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ON THE VALIDITY OF THE EXPONENTIAL SIZE DISTRIBUTION FUNCTION OF INTERSTELLAR DUST

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Abstract. The equation of dust formation is analyzed. It is shown that the exponential size distribution function is valid only when destructed particles completely vanish. The corrected size distribution function for core-mantle particles is presented.

Key words: interstellar dust: size distribution function

1. Introduction

Two well known interstellar dust size distribution functions (SDF) are usually used for extinction calculations. One is obtained from the inverse interstellar extinction problem (Mathis, Rumpl and Nordsieck 1977), the other is derived on purely theoretical grounds (Greenberg 1968, Hong and Greenberg 1978). The primary test of the validity of the obtained SDF might be their consistency with the dust formation processes and physical conditions in the system consisting of dust grains and formation medium.

It is known that a complete mathematical model of dust formation is very complicated (see, for example, Liffman 1990, Liffman and Clayton 1990 and references therein). However, even a formal analysis of formation equations gives some insight into the behaviour of the interstellar SDF.

This paper will concentrate on basic assumptions of the models of dust formation and on resulting limitations of the derived SDF.

2. Formation of bare particles

Usually grains are assumed to be spheres. Therefore, we limit ourselves to the phenomena where shape effects are insignificant.

Let us denote by $n(r,t)dr$ the number of grains per unit volume within the radii range r, $r+dr$ at time t. Further it is more convenient to analyze total mass, $m(r)n(r,t)$, of all grains with the radii between r and $r+dr$ per unit volume at the time moment t, $m(r)$ being the mass of a grain of the radius r.

An interstellar dust particle can be destroyed by evaporating continuously or by breaking into a number of fragments, including complete evaporation. Following Hellyer (1970), let us define the function of the destruction efficiency

$$D(\Delta t, r, r') = p(r)\Delta t \; \xi(r, r'), \tag{1}$$

where $p(r)\Delta t$ is the destruction probability of a grain of the radius r during the time interval Δt. Then $\xi(r,r')$ is the mean SDF resulting after destruction of the particle of the radius r, i.e., $\xi(r,r')dr'$ is the number of grains in the radii range between r' and $r'+dr'$. Obviously, the mass conservation law is valid:

$$\int_0^r m(r') \; \xi(r,r')dr' = m(r). \tag{2}$$

Similarly, we introduce the function of the growth efficiency

$$A(\Delta t, r, r') = q(r, \Delta t) \; \eta(r, r', \Delta t), \tag{3}$$

where $q(r,\Delta t)$ is the probability for a particle of the radius r to increase its radius during the time Δt, and $\eta(r,r',\Delta t)$ is the mean SDF resulting after growth event. Let us take into account only a continuous growth, neglecting coagulation effects. The growth event is the necessary event, thus $q(r,\Delta t)$ is equal to 1. The SDF $\eta(r,r',\Delta t)$ will peak as the Dirac δ-function for the particles with the radii $r'-r=\frac{dr}{dt}\Delta t$. Here $\frac{dr}{dt}$ is the growth rate of the particle. Thus,

$$A(\Delta t, r, r') = \delta(\frac{dr}{dt}\Delta t - (r'-r)). \tag{4}$$

The description of the process of continuous destruction can be included into the description of continuous growth process. It should be noted that both A and D are obtained averaging over all possible growth and destruction events and processes.

The dust formation mass balance equation for the time moments $t+\Delta t$ and t can be written as

$$m(r)n(r,t+\Delta t) = m(r)n(r,t) + m(r)R_1(r)\Delta t$$
$$-m(r)R_2(r)\Delta t + m(r)\int_r^\infty D(\Delta t, r', r)n(r',t)dr'$$
$$+m(r)\int_0^r A(\Delta t, r', r)n(r',t)dr' - \int_0^r D(\Delta t, r, r')m(r')n(r,t)dr'$$
$$-m(r)\int_r^\infty A(\Delta, r, r')n(r,t)dr', \tag{5}$$

where $R_1(r)$ and $R_2(r)$ are some mean SDF correspondingly injected and lost by the system where formation processes occur.

In order to make some qualitative conclusions, let us assume that:

1) the SDF has reached the steady state ($\frac{\partial n}{\partial t} = 0$);

2) the system is conservative ($R_1(r) = 0,\ R_2(r) = 0$);

3) the growth rate of the grain does not depend on the radius ($\frac{dr}{dt} = \text{const}$). The latter requirement assumes that the growth of grains starts from the radius $r = 0$.

