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ON THE POSSIBILITY OF DETERMINING THE DISTANCE TO THE GALACTIC CENTER FROM THE GEOMETRY OF SPIRAL ARM SEGMENTS

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Abstract. A new approach to determining the solar galactocentric distance, R_0, from the geometry of spiral-arm segments is proposed. Geometric aspects of the problem are analyzed and a simplified three-point method for estimating R_0 from objects in a spiral segment is developed in order to test the proposed approach. An estimate of $R_0 = 8.44 \pm 0.45$ kpc is obtained by applying the method to masers with measured trigonometric parallaxes, and statistical properties of the R_0 estimation from spiral segments are analyzed.

Key words: Galaxy: structure – Galaxy: fundamental parameters – masers

1. INTRODUCTION

The study of the spiral structure of the Milky Way is of fundamental importance for galactic astronomy and dynamics. Till recently, spatial modeling of tangent and other concentrations of gas and young objects – one of techniques used in the field – was principally based on the *positions* of spiral-arm segments (short as a rule) rather than on the *geometry* of individual segments. Unfortunately, this approach yields discrepant results (cf., e.g., Efremov 2011; Francis & Anderson 2012). However, recent high-precision measurements of heliocentric distances (parallaxes) for fair tracers of *long* segments of spiral arms provide the possibility of spatial modeling of individual spiral segments (e.g., Xu et al. 2013; Reid et al. 2014; Bobylev & Bajkova 2014).

The latter approach is a direct method for estimating the pitch angles for the segments (see, e.g., Bobylev & Bajkova 2014). However, we can take a step further trying to treat the distance to the center of the Galaxy, R_0, as a free parameter in spatial modeling of spiral segments. In case of success, we gain (1) a more comprehensive modeling given the close interrelation between the pitch angle and R_0 (according to our calculations) and (2) a new method for determining R_0 from objects of the flat Galactic subsystem. The proposed technique is by nature a spatial and absolute (if it uses maser parallaxes) method of R_0 measurement (see Nikiforov 2004).

In this paper, we consider geometric aspects of the problem (Section 2), apply our new approach to masers with trigonometric parallaxes (Section 3), and examine numerically the statistical properties of the R_0 estimation from spiral segments (Section 4).

2. DETERMINATION OF THE GEOMETRIC PARAMETERS OF THE LOGARITHMIC SPIRAL FROM POINTS OF ITS SEGMENT

We assume that the spiral arm is a logarithmic spiral with a constant pitch angle i. The Galactic center is assumed to be situated at the pole of the spiral and the solar galactocentric distance R_0 is considered to be equal to the distance to this pole. The galactoaxial distance R of a point on the logarithmic spiral is defined by the equation

$$R(\lambda) = |R_0| e^{k(\lambda - \lambda_0)}, \tag{1}$$

where $\lambda \in (-\infty; +\infty)$ is the *rotary* galactocentric longitude of the point and $k \equiv \tan i$, λ_0 is the positional parameter. Longitude λ is counted clockwise as seen from the North Galactic Pole; $\lambda = 0 \pm 2\pi n$, $n \in \mathbb{Z}$, in the direction to the Sun. We suppose that the direction to the Galactic center is known. Although trailing spiral arms are expected, we accept solutions irrespective of whether the resulting pitch angle is negative or positive.

First we consider the problem on the number of points that define the logarithmic spiral as a geometric figure. Given that the logarithmic spiral has three parameters (R_0, i, and λ_0) we try to determine them from three points M_i, $i = 1, 2, 3$, supposed to belong to the spiral. We solve the problem in terms of the *nominal* galactocentric longitudes, Λ_i, which specify the positions of points on the Galactic plane. The longitudes $\Lambda_i \in (-\pi; +\pi]$, $i = 1, 2, 3$, can be found from the following equations:

$$\sin \Lambda_i = Y_i / R_i, \quad \cos \Lambda_i = (R_0 - X_i)/R_i\,, \tag{2}$$

$$R_i^2 = R_0^2 + r_i^2 \cos^2 b_i - 2R_0 r_i \cos b_i \cos l_i\,, \tag{3}$$

where r_i is the heliocentric distance of the ith point; l_i and b_i are its Galactic longitude and latitude respectively; X_i and Y_i are the Cartesian heliocentric coordinates, which are defined by the equations

$$X_i = r_i \cos b_i \cos l_i\,, \quad Y_i = r_i \cos b_i \sin l_i\,. \tag{4}$$

The X-axis points toward the Galactic center and the Y-axis points in the direction of Galactic rotation. The Sun is located at the origin of the coordinate system.

