the set of axioms is granted, the measurement rules can be developed in a purely mathematical way without requiring any empirical judgment. Furthermore, though the truth of the axioms cannot be proved (in the purely philosophical sense), they are empirically supported since the axioms are abstracted from what accountants have been doing in their daily practices. In addition, the number of axioms is minimized to avoid any redundant axioms coming into the set; i. e., the set of axioms developed here is necessary and sufficient to support the set of measurement rules, which is another half of the whole measurement system. Therefore, our set of axioms is *not a mere listing of concepts* that we think necessary for conventional accounting, but is tied in logically and mathematically with the set of measurement rules and hence with the whole measurement system.

II Axioms

A Introduction

1. The set of axioms necessary for the System of accounting measurement that will approximate conventional accounting measurement consists of Axiom of Quantities, Axiom of Ownership, and Axiom of Exchanges. Since these three axioms are related to each other, one of the three axioms cannot be explained fully without having explained the rest of the axioms. Hence, we shall first explain each axiom separately in a less precise form and then state the three axioms in a more precise form.

B Axiom of quantities

1. Essentially, this axiom states that all objects which may be pertinent to accounting are quantifiable – i. e., measurable by some methods such as amount, volume, weight, length, time (service life), etc.

2. To state it more precisely:
 - i. Objects dealt with in accounting can be partitioned into a countable collection of classes (i. e., each object is classified into one and only one class).
 - ii. For each class, there exists a function called a *physical measure* by which a unique number (quantity) is determined for the set of all objects in the class or for any subset of it.
 - iii. For each class, if two sets of objects in the same class are of the same quantity derived by a physical measure for the class they are mutually substitutable for the purpose of exchanges.⁵

The above statements will be made precise after the axioms of ownership and exchanges are introduced and similarly explained. For the moment, however, we may proceed to elucidate this axiom further, independently from the rest of the three axioms.

- 3 This axiom may be regarded as an axiom of classification based upon substitutability of objects. The physical measure defined for each class is used to define substitutability quantitatively.⁶
- 4 If an object is not substitutable for any other object (e. g. antiques), a class must be provided for the object with the physical measure whose value (quantity) is, say, 1 if the object is included in the class and, otherwise, zero.
- 5 In addition to the above requirements (i) – (iii), we require that a physical measure in each class satisfy all the requirements for a measure in mathematics, i. e.,
 - iv. For each class, the physical measure defined for the class must be always non-negative, zero if the class is empty, and countably additive.⁷

5 Note that the axiom also implies that an object a is substitutable for itself (reflexive); if a is substitutable for a' , then a' is substitutable for a (symmetrical); and if a is substitutable for a' , and a' is substitutable for a'' , then a is substitutable for a'' (transitive).

6 The substitutability must be recognized not only by the subject who owns the objects but also by other subjects who enter into the process of exchanges (economic and physical) of the objects. For example, Brand A regular gasoline and Brand B regular gasoline may be classified into the same class if they are to be used in a factory and if the differences in quality of the two brands are immaterial, since in such a case a given number of gallons of Brand A gasoline is perfectly substitutable for the same gallon of Brand B gasoline. On the other hand, if they are owned for resale purposes and if for customers the two brands are not indifferent, though their quality difference may be insignificant, they should be classified into different classes.

7 See, for example, Halmos (Halmos, 1950) or Royden (Royden, 1963) for the properties of a measure.

This means that quantities derived by the physical measure are always positive or zero, and zero if the class contains no objects.⁸ The requirement that the physical measure must be countably additive means that the quantity of distinct objects A and B combined together is the sum of the quantity of A and the quantity of B .⁹

This is to assure that the quantity of 100 chairs is 100 no matter how we count them, or the quantity of 1,000 gallons of gasoline is 1,000 no matter how we measure it. On the other hand, so far as these requirements for a measure are satisfied, it does not matter whether gasoline is measured by gallons or pints or ounces.¹⁰ Also it should be noted that the value of a physical measure is allowed to take positive infinity as in the example discussed in the next paragraph.

- 6 An object itself may be classified into a class different from one to which the *services* or *utilities* of the object belong. For example, land itself may be measured by its area (provided that two lots with the same area are substitutable—otherwise they belong to different classes); whereas services obtained from the land may be measured by time. If the service life of land is considered to be infinite, the quantity of services of land is infinite. However, this does not disturb our measurement system at all as becomes clear later.
- 7 One of the three amazing human abilities assumed in the three axioms is that a person is able to know that he is indifferent about two sets of objects when in fact the two are not identical. How can human beings acquire such an ability? Perhaps, only by experiences. However, we shall not go into this psychological or rather philosophical question, but simply assume that such an ability has been granted by setting it in the axiom.
- 8 Though substitutability is essential in this axiom, what we need is not a simple substitutability but an *additive* substitutability. As mentioned in the

⁸ Mathematically, the physical measure can take zero on non-empty sets of objects in a class so far as other requirements for a measure are satisfied. However, since normally a non-empty set of objects and an empty set of objects are not substitutable, by the requirement iii) above such a trivial physical measure can be ruled out.

⁹ More precisely, a physical measure q_i defined for the i th class must satisfy

$$q_i \left(\sum_{j=1}^{\infty} A_j \right) = \sum_{j=1}^{\infty} q_i(A_j) \quad (i=1, 2, \dots) \quad (2.1)$$

for any sequence of disjoint subsets of A .

¹⁰ Clearly a function whose value is always equal to a multiple by a positive constant of the value of another measure is also a measure.

footnote n. {11}, the substitutability must be reflexive, symmetrical, and transitive. However, in addition to this, we need the property that if a_1 is substitutable for b_1 and a_2 is substitutable for b_2 , then a_1 and a_2 together are substitutable for b_1 and b_2 together. This is what is meant by an additive substitutability. Here is the need for introducing quantities or physical measures that generate quantities. Note that quantities discussed here presuppose substitutability. That is, *we are interested in quantities only insofar as they are supported by substitutability*. For example, paint and wine may be measured by the same unit of measure, e. g., by gallons. But “100 gallons of paint or wine” has no meaning at all in our accounting measurement system (even though the figure may be useful for some other purposes) since the figure does not base upon substitutability.