Substituting expressions (1) and (4) into equation (5) and using relation (2), for $\Delta t \to 0$ we obtain

$$\frac{\partial n}{\partial r}\frac{dr}{dt} = -p(r)n(r) + \int_r^\infty p(r')\ \xi(r', r)n(r')dr'. \tag{6}$$

Comparing equation (6) with the corresponding formulae of Greenberg (1966, 1968) we can notice an implicit assumption made by the author: not only destroyed grains but even their debris are completely excluded from the balance. It should be noted that this assumption was made already by Oort and van de Hulst (1946) in the first theoretical study of dust formation. Meanwhile, the additional term on the right hand side of (6) suggests that debris of disrupted particles are added to the smaller grains of the corresponding radii. This will increase steepness of the SDF for small radii, implying relatively larger numbers of small particles than the exponential law function suggests.

3. Core-mantle grains

In dense molecular clouds mantles can form on interstellar grains. For core-mantle grains with single radius cores, Greenberg and associates usually use the SDF

$$n(r_m, r_c) = Ne^{-k(r_m - r_c)^3}, \quad r_c < r_m < r_{\max}. \tag{7}$$

Here r_c is the radius of the core, r_m is the radius of the mantle, $r_{\max}$ is the largest available radius of mantles, N is the normalization multiplier and k is the constant depending on grain material and properties of the surrounding medium (see Greenberg 1978, Schutte and Greenberg 1991). Let us differentiate equation (7). This yields the differential equation

$$\frac{\partial n(r_m, r_c)}{\partial r_m} = -3k(r_m - r_c)^2 n(r_m, r_c), \quad r_c < r_m < r_{\max}. \tag{8}$$

Most of the grain destruction processes have efficiencies proportional to the grain geometrical cross-section. It follows from equation (8) that grains have radii r_m, but they are destroyed less effectively than the particles having radii $r_m - r_c$.

Also an inherent assumption is made that destructed mantles evaporate completely. This is possible when the mantle material is bounded rather loosely. Let us also assume that the binding energy of cores is high enough to neglect their destruction in molecular clouds. Then the differential equation for the formation of mantles should be

$$\frac{\partial n(r_m, r_c)}{\partial r_m} = -3kr_m^2 n(r_m, r_c), \quad r_c < r_m < r_{\max}. \tag{9}$$

The solution of equation (9) is

$$n(r_m, r_c) = N_1 e^{-kr_m^3}, \quad r_c < r_m < r_{\max}, \tag{10}$$

where N_1 is the normalization constant.

Let us suppose that all cores have the same radii and all they are the accretion centers. The normalization constant must be established from the relation

$$\int_{r_c}^{r_{\max}} n(r_m, r_c) dr_m = N_0, \tag{11}$$

where N_0 is the total number of cores.

The constant N_1 is

$$N_1 = \frac{N_o}{\int_{r_c}^{r_{max}} e^{-kr^3} dr}. \tag{12}$$

Greenberg's SDF has the normalization constant

$$N = \frac{N_o}{\int_{r_c}^{r_{max}} e^{-k(r-r_c)^3} dr}. \tag{13}$$

Size distribution functions (7) and (10) for the same number of single radius cores are shown in Fig. 1.