The solution of the problem is given by the following equations:

$$(\Lambda_3 - \Lambda_2) \ln R_1 + (\Lambda_1 - \Lambda_3) \ln R_2 + (\Lambda_2 - \Lambda_1) \ln R_3 = 0; \tag{5}$$

$$k = \ln\left(R_i/R_j\right) \big/ (\Lambda_i - \Lambda_j), \quad i, j = 1, 2, 3, \;\; i \neq j; \tag{6}$$

$$\lambda_0 = \Lambda_i - \ln\left(R_i/|R_0|\right)/k, \quad i = 1, 2, 3. \tag{7}$$

The parameter R_0 can be determined from Equation (5), and k and λ_0 can then be calculated from Equations (6) and (7), respectively. Given the transcendental nature of Equation (5), below we place an emphasis on the number of its roots.

Here we use the model spiral with $R_0 = 8$ kpc, $i = -18.^\circ7$, $\lambda_0 = -30.^\circ0$ (Nikiforov & Shekhovtsova 2001) to illustrate our findings. In the case where all points M_i, $i = 1, 2, 3$, are located on the same side of the X-axis, Equation (5) has always two roots and hence two spirals pass through the three points (see the left panel in Fig. 1). In the case where the points are on different sides of the X-axis, Equation (5) can have one, two or three roots, however, in most of the cases there is only one unique root (see Fig. 1, right panel). The equation has three roots

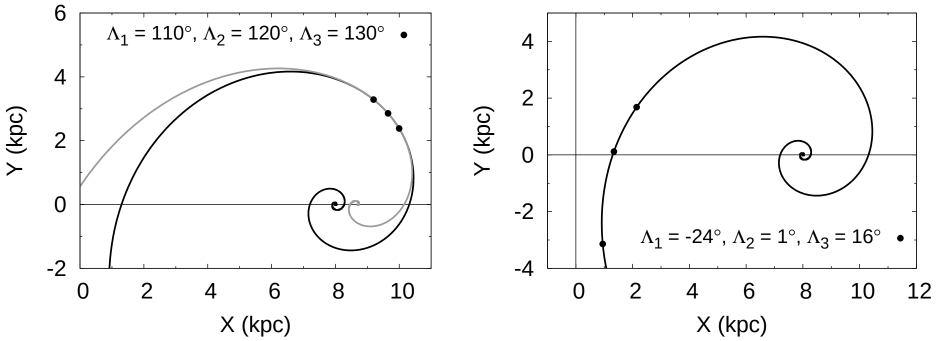

Fig. 1. Spirals passing through the three given points. The black line shows the model spiral. The gray line (left panel) shows the second spiral which passes through the given points in the case if they are located on the same side of the X-axis.

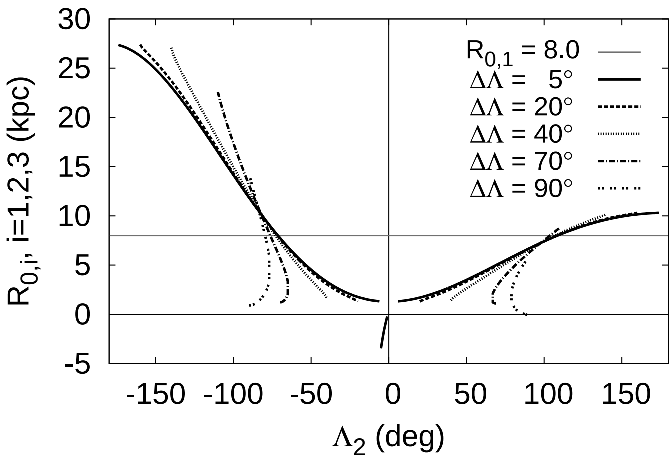

Fig. 2. Dependence of the roots of Equation (5) on the longitude of the point M_2. Here $\Delta\Lambda$ is the galactocentric angle between the adjacent points in a triplet. The gray horizontal line shows the root that is equal to the initial R_0 value, $R_{0,1} = 8$ kpc. The curved lines show the branches of (one or two) additional roots for each $\Delta\Lambda$ (see text).

if $(\Lambda_3 - \Lambda_1) \gtrsim 100°$ and $|\Lambda_2| \lessapprox (\Lambda_3 - \Lambda_1)/2$ (see Fig. 2). Note that, since there is always one root equal to the initial value ($R_{0,1} = 8$ kpc), the total number of roots for given $\Delta\Lambda$ and Λ_2 is determined by the number of intersections of the line $\Lambda_2 =$ const with the branch of additional roots for this $\Delta\Lambda$ in Fig. 2. One or two such intersections mean two or three roots, correspondingly; if there is no intersection, the root $R_{0,1}$ is the only one.