It seems to be extremely important to note this point. Quantities based upon substitutability are the essential factors in conventional accounting measurement.¹¹

- 9 The set of objects that is dealt with in conventional accounting (we shall call it an *accounting set*) is not a simple collection of objects but a set with such properties as those described above. The axiom of quantities is to assure that there exists such an accounting set with which to start the construction of a measurement system.

C Axiom of ownership

1. The axiom of ownership, or more precisely the axiom of ownership identifiability, involves four factors in its foundation: a subject, objects, a relation called ownership, and time.
2. *A Subject*: A subject in this axiom is an identifiable thing that has a capacity for owning objects. A subject may be a person, a partnership, a corporation, or may even be an object or a collection of objects (estate). Conventional accounting exists only for one subject at a time. It is not defined for a simple collection of subjects (e. g. a mob) unless such a collection is also considered as constituting a subject (a holding company). It is not defined for objects without a subject (minerals in the moon, unless the moon itself is

¹¹ We may say that two sets of objects, a and a' , are *equal*, denoted by $a = a'$, if and only if both sets belong to the same class and the measure on a and the measure on a' are equal. This is another way of saying that two sets of objects are equal if and only if they are substitutable.

considered as a subject). Therefore, except when there is a possibility of confusion, we shall always assume that we have a given subject.

3. *Objects*: Objects are any identifiable things that have a capacity for being owned by (or belonging to) a subject. Objects in conventional accounting include not only goods as such but also utilities that are generated from goods. It is an important accounting problem to decide what types of objects are to be included in the accounting set U . However, from the viewpoint of measurement we shall assume that all issues concerning the objects in U have been settled (otherwise no measurement systems can be developed) and the only property that we need to require is that objects in U be identifiable.
4. *Ownership Relation*: The relation between a subject and objects is described as the latter “belonging” to the former. The belongingness need not be a legal ownership though we use legal terms for convenience. Any mechanism can be used in defining ownership relation so far as at any point in time every element in the accounting set U is uniquely characterized either as belonging to the subject or not belonging to the subject (but not both). Therefore, it may be a legal ownership, an economic ownership, organizational responsibility, or even a physical possession, or a physical attachment to a subject. The essential point of the axiom is that the set of all objects that “belong” to a given subject at any given point in time must be uniquely identified. If there is any dispute about the belongingness of an object, whatever the definition of ownership may be, such a dispute must be settled before applying the measurement system.
5. *Time*: Since we are interested in changes over time in the set of objects that belong to a given subject, the set of all objects that belong to the subject at any given point in time must be identifiable at least at that time or later. (It need not be identifiable earlier than that time.) This also implies that any changes in the set of all objects that belong to the subject can also be recognized. We shall define the set of all objects that belong to the subject at time t to be the *property set* of the subject at time t , denoted by A_t , where A_t is a subset of U for any t .
6. The second amazing human ability assumed in the three axioms is the ability to identify an object as belonging to a given subject. Particularly in the case of a legal ownership, many objects do not have labels that identify the owner; nevertheless the objects are recognized as belonging to a subject not only by the subject himself but also by the rest of the people in the society. Without the axiom of ownership we are not able to identify A_t on which we want to apply our measurement system.

7. By combining the axiom of ownership with the axiom of quantities, the set of all objects that belong to the subject at time t , i. e., A_t , can be uniquely identified, once we fix the order of the classes, by a vector

$$q^t = (q_1^t, q_2^t, \dots) : q_i^t \geq 0 \quad (i = 1, 2, \dots) \quad (2.2)$$

where q_i^t is the quantity of the objects that belong to the i th class and that are owned by the subject at time t . Then, the problem for an accounting measurement system is, using the set of quantities q^t , to generate a unique real number for any given time t . Such a number $m(A_t)$ is used as a measure on the set of objects that belong to the subject at time t .

8. A remarkable characteristic of the measurement system of conventional accounting is the fact that it is based on activities called exchanges. In other words, $m(A_t)$ is not determined arbitrarily, nor by the weight of the weighable objects in A_t , nor by the market value of the total objects in A_t , nor by anything else-but by the method called *measure imputation through exchanges*.

D Axiom of exchanges

1. By an exchange we mean a phenomenon which decreases the property set of a given subject at a given point in time by a set of objects (called outgoing objects), denoted by d^- , and in return increases the property set by another set of objects (called incoming objects), denoted by d^+ . Thus, an exchange involves two sets of objects which are related with each other by a “cause-and-effect” relationship. An exchange may occur by an activity of the subject to whom the property set belongs or it may occur by some other forces (by nature or by other subjects).
2. The third amazing human ability is that a person is able to know a set of objects d^+ is obtained in exchange for another set of objects d^- . For example, a warehouse manager can see only goods coming in and going out, while a cashier can see only cash balance increased or decreased. But when we see the firm’s operation as a whole, we are able to say that cash is increased in *exchange* for goods delivered. That is, we have a mechanism outside the measurement system, by which we are able to say that d^+ is obtained as a result of foregoing d^- , or in exchange for d^- . The axiom of exchanges is provided essentially to assure us that we can recognize such exchanges as they take place. We shall denote an exchange d by a pair of ordered sets,

$$d = [d^+, d^-] \quad (2.3)$$

in which the first element shows the set of incoming objects and the second element the set of outgoing objects. When this concept of exchanges is applied to conventional accounting, it will be noted that this concept of exchanges is applicable for both *economic exchanges*, i. e., exchanges through market, and *physical exchanges*, i. e., exchanges through production processes. In our measurement system the two types of exchanges are not distinguished since there is no need for this from the viewpoint of accounting measurement.

3. For the reason that will become clear in the discussions of measurement rules, we shall require that the set d^+ of the incoming objects belongs to a single class only. If d^+ consists of objects in two or more classes, it is assumed that by some methods the exchange can be partitioned into a countable collection of (sub-)exchanges each of which contains incoming objects that belong to a single class only.
4. The time dimension of an exchange has been deliberately postponed up to this point for a simpler explanation of the concept. We shall discuss this now in detail.