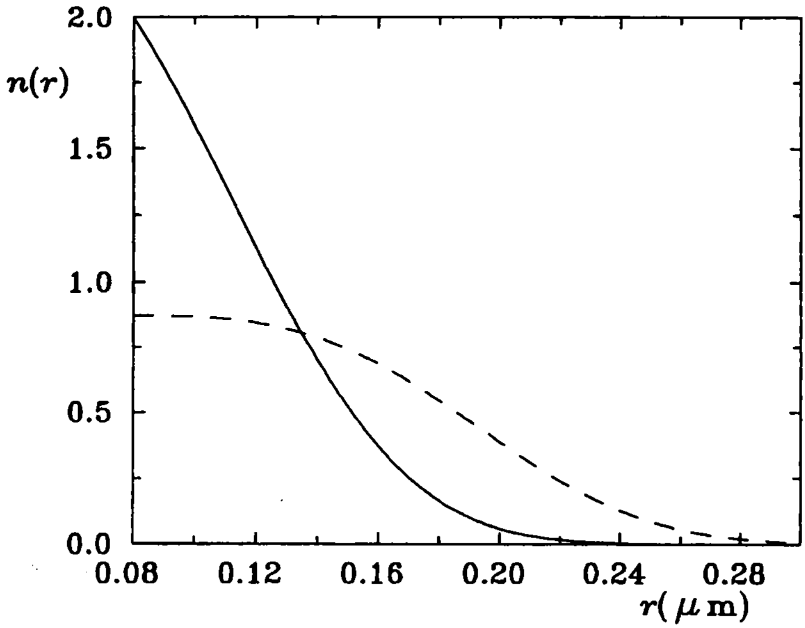

Fig. 1. The size distribution functions of core-mantle grains for the same number of single radius cores ($N_0 = 1$). The dashed line is Greenberg's SDF and the solid line is a modified SDF.

It is interesting to compare the extinction curves for ensembles of grains with SDF (7) and (10). The extinction of core-mantle grains was calculated using the Mie theory (Bohren and Huffman 1983) for dust grains with astrosilicate cores (Draine and Lee 1984), dirty ice mantles (Isobe 1971) and "large grain" parameters ($r_c = 0.08$ μm,

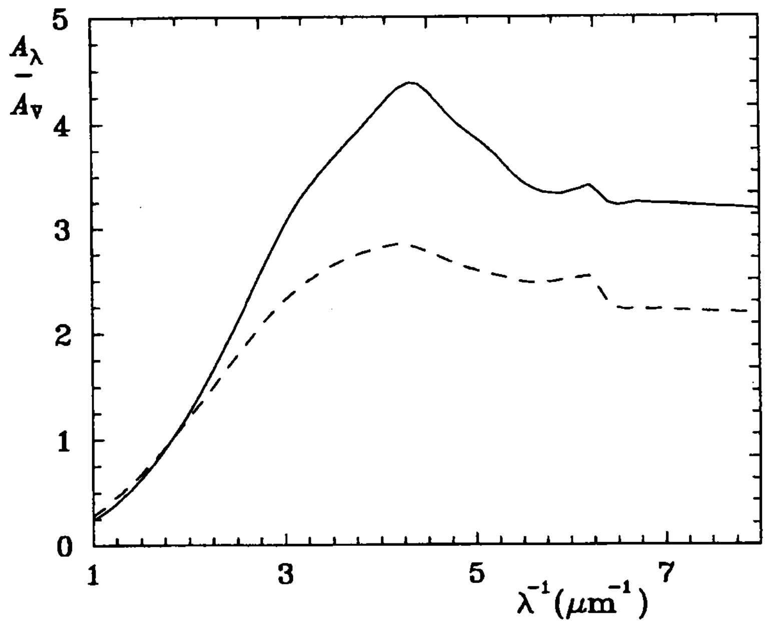

Fig. 2. The extinction curves for Greenberg's SDF (dashed line) and modified SDF (solid line). They are normalized to 1 at $\lambda = 550$ nm.

$r_{max} = 0.4\ \mu$m, $k = -5/0.22^3\ \mu m^{-3}$, see also Schutte and Greenberg 1991). The extinction then was normalized to 1 at $\lambda = 550$ nm (see Fig. 2).

The ensemble of particles with the corrected SDF yields an increased ultraviolet extinction due to the relatively larger amount of small grains. The extinction at $\lambda = 550$ nm for the same number of cores for Greenberg's SDF is about 2 times higher than that for the modified SDF. Therefore, the modified SDF requires more core material to cause the same extinction at $\lambda = 550$ nm as does Greenberg's SDF. However, several fractions of grains exist in the interstellar medium. Apparently for this reason, the increase of the ultraviolet extinction will also be less appreciable than it can be seen from Fig. 2.

The thickness of the mantles for core-mantle grains will follow a near exponential SDF rather than the power law function: no mechanism exists forming several grains with the mantles from one big dust grain.

4. Conclusions

(1) Formal analysis of the dust formation equation yields that the exponential SDF is valid only if demolition of a particle is complete – there remains no debris of the destroyed particle. The resulting SDF for bare particles should follow the SDF at small radii steeper than the exponential function.

(2) Greenberg's SDF for core-mantle grains becomes invalid if the destruction probability is assumed to be proportional to the grain geometrical cross-section. Greenberg's particles have radii r_m, but they are destroyed as having radii $r_m - r_c$. Modifications of Greenberg's SDF will somewhat affect the ultraviolet extinction and the total amount of absorbing matter along the line of sight.

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