The parameters of the model spiral could be uniquely determined from four points belonging to the spiral. In practice, one cannot draw a one-turn spiral segment through four arbitrary points, and we therefore apply the three-point method to real and simulated data. In the cases where the spiral segment is presented by more than three objects, we estimate the parameters of the segment as the medians of the sets of values found from all possible triplets of objects.

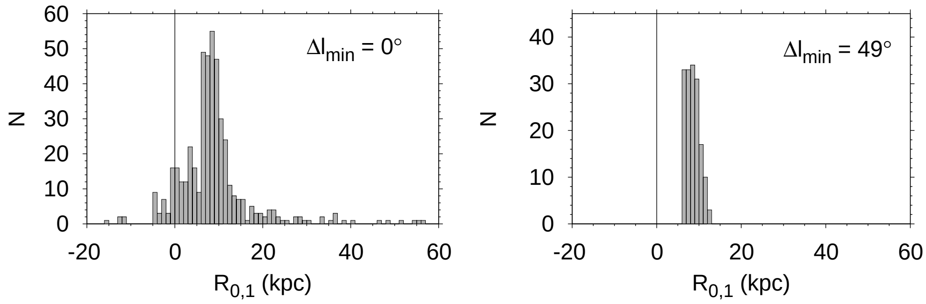

Fig. 3. Distribution of the R_0 estimates found from all possible triplets of objects belonging to the Perseus arm.

3. APPLICATION OF THREE-POINT METHOD TO MASERS

We use the catalog of masers by Reid et al. (2014) as a source of observational data. Ninety masers from this list were assigned by Reid et al. (2014) to five segments of the Galaxy's spiral arms.

In the cases where most of the masers of a segment are located on the same side of the X-axis (as for the Scutum and Sagittarius arms), most of the triplets determine pairs of spirals passing through the given triplet. In such a situation, we choose the spiral with the smallest deviation from the segment masers. In the cases where most of the triplets have a unique spiral passing through all their three points we exclude the triplets with two possible spirals from consideration.

We tested various values of the minimum heliocentric angle $\Delta l_{\min}$ between the adjacent points of the triplet. As illustrated in Fig. 3, the distribution of the solar galactocentric distance estimates R_0 calculated from triplets of the Perseus arm has the stable dominant peak and the variance which is dependent on $\Delta l_{\min}$. We choose the value of $\Delta l_{\min}$ characterized by the lowest standard error of the mean of R_0 values.

Table 1 lists the median values, $\mathsf{Me}\,R_0$, of R_0 estimates for five spiral segments. We used the jackknife technique to compute the standard deviations and bias estimates. The weighted average of the bias corrected estimates, $R_{0,\mathrm{corr}}$, for the Perseus and Scutum arms with the weights inversely proportional to the jackknife variances, $\sigma^2_{R_{0,\mathrm{J}}}$, is $R_0 = 8.44 \pm 0.45$ kpc. Three other segments are excluded from consideration for the following reasons: the Sagittarius arm has obviously a bimodal distribution of R_0 values, the Outer arm has a very small sample of triplets, the model spiral for the Local arm is in rather poor agreement with the position of masers. Hence we estimate the distance to the Galactic center from masers in spiral arms to be

$$R_0 = 8.44 \pm 0.45 \text{ kpc}. \tag{8}$$

Our simplified three-point method gives an R_0 estimate that agrees with those derived from maser kinematics: $R_0 = 8.34 \pm 0.16$ kpc (Reid et al. 2014) and $R_0 = 8.03 \pm 0.12$ kpc (Bajkova & Bobylev 2015).

We also estimated the parameters i and λ_0 for each segment as the medians of the sets of values determined from all possible pairs of objects using Equations (6) and (7). The estimates obtained are listed in Table 2 and the resulting spiral structure is shown in Fig. 4.