If in every exchange d , an addition of d^+ to a property set A and a subtraction of d^- from the property set A are made *simultaneously*, the measurement system can be much simpler. If this is true, we can define an exchange at time t , denoted by d_t , to be a phenomenon that adds to A_t a set d_t^+ that is not in A_t , and subtracts from A_t a set d_t^- that is in A_t after d_t^+ has been added.¹²

However, in many cases, exchanges that are dealt with in conventional accounting contain objects that are to be added or subtracted in the future. For example, a sale on account is an exchange between goods to be subtracted

¹² More precisely, $d_t = [d_t^+, d_t^-]$ is an operation which changes A_t into $A_{t'}$, where

$$A_{t'} = A_t \cup d_t^+ - d_t^- \quad (2.4)$$

subject to

$$d_t^+ \cap A_t = \Phi \quad (2.5)$$

$$d_t^- \subset (A_t \cup d_t^+) \quad (2.6)$$

Here $A_{t'}$ is the set of objects that belong to the subject immediately after the exchange d_t , or more precisely,

$$A_{t'} = \lim_{\Delta t \rightarrow 0} A_{t+\Delta t} = \lim_{\Delta t \rightarrow 0} A_{t'} + \Delta t \quad (2.7)$$

(I am indebted to Professor H. Uzawa of the University of Chicago for his remarks on this point.)

now and cash to be added in the future. In order to take into account such exchanges, let us allow an addition of d^+ to occur at time t^+ and a subtraction of d^- to occur at time t^- with $t^+ \geq t$ and $t^- \geq t$ where t is the time at which the exchange has occurred. Because of such an extended notion of exchanges, objects to be added or subtracted in the future as the result of exchanges that have occurred in the past must be recorded and identified at any later time.

- 5 Let us, then, define an exchange in the following way: An exchange $d_t = [d^+, d^-]$ is a phenomenon at time t which results in adding the set d^+ of incoming objects (belonging to a single class) to the property set A_{t^+} and subtracting the set d^- of outgoing objects from the property set A_{t^-} where $t^+ \geq t$ and $t^- \geq t$. With this definition of exchanges we need an axiom by which it is guaranteed that any object that is to be added to or subtracted from the property set A can be associated with an exchange uniquely so that, for example, when cash is received a month later it can be recognized as a payment for the goods delivered earlier and not a simple increase in cash. Furthermore, we require that all exchanges that have occurred be countable and can be ordered completely and uniquely according to the time of their occurrence for the purpose of recording.
- 6 Considering all these factors, the axiom of exchanges is defined as follows: For any object that is added to or subtracted from the property set A , an exchange that has caused the addition or subtraction of the object can be uniquely identified; and all exchanges that have occurred are identifiable, countable, and can be ordered completely and uniquely according to the time of their occurrence.

E Definitions and three axioms

1. The above discussions are now summarized in the following form of definitions and axioms:

Definitions:

- i. A *subject* is any identifiable thing that is capable of owning other things.
- ii. *Objects* are any identifiable things that are capable of being owned by a subject.
- iii. *Time* is a real variable; a smaller value of time means an earlier time and a larger value a later time.
- iv. A *physical measure* is a non-negative set function that is defined on a class of objects and all of its subsets such that it is countably additive, it takes zero on the empty set, and that two sets of objects in a same class

are substitutable if they are of a same value of the physical measure. A class with such a function is called a measurable class.

- v. An *accounting set* is a set of objects that may be partitioned into a countable collection of measurable classes.
- vi. *Ownership* is a well-defined relationship between a subject and objects at a given time by which for any object it is uniquely determined whether or not the object belongs to the subject at the given time.
- vii. A *property set* of a subject at time t is a subset of an accounting set and consists of all objects that belong to the subject at time t .
- viii. An *exchange* at time t is a phenomenon at time t which results in adding a set of incoming objects (all belonging to a single class) to the property set A_{t+} and subtracting a set of outgoing objects from the property set A_{t-} , where $t^+ \geq t$ and $t^- \geq t$.

Axioms:

Axiom of Quantities: There exists an accounting set U , that is, a set of objects that may be partitioned into a countable collection of measurable classes.

Axiom of Ownership: The property set A_t of a given subject at any time t can be uniquely determined at that time or later.

Axiom of Exchanges: For any object that is added to or subtracted from the property set A_t , an exchange that has caused the addition or subtraction of the object can be uniquely identified; and all exchanges that have occurred are identifiable, countable, and can be ordered completely and uniquely according to the time of their occurrence.

- 2 With this set of axioms, the set of measurement rules can be developed in a purely logical and mathematical way as we shall see in the next section.

III Measurement rules

A A uniform measure

1. By the axiom of quantities and the axiom of exchange, the property set can be described by a set of quantities, i. e., values of physical measures. Then, the problem for accounting measurement is to derive a measure of the property set by which a unique real number is assigned for any property set. The set of measurement rules to be discussed below is designed to generate such a measure which will be called a uniform measure or simply a u -measure from a set of physical measures by means of exchanges.

2. For this purpose, we first select a class as a basic class and use the physical measure defined on the class to generate a u -measure. It is convenient to choose a class whose objects are often used in exchanges but this is not essential from the viewpoint of mathematical measure. However, we shall require the class to be of finite measure, i. e., at any time t the set of objects in the property set A_t that belong to the class is always of a finite value of the physical measure defined on the class.¹³
3. Then,
 - i. The u -measure of any set of (current and future) objects in the basic class is set equal to its quantity measured by the physical measure of the basic class.
 - ii. The u -measure of the empty set is set equal to zero.

If cash is selected for a basic class, the u -measure of any set of cash, regardless of whether it is cash on hand (called current cash) or cash to be received or paid in future (called future cash), is given by its quantity measured by the physical measure of cash, e. g., dollars, and all receivables and payables as well as deposits and loans belong to the basic class. If gasoline is selected for a basic class, all receipts or deliveries of gasoline committed by exchanges are considered to belong to the basic class.