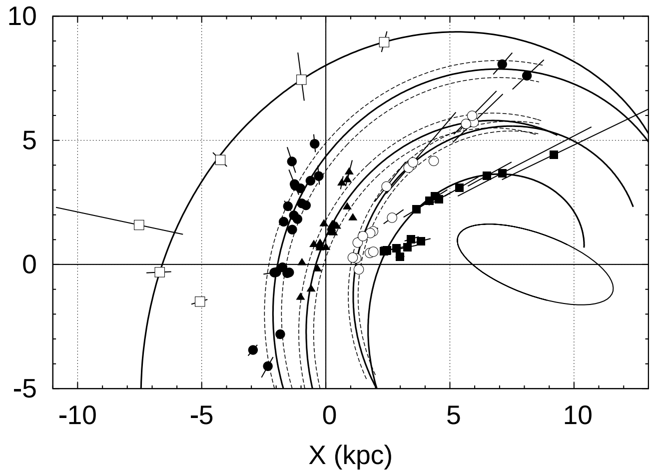

Fig. 4. Spiral pattern and the projected distribution of masers onto the Galactic plane: Outer arm (open squares), Perseus arm (solid circles), Local arm (solid triangles), Sagittarius arm (open circles), and Scutum arm (solid squares). The galactic bar is shown as an ellipse (Jílková et al., 2012).

Table 1. Estimates of R_0 for five spiral segments. The number of triplets with the minimum heliocentric angle $(\Delta l)_{\min}$ is designated as $N_{\therefore}$. Me R_0 denotes the median of R_0 values found from all triplets. The $R_{0,\mathrm{corr}}$ and $\sigma_{R_{0,\mathrm{J}}}$ columns give the bias corrected solar galactocentric distance estimate and its standard deviation, respectively, found using the jackknife technique, and $\Delta R_{0,\mathrm{corr}}$ is the difference between $R_{0,\mathrm{corr}}$ and Me R_0.

Arm	$N_{\therefore}$	$(\Delta l)_{\min}$	Me R_0 (kpc)	$R_{0,\mathrm{corr}} \pm \sigma_{R_{0,\mathrm{J}}}$ (kpc)	$\Delta R_{0,\mathrm{corr}}$ (kpc)
Outer	7	0°	$8.4^{+5.3}_{-19.6}$	16.1 ± 7.5	+7.7
Perseus	161	49°	$8.43^{+0.19}_{-0.20}$	8.36 ± 0.53	−0.07
Local	328	0°	$2.93^{+0.35}_{-0.19}$	2.17 ± 1.00	−0.76
Sagittarius	306	0°	$9.92^{+0.36}_{-0.34}$	10.62 ± 0.69	+0.70
Scutum	267	0°	$9.01^{+0.30}_{-0.15}$	8.62 ± 0.81	−0.39

Table 2. Estimates of i and λ_0 for five spiral segments with $R_0 = 8.44$ kpc. Here $\sigma_{i,\mathrm{J}}$ and $\sigma_{\lambda_0,\mathrm{J}}$ are the standard deviations calculated using the jackknife technique, and $(\sigma_\mathrm{w})_0$ is the estimated segment's width.

Arm	Me i	$\sigma_{i,\mathrm{J}}$	Me λ_0	$\sigma_{\lambda_0,\mathrm{J}}$	$(\sigma_\mathrm{w})_0$ (kpc)
Outer	$-18.^\circ6^{+6.^\circ7}_{-5.^\circ6}$	$0.^\circ80$	$+98.^\circ3^{+25.^\circ5}_{-10.^\circ8}$	$2.^\circ0$	
Perseus	$-10.^\circ6^{+0.^\circ55}_{-0.^\circ35}$	$1.^\circ08$	$+63.^\circ3^{+4.^\circ3}_{-2.^\circ1}$	$9.^\circ4$	0.34 ± 0.05
Local	$-16.^\circ5^{+1.^\circ4}_{-2.^\circ2}$	$5.^\circ1$	$+9.^\circ0^{+0.^\circ29}_{-0.^\circ16}$	$0.^\circ55$	0.29 ± 0.04
Sagittarius	$-9.^\circ9^{+1.^\circ8}_{-0.^\circ80}$	$3.^\circ6$	$-50.^\circ8^{+6.^\circ5}_{-16.^\circ7}$	26°	0.20 ± 0.04
Scutum	$-21.^\circ4^{+0.^\circ58}_{-1.^\circ04}$	$1.^\circ8$	$-43.^\circ9^{+2.^\circ8}_{-5.^\circ7}$	$10.^\circ5$	