4. On the other hand, objects that belong to a non-basic class, a class other than the basic class, are recognized in the measurement rules only at the time they are actually received or delivered. For example, if current cash is received in exchange for merchandise to be delivered 30 days from now, the exchange needs to be modified as if it were an exchange between current cash and future cash (advance receipt) and an exchange between current cash and current merchandise at the time when merchandise is actually delivered. Procedures for such modifications of exchanges will be discussed later in this section in more detail. Therefore, we shall proceed, assuming that no exchanges involve future non-basic objects.
5. In order to start the measurement system it is necessary to have the values of the u -measure given for each class of objects in the property set A at the beginning of a period for which we want to account. Therefore, we must require A to be empty or consist only of objects in the basic class, or values of the u -measure for each class be given as, for example, in a beginning balance sheet. If none of these cases apply, the u -measure of objects in a non-basic class is arbitrarily set equal to zero in order to supply an artificial start. Initial proprietary investments

¹³ This is necessary since we are going to have an operation called a subtraction of a set d' from A and if the u -measure of A and d' are both infinite, $\infty - \infty$ is undefined.

of non-basic objects belong to this category which must be solved by means of evaluation by, say, market values. The set of three axioms is not capable of providing market values. This will be discussed later with other cases in which market values are used in conventional accounting.

B Allocations, imputations, and comparisons of measures

1. The basic part of measurement rules consists of the following three rules.
 - i. *Measure Allocations*: Allocate the u -measure of a non-basic class to the set of outgoing objects of the class and to the set of remaining objects in proportion to their physical measures.
 - ii. *Measure Imputations*: If incoming objects belong to a non-basic class, assign as the u -measure of the incoming objects the sum of the u -measures of all out-going objects in the exchange. Increase the u -measure of the class to which the incoming objects belong by the u -measure of the incoming objects.
 - iii. *Measure Comparison*: If the set of incoming objects is empty or belongs to the basic class, calculate a measure gain (or loss) by subtracting the sum of the u -measures of outgoing objects from the u -measure of the incoming objects.
2. The rule of measure allocations states the moving average method, but rules for other methods of measure allocations used in conventional accounting can be well established. For example, rules for FIFO or LIFO may be determined by setting up a set of ordered subclasses in each class with each subclass having a pair of a physical measure and a uniform measure. Then, by specifying the order in which these subclasses are used for measure allocation together with the above rule for the moving average method, we can derive rules for FIFO or LIFO method.
3. The rule of measure imputations is a fundamental rule for the historical cost basis. The rule of measure comparisons, then, requires that a measure gain (or loss) is recognized if and only if incoming objects belong to the basic class (the revenue realization principle). Of course, all these operations for measure allocations, imputations, and comparisons are supported by the three axioms.

C Modifications of exchanges

1. When an exchange involves non-basic future objects, i. e., objects in a non-basic class that are to be received or delivered at some time later, the

exchange must be modified before applying the above rules. This is done in the following way.

2. *Two Sets of Future Objects:*

- i. In an exchange $[d_1, d_2]$,¹⁴ if both d_1 and d_2 are sets of future objects, do not recognize the exchange until one set is actually received or delivered; unless *both* d_1 and d_2 are sets of *basic* future objects, in which case recognize the exchange.

For example, even if a purchase order has been issued, it is the practice in conventional accounting not to make an entry until either goods have been received or cash has been paid. However, if a note is issued by a customer who owes the firm, an entry is immediately made by increasing notes receivable and by decreasing accounts receivable. To approximate these practices, the above rule was set up.

3. *Increase in Non-basic Future Objects:*

- ii. In an exchange $[d_1, d_2]$, if d_1 is a set of non-basic future objects and d_2 is a set of current objects (basic or non-basic), modify the exchange as if it were $[d_3, d_2]$ where d_3 is a set of basic future objects with $m(d_3) = m(d_2)$, where $m(d_i)$ is the *u-measure* of the set d_i , and recognize an exchange $[d_1, d_3]$ when d_1 is actually received.

Suppose that a purchase order has been issued and cash has already been paid but goods have not yet been delivered. Ordinarily, the cash paid is recorded as an advance payment until goods are received, even though it is possible, and actually done in some cases, to set up an account for future inventory. This is a rather trivial point in view of measurement of the property set since both methods yield exactly the same result. In any case, since the *u-measure* of current objects is uniquely determined by the rule of measure allocations, regardless of whether d_2 is basic or non-basic, there is no problem in deriving d_3 .

4. *Decrease in Non-basic Future Objects I:*

- iii. In an exchange $[d_1, d_2]$ if d_2 is a set of non-basic future objects and d_1 is a set of basic current objects, modify the exchange as if it were $[d_1, d_3]$ where d_3 is a set of basic future objects with $m(d_3) = m(d_1)$, and recognize an exchange $[d_3, d_2]$ when d_2 is actually delivered.

This is exactly the opposite case from the previous one. Suppose an order is sent from a customer with an advance payment; the cash received is treated as an advance receipt until goods are delivered. This is in line with the realization principle.

¹⁴ In our notation, the first element in the bracket always denotes the set of incoming objects and the second element, the set of outgoing objects.

5. *Decrease in Non-basic Future Objects II:*

- iv. In an exchange $[d_1, d_2]$ if d_2 is a set of non-basic future objects and d_1 is a set of non-basic current objects, modify the exchange as if it were an exchange $[d_1, d_3]$ where d_3 is a set of basic future objects with $m(d_3) = \bar{m}(d_2)$, where $\bar{m}(d_2)$ is the expected u -measure of d_2 , and recognize an exchange $[d_3, d_2]$ when d_2 is actually delivered.

Here, the expected u -measure of d_2 is given as the sum of the expected u -measures calculated for each class to which a subset of d_2 belongs, by multiplying the physical measure of the subset of d_2 by the average u -measure, i. e., the u -measure of the class divided by the (non-zero) physical measure of the class, and zero if the physical measure of the class is zero.