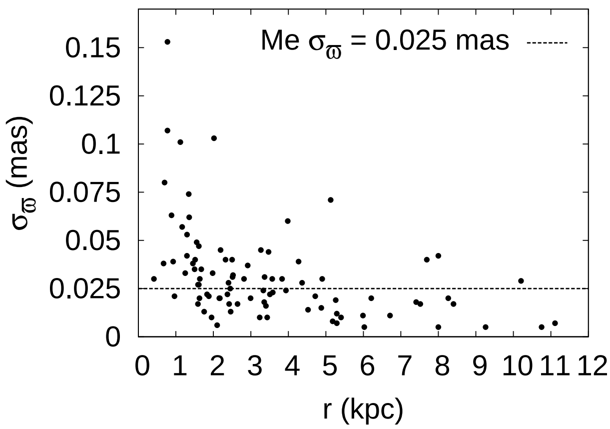

Fig. 5. Dependence of absolute parallax error on the heliocentric distance to the maser.

4. NUMERICAL INVESTIGATION OF THE STATISTICAL PROPERTIES OF THE R_0 ESTIMATES BASED ON SPIRAL SEGMENTS

We use Monte Carlo technique to investigate the statistical properties of the R_0 estimates based on the spiral segment's geometry. Here we consider a basic model representing the Perseus arm with the following adopted geometric parameters: $R_0 = 8.0$ kpc, $i = -10.^\circ0$, and $\lambda_0 = +61.^\circ0$. The galactocentric longitudes of the segment ends are equal to $\lambda_1^s = -21.^\circ0$ and $\lambda_2^s = +88.^\circ0$. We denote the angular length of the segment as $\Delta\lambda$. The simulated segment is populated by N=24 objects outlining it and its width σ_w is set equal to 0.34 kpc. The absolute and fractional parallax errors are set equal to $\sigma_\varpi = 0.025$ mas and $\sigma_\varpi/\varpi = 0.06$, respectively. The absolute parallax error varies from maser to maser and decreases with increasing heliocentric distance (see Fig. 5) and that is why we consider both absolute and fractional parallax errors.

We vary one parameter with all other parameters fixed, and investigate the variation of the standard deviation and bias of the estimator. We generated a total of 10 000 simulated catalogs of objects for every set of parameters by varying two consequent offsets: the offset crosswise the arm and the offset of parallax. Both offsets are normally distributed with the standard deviations equal to σ_w and σ_ϖ (or σ_ϖ/ϖ), respectively.

Fig. 6 illustrates the dependence of the median R_0, the standard deviation of R_0 estimate, and the uncertainty of Me R_0 on the angular length $\Delta\lambda$ of the segment. The standard deviation of R_0 decreases by a factor of ten over the $\Delta\lambda$ interval from 50° to 190°. The bias is insignificant for the segment lengths $\Delta\lambda$ greater than 60°.

Fig. 7 shows the dependence of the same statistical characteristics on the absolute (σ_ϖ) and fractional (σ_ϖ/ϖ) parallax error. The standard deviation of the R_0 estimate increases by a factor of three over the interval from $\sigma_\varpi = 0$ to 0.05 mas.

The largest biases are found in the cases of large uncertainty σ_ϖ and small number of objects in the segment (see Table 3). The standard deviation is mostly influenced by the angular length $\Delta\lambda$ and absolute uncertainty of parallax σ_ϖ.

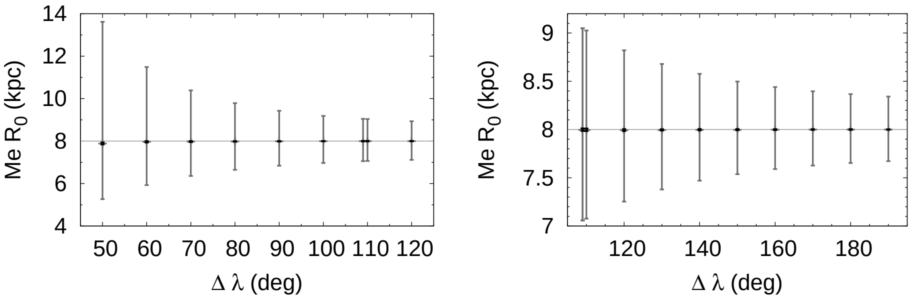

Fig. 6. Median and standard deviation of R_0 estimate (the gray dashes and error bars) and the uncertainty of $\mathsf{Me}\,R_0$ (the black error bars) vs. $\Delta\lambda$ in the case of constant λ_1^s (left panel) and λ_2^s (right panel).