A problem arises when d_1 , the set of incoming objects, is current objects belonging to a non-basic class, since in such a case a u -measure cannot be determined for d_2 or d_1 . For example, 500 barrels of crude oil received in exchange for 1,000 gallons of gasoline to be delivered in 30 days from now. (Note that this is different from the case in which crude oil was received on a consignment basis. In such a case, we need not consider that crude oil “belongs” to the subject.) The most likely treatment in conventional accounting is to use a so-called replacement cost of d_1 , that is, the amount of cash to be paid if the exchange were made with cash. In our system, however, we do not have a mechanism to generate such a replacement cost or any other type of market values. Hence, as an approximation we use the average costs, at the time of the exchange, of goods on hand in the classes that d_2 belongs to, provided that the average costs are defined for these classes. If they are not defined for a class that a subset of d_2 belongs to, we reluctantly set the u -measure of the corresponding subset of d_2 equal to zero. We will comment on this later again together with the discussions on market values.

D A uniform measure on property sets

1. By means of these rules, we can uniquely determine the u -measure for any property set that a subject has owned, by adding the u -measures for each class in the property set. Future objects that are not included in the property set are all stated in terms of basic objects, hence the u -measure on these objects can be derived by the axioms of quantities and exchanges at any time. Let the u -measure at time t of all basic objects to be received (or added to a property set) in the future be represented by $m(B_t^+)$ and let the u -measure at time t of all basic objects to be delivered (or subtracted from a property set) in the future be represented by $m(B_t^-)$. Also let us define an

extended property set at time t , denoted by $\bar{A}_t$ as meaning the property set A_t together with the above two sets of basic future objects, i. e.,

$$A_t = \{A_t, B_t^+, B_t^-\}. \quad (3.1)$$

Then, since $m(\bar{A}_t)$, $m(B_t^+)$, and $m(B_t^-)$ can all be determined uniquely by the above procedures, $m(\bar{A}_t)$ is well-defined by

$$m(\bar{A}_t) = m(A_t) + m(B_t^+) - m(B_t^-).^{15} \quad (3.2)$$

- By the rule of measure imputation, $m(\bar{A}_t)$ changes its value if and only if a non-zero measure gain g_t occurs, and $m(\bar{A}_t)$ after an exchange involving g_t has occurred is equal to the sum of $m(\bar{A}_t)$ before the exchange plus g_t generated by the exchange. Therefore, we always have an increase in the u -measure between two points in time t_1 and t_2 , i. e., $m(\bar{A}_{t_2}) - m(\bar{A}_{t_1})$, exactly equal to the sum of all measure gains during the period. We may compare this with a net profit figure derived from a balance sheet ($m(\bar{A}_{t_2}) - m(\bar{A}_{t_1})$) and a net profit figure derived from an income statement

$$\left(\sum_{t_1 \leq t < t_2} g_t \right),$$

in conventional accounting.

- It should be remembered that all of these rules are strictly based upon what is provided by the three axioms and nothing more. Of course, by increasing the number of axioms these rules can be made more complicated for better approximations of principles and practices in conventional accounting. However, in order to focus on the fundamental factors, we have deliberately neglected minor points needed for further improvement in approximation.

IV A mathematical representation of conventional accounting measurement¹⁶

- The above discussions may be further clarified by the following mathematical representation of the axioms and the measurement rules.

¹⁵ $m(\bar{A}_t)$ is no longer guaranteed to be non-negative. However, it satisfies all the requirements for a *signed measure* in measure theory, since it is countably additive, of finite values, and takes zero on the empty set. Refer to Halmos (Halmos, 1950) or Royden (Royden, 1963).

¹⁶ This section may be skipped without loss of continuity of the materials.

2. By the axiom of ownership, we have for any given subject a unique set-valued function f of a real variable t ,

$$A = f(t) \quad (4.1)$$

where A is a subset of a universal set U for all t .

3. By the axiom of quantities A can be represented by a vector q for any t . Hence, we have a vector-valued function g of a real variable t

$$q = g(t). \quad (4.2)$$

4. By the axiom of exchanges together with the rules of modifying exchanges discussed above, we can impose the following properties on the function $g(t)$:

The function $g(t)$ changes its values only at countable points of t . Hence the function is completely described by a sequence

$$(q^1, q^2, \dots) \quad (4.3)$$

together with a sequence

$$(t^1, t^2, \dots) \quad (4.4)$$

where¹⁷

$$q^n = g(t) \quad \text{for all } t^{n-1} < t \leq t^n \quad (n=1, 2, \dots) \quad (4.5)$$

and

$$t^n < t^{n+1} \text{ for all } n=1, 2, \dots \quad (4.6)$$

with

$$t^0 = -\infty \quad \text{and} \quad q^1 = 0. \quad (4.7)$$

Also if the first element in the vector q , denoted by q_1 , represents the physical measure for the basic class, and all the subsequent elements q_i ($i=2, 3, \dots$) the physical measures for non-basic classes, we have

$$q_i^n \geq 0 \quad \text{for all } i=2, 3, \dots; n=1, 2, \quad (4.8)$$

whereas q_i^n ($n=1, 2, \dots$) is allowed to take a negative value due to negative future basic objects, and

$$q_i^{n+1} - q_i^n > 0 \quad (4.9)$$

for at most one i ($i=1, 2, \dots; n=1, 2, \dots$) since the set of incoming objects always belongs to a single class.

¹⁷ Here, the function g is assumed to be left continuous. However, the whole development may very well be done based on a right-continuous function.

5. Then, the measurement rules are stated as follows:

$$\text{For all } n = 1, 2, \dots \quad p_1^n = q_1^n, \quad (4.10)$$

and for all $n = 1, 2$, and $i = 2, 3, \dots$

$$p_i^n = 0 \quad \text{if } q_i^n = 0 \quad (4.11)$$

$$p_i^{n+1} = \frac{p_i^n}{q_i^n} \cdot q_i^{n+1} \quad \text{if } q_i^{n+1} < q_i^n \quad (4.12)$$

$$p_i^{n+1} = p_i^n \quad \text{if } q_i^{n+1} = q_i^n \quad (4.13)$$

$$p_i^{n+1} = p_i^n + \sum_{j \neq 2i} (p_j^n - p_j^{n+1}) \quad \text{if } q_i^{n+1} > q_i^n \quad (4.14)$$

The u -measure $m(\bar{A}_t)$ of any extended property set A at time t is then given by

$$m(\bar{A}_t) = \sum_i p_i^n \quad (4.15)$$

where n is the index which satisfies

$$t^{n-1} < t \leq t^n \quad (n = 1, 2, \dots). \quad (4.16)$$

6. Therefore, with these axioms and measurement rules we have a unique real-valued function h of a real variable t which generates the u -measure of an extended property set at time t , i. e.,

$$m(\bar{A}_t) = y(t). \quad (4.19)$$

We may consider a vector-valued function $g(t)$ as the input to the accounting measurement system and a real-valued function $h(t)$ as the output from the system.¹⁸ Also, as pointed out earlier, note that in conventional accounting the quantity

$$h(t_2) - h(t_1) \quad (4.19)$$

18 Let G be the set of all vector-valued functions g of a real variable t which satisfy the requirements in the paragraph 4 above, and let H be the set of all real-valued functions h of a real variable t . Then, the accounting measurement system m is a function which maps G into H , or

$$G \xrightarrow{m} H. \quad (4.18)$$

Note that the discrete nature of the functions in G is carried over to the functions that are in the range of the function m contained in H .

is used as an “evaluation” of the set of all activities that have taken place between time t_1 and t_2 (i. e., during the time period $t_1 \leq t < t_2$).

V Applications

A Goods and utilities

1. We shall now discuss some of the problems which may arise when we apply the above system in a practical situation. First of all, the distinction between goods and utilities that was mentioned earlier will be discussed in some detail.
2. Throughout our process of abstracting phenomena which are dealt with in conventional accounting, we have paid different consideration to *goods* themselves and *utilities* which may be generated by the goods. For example, suppose that a firm K rents a lot of its land to another firm L . The ultimate right on the land still remains in the hands of K , hence K may get a better title on the land than in a case in which K has only an obligatory right on L to deliver the land at a given future time. However, we need not distinguish the two cases from the standpoint of accounting measures. The land itself is in the hands of L anyway and K has a right to receive the land in the future in both cases; hence we may consider both of them as a case of a future object “belonging” to K .
3. Now consider a rent on the land. This is actually a return for the utilities generated from the future object. In such a case, we do *not* consider that the current land is exchanged for the future land *plus* a given amount of money as a rent. If we considered so, the u -measure of the incoming objects m (the future land + cash) would be imputed from the u -measure of the outgoing object, m (the current land), by following the historical cost basis in conventional accounting that is reflected in our rule of measure imputation. However, this is not the method used in conventional accounting. Instead, the measure on the land is held constant and the measure on incoming cash is recorded as income which corresponds to our concept of a measure gain. The only way that this procedure can be interpreted consistently with other procedures is to recognize the exchange

$$d = [\text{the future land, the current land}], \quad (5.1)$$

i. e., the future land was obtained in exchange for the current land (hence the measure of the future land is set equal to the measure of the current land), and

then recognize another exchange, say,¹⁹

$$d' = [\$1,000 \text{ cash}, \\ 1 \text{ year service of the land}]. \quad (5.2)$$

That is, K obtained \$1,000 cash in exchange for “1 year service of the land.” However, ordinarily the total service life of land is unlimited; namely the physical measure of the services of the land is infinite. Since the u -measure for this class ought to be a finite number (by definition of the basic class measure), the u -measure that is to be allocated to the 1 year services of the land is zero, according to the rule of measure allocation discussed above. On the other hand, the u -measure of \$1,000 cash is 1,000; hence we recognize a measure gain of 1,000 in this exchange, i. e.,

$$\begin{aligned} & m(\$1,000 \text{ cash}) \\ & - m(\text{the 1 year services of the land}) \\ & = 1,000. \end{aligned} \quad (5.3)$$

4. If, however, K rented a machine with 10 years of remaining life and if the services of the machine in one year are comparable to the services of the machine in any other year (which is the basis for a straight line depreciation method), one-tenth of the u -measure of the machine should be matched with the u -measure of the \$1,000 cash.²⁰ This in effect is what has been done in conventional accounting. (Although rent income and depreciation are calculated separately, the net effect of the u -measure of the property set is the same as the method described here.)
5. Now let us extend this notion of future objects to the cases involving future cash. When \$1,000 of current cash is deposited in a bank now and \$1,050 of cash is returned a year from now, we should *not* consider this as an exchange,

$$d = [\$1,050 \text{ future cash}, \\ \$1,000 \text{ current cash}]. \quad (5.4)$$

Instead we recognize the exchange,

$$d = [\$1,000 \text{ future cash}, \\ \$1,000 \text{ current cash}], \quad (5.5)$$

¹⁹ Though the service of the land is delivered continuously throughout the year, we can recognize it at discrete points in time by the axiom of exchanges.

²⁰ If, say, one-tenth of the services in the first year is comparable to the services in the tenth year, we should allocate the u -measure of the machine according to a physical measure that reflects the difference in services, e. g. by the sum-of-years-digit method.

and at the end of 1 year period, the exchanges,

$$d' = [\$50 \text{ current cash}, \\ 1 \text{ year service of } \$1,000 \text{ cash}] \quad (5.6)$$

$$d'' = [\$1,000 \text{ current cash}, \\ \$1,000 \text{ future cash}]. \quad (5.7)$$

Since, as in the case of land, the “service life” of cash is unlimited, the u -measure of 1 year service of \$1,000 cash is zero. Hence, a measure gain of 50 is recognized in the exchange d' by the rule of measure comparison discussed above.²¹

A confusion arises in this case because of the fact that current cash is received in exchange for both cash delivered before and 1 year service of cash; but if we compare this with the case of land, it will become clear that our way of interpreting interest is more consistent with the conventional accounting procedures than a way of looking at the exchange as the current cash foregone in exchange for an increased amount of cash in the future. We are not saying that this is the way that interest in general should be interpreted, but that by interpreting interest this way we can get a more *consistent* interpretation of conventional accounting principles and procedures with a simpler set of axioms and measurement rules.