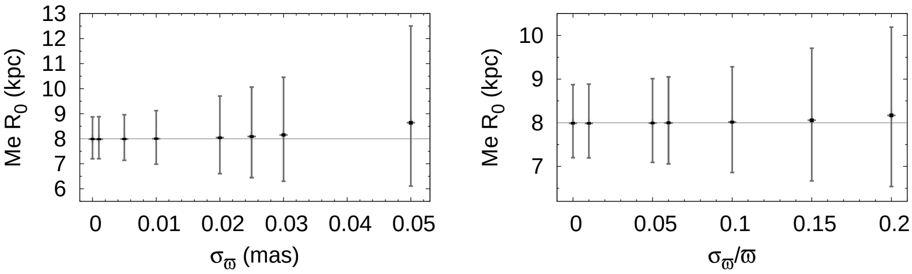

Fig. 7. Dependence of the median and standard deviation of R_0 estimate (the gray dashes and error bars) and the uncertainty of $\mathsf{Me}\,R_0$ (the black error bars) on absolute parallax error σ_ϖ (left panel) and fractional parallax error σ_ϖ/ϖ (right panel).

Table 3. Results of the numerical simulations. Each parameter p varies from $p_{\min}$ to $p_{\max}$. Here σ_{R_0} and ΔR_0 denote the standard deviation of the R_0 estimate and the bias of the estimator, respectively.

p	$p_{\min}$	σ_{R_0} (kpc)	ΔR_0 (kpc)	$p_{\max}$	σ_{R_0} (kpc)	ΔR_0 (kpc)
$\Delta\lambda$, $\lambda_1^s = -21°$	50°	$^{+5.7}_{-2.6}$	-0.12 ± 0.05	120°	$^{+0.94}_{-0.89}$	-0.00 ± 0.01
$\Delta\lambda$, $\lambda_2^s = +88°$	109°	$^{+1.05}_{-0.94}$	$-0.00^{+0.02}_{-0.01}$	190°	$^{+0.34}_{-0.33}$	-0.00 ± 0.04
σ_ϖ (mas)	0.00	$^{+0.89}_{-0.79}$	$+0.01 \pm 0.01$	0.05	$^{+3.9}_{-2.5}$	$+0.65^{+0.03}_{-0.05}$
N	3	$^{+3.3}_{-2.1}$	$+0.61 \pm 0.03$	60	$^{+0.66}_{-0.61}$	$+0.00 \pm 0.01$
$\sigma_{\rm w}$ (kpc)	0.00	$^{+0.54}_{-0.52}$	$+0.00 \pm 0.01$	0.60	$^{+1.7}_{-1.3}$	$+0.04 \pm 0.02$
σ_ϖ/ϖ	0.00	$^{+0.89}_{-0.79}$	$+0.01 \pm 0.01$	0.20	$^{+2.0}_{-1.6}$	$+0.17 \pm 0.02$
i	−20°	$^{+0.94}_{-0.85}$	$+0.00 \pm 0.01$	0°	$^{+1.10}_{-0.96}$	$-0.02^{+0.02}_{-0.01}$

5. CONCLUSIONS

We have for the first time determined the solar galactocentric distance R_0 from the spatial distribution of objects tracing the spiral arms of the Galaxy. We used the data for masers with measured trigonometric parallaxes to yield an estimate of $R_0 = 8.44 \pm 0.45$ kpc. Parameters of five Galactic spiral segments were evaluated. The results of numerical simulations support the efficiency of our new approach.

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REFERENCES

Bajkova A. T., Bobylev V. V. 2015, Baltic Astronomy, 24, 43

Bobylev V. V., Bajkova A. T. 2014, MNRAS, 437, 1549

Efremov Yu. N. 2011, Astronomy Reports, 55, 108

Francis C., Anderson E. 2012, MNRAS, 422, 1283

Jílková L., Carraro G., Jungwiert B., Minchev I. 2012, A&A, 541, AA