6. But how about risks attached to future objects like accounts receivable? How can we interpret the practices of setting up reserves for bad debts? If, for example, in an exchange

$$d = [\$1,000 \text{ future cash}, \\ 4,000 \text{ gallons of gasoline}] \quad (5.8)$$

where the u -measure of 4,000 gallons of gasoline is, say, 400, the firm expects, *at the time of the exchange*, that it can get only \$800 cash, then the firm should recognize the exchange of

$$d' = [\$800 \text{ future cash}, \\ 4,000 \text{ gallons of gasoline}], \quad (5.9)$$

yielding a measure gain of 400. (Note that we do not care whether this is done by setting a reserve or by directly subtracting from the receivable so far as the net effect on the measure is the same.) This is exactly in line with the historical cost basis in conventional accounting. However, reserves for bad debts set up later, due to the changes in financial situations of customers or due to some other reason, must be considered as a departure from the

²¹ A similar interpretation can also be made for interests on payables.

historical cost basis that is similar to the application of market price in inventory when a market price becomes lower than the historical cost. (See discussions on market prices later.)

7. The rule of measure imputations developed above is directly in line with procedures in cost accounting. On the other hand, administrative expenses may be interpreted as exchanges which have no apparent incoming objects ($d^+ = \emptyset$), and since $m(\emptyset) = 0$, a measure loss is recognized at each exchange. A part of selling expenses may have direct incoming objects (reputation, good will, better salesmen) but we may consider that all such incoming objects are consumed when expenses occur. If so, there is no difference in the total measure gain. If they are not consumed at the time when expenses occur, we should, at least in principle, recognize the incoming objects and apply the rule of measure imputations.
8. It will also be noted that by utilizing the notion of “future cash” the principle of revenue realization and the accrual basis can be interpreted consistently with our measurement rules.
9. Finally, we would like to emphasize the fact that the concept of profit in conventional accounting cannot be derived independently from the concept of assets (or the property set in our system).

Profit in conventional accounting is derived as a measure (or evaluation) of the set of activities that have taken place during the given period of time. Unlike job evaluation or grading students' performance in a class, however, in conventional accounting we do not evaluate activities as such but evaluate them *through their effects on assets* (or a property set). The tendency of stressing the income determination side of accounting problems in current accounting theories should not be misinterpreted as if profit figures were derived as measures of activities independently from their effects on assets. In addition, just like the speed of an object is derived as the difference of the positions of the object at two different points in time relative to the length of the time period, profit is something that is derived as the difference of the u -measures of the property sets at two different points in time relative to the length of the time period, i. e. profit during the period between time t_1 and time t_2 , denoted by $P(t_1, t_2)$, is given by

$$P(t_1, t_2) = m(\bar{A}_{t_2}) - m(\bar{A}_{t_1}). \quad (5.10)$$

Therefore, so far as the historical cost principle is adhered to, asset valuation and income determination are the two sides of the same coin, both concerned with *physical measures*, not with the uniform measure, when quantities of objects are hard to measure physically.

B Some problems in applications

1. We shall now discuss principles and procedures that are used in conventional accounting but cannot be handled in the same way in our system.
2. *Market Values*: The conventional accounting system that we have tried to approximate by our mathematical measure system is the one which is based upon such principles as the historical cost basis, the realization principle, and the accrual basis, etc. Therefore, we have neglected such concepts as *market values*, *replacement costs*, *net realizable values*, etc. These concepts come into the conventional accounting systems and procedures in somewhat inconsistent ways. Some such examples may be listed as follows:
 1. Cost-or-market-whichever-lower method applied to inventory and marketable securities.
 2. Donated assets or assets obtained at unusually low prices.
 3. Initial proprietary investment involving non-cash assets.
 4. Allocation of joint costs based upon market values of joint products.²²
 5. Reserves for bad debts.

These are not incorporated into our system for an important reason that will be amplified more in the next concluding section.

3. *Proprietary Investments*: An exchange involving proprietary investment must be regarded as an exchange between current cash and future cash to be delivered at an indefinite time in the future, in order to operate the mathematical system developed here and generate a measure that approximates the one generated by the conventional accounting system, even though proprietary investment is different from loans and other payables from the standpoint of legal claims against the subject, the entity.
4. *Capital Surplus*: An exchange involving capital surplus cannot be distinguished from an exchange involving earned surplus by our measure system. We consider, however, that the need to distinguish capital surplus and earned surplus comes from legal and managerial requirements concerning the distribution of income. This is a problem of allocation *policy* that arises after the total measure gain has been determined. We are, of course, not denying the importance of the contribution by conventional accounting in

²² Note that in the axiom of exchanges we required that the set of incoming objects belongs to a single class in any exchange to avoid this case. If an exchange involves incoming objects that belong to more than one class, the exchange must be decomposed before the measurement rules are applied on it.

such an area, but we have set aside this area from our consideration of constructing a mathematical measure system.

5. *Reverse Exchanges*: An exchange that is made in order to cancel another exchange that was made earlier (sales return, purchase return, etc.) is also unable to be handled in our system. Exchanges of this type presume the ability of human beings to identify an exchange as a reverse exchange of another one. While it is possible to set up an axiom to take care of this point, reverse exchanges themselves are rather unimportant and hence we decided not to include such an axiom in our set of axioms in order to stress the most fundamental factors in conventional accounting.

VI Summary and conclusion

1. We have shown that if the set of three axioms, Axiom of Quantities, Axiom of Ownership, and Axiom of Exchanges is granted, we can generate, by a set of measurement rules, a uniform measure from a set of physical measures by means of measure imputations through exchanges, in a purely mathematical way without appealing to empirical judgment.
2. Furthermore, the empirical support for the measurement system imbedded in the set of three axioms is simple and very plausible; loosely speaking, all that is required in the axioms is (i) objects can be quantified based on substitutability, (ii) the set of all objects that belong to a subject can be identified, and (iii) changes in such a set can be made through exchanges that are identifiable, or more simply, “I can identify things that are substitutable, things that I have, and things that are exchanged.” As an example of simplicity, note that objects are defined to be simply identifiable things that can be “owned” by a subject, where ownership is attained through exchanges. It is, therefore, not necessary for objects to have any other properties such as usefulness for a subject in order to be dealt with in this measure system. We have used the term utilities to indicate things that may be generated from goods, but such utilities may very well be dis utilities insofar as they are obtained through exchanges and can be quantified by a physical measure.

If, as we believe, the measurement system developed in this paper is a good abstraction of the measurement system of conventional accounting, then we may say that this simplicity of the empirical foundation of the measurement system is the reason for conventional accounting having been able to provide profit figures over the past several centuries.

3. In the last section, we discussed the fact that our system cannot handle the accounting practices of using *market values*. Here, we would like to point out an important difference between accounting measurement based upon historical cost (or simply “historical cost measurement”) and accounting measurement based upon market values, replacement cost, net realizable values, discounted future cash flows, etc. (or simply “market value measurement”). First of all, the basis for historical cost measurement is phenomena that *have occurred*, whereas the basis for market value measurement is phenomena that *could have occurred*, or *can occur*, or *will occur*.²³ Consider a tree as a representation of paths that a subject could have taken or can take in the future. At each branching point, regardless of the number of branches at that point, the branch that the subject has taken can be uniquely determined. Hence, the whole path that the subject has taken up to the present time can also be uniquely determined. On the other hand, in most cases the paths that the subject could have taken or can take in the future are numerous, inexhaustible, and indeterminable (since we have to take into account possible reactions from the rest of the world which are mostly unknown).

This means that in historical cost measurement the path that is to be measured is uniquely given, and all issues are directed toward searching for the path, interpreting it, and measuring it; whereas in market value measurement, we must first agree on which path we are going to measure, and then resolve those same issues that are raised in historical cost measurement.^{24,25} But how can we agree on a choice of a path among numerous alternatives, some of which are indeterminable, even if we decide to choose, say, an “optimum path” based upon agreed criteria? (Even in the game of chess that is far simpler than phenomena in the real world, we have not been able to find an “optimum path.”)

Furthermore, information on market values, replacement cost, net realizable values, discounted future cash flows, etc., have two factors in common with other forecasted data; their usefulness depends very much upon the ability of

²³ For example, market values at which a firm could have replaced the machine or could have sold the securities at the date of the balance sheet.

²⁴ In terms of the mathematical representation of accounting measurement given in Section IV – {A Mathematical Representation Of Conventional Accounting Measurement}, this is equivalent to saying that in historical cost measurement, the value of the function $g(t)$ is uniquely determined, at time T , for all $t \leq T$, whereas in market value measurement this is not true.

²⁵ Of course, the uniqueness of the path does not necessarily imply that the path is recognized and interpreted uniquely by all accountants. We are, however, pointing out that the two measurement systems are in different dimensions of complexity.

the forecaster and their usefulness is limited in time.^{26,27} Considering these factors together with the point discussed in the above paragraph, it is clear that a system based upon market value measurement is not appropriate as an accounting measurement system that is to be applied continuously over a long period of time.

4. We do not deny the usefulness of information based on market values, etc. as *ad hoc* information for a particular decision making by a particular decision maker and perhaps as supplementary information to those provided by an accounting system based upon historical cost measurement. However, we should clearly recognize these fundamental differences between historical cost measurement and market value measurement and should not mix them together in one system at the same dimension.
5. It goes without saying that information generated from an accounting system must be *useful* for users. Therefore, the usefulness of the uniform measure provided by an accounting system which was developed here as an abstraction of conventional accounting measurement must also be carefully looked at in view of users of the data provided by the system.²⁸

Then, for example, we may very well change the basic class from cash to some other objects, say, a class of inventories if the user's needs are better satisfied by a measurement system based on inventories as a basic class as in the case of inflation.²⁹ In our measure system there is nothing to be changed in order to have a class of inventory for the basic class since the only requirement placed on a basic class is that the physical measure defined on the class is always of finite value.

Furthermore, we may very well change the set of measurement rules completely if this improves the usefulness of the information produced from the system. So far as we base our system upon the same set of axioms, the implementation of new measurement rules are relatively easy since they are purely mathematical (i. e., a part of the system which can be completely computerized) without requiring human ability or judgment. Such changes in

²⁶ Nobody reads tomorrow's weather forecast the day after tomorrow, whereas historical records on weather are permanent.

²⁷ It is true that even in historical cost measurement there exist needs for estimating the quantities of future objects or the quantities of objects that are consumed (e. g. depreciation). However, such needs exist only for objects involved in exchanges that a subject *has already committed*, unlike forecasting on alternatives that the subject *can* take.

²⁸ An analysis of mathematical relationship between a management goal and an accounting indicator for the goal by Ijiri (Ijiri, 1963) is an attempt to uncover this area.

²⁹ If a class of inventories is chosen as a basic class, cash is measured in terms of the physical measure defined on the basic class. We may apply FIFO, LIFO, or moving average method on cash.

measurement rules in order to generate more useful data ought to be explored in the future as a very prospective direction in which conventional accounting measurement can be fruitfully developed.

6. We believe that accounting in the future will be developed on a more general foundation without being restricted to monetary units in order to meet the ever expanding needs for information by management as well as stockholders and other interested parties.³⁰ The idea of constructing a uniform measure from a set of physical measures by means of exchanges that have taken place will certainly supply very useful foundations for a development of accounting into such new areas.

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³⁰ Professor W. W. Cooper of Carnegie Institute of Technology has been stressing the need for and possibility of designing an accounting system based upon non-monetary units as well as axiomatic approaches to such an extended accounting system; it is this need which has stimulated the present work.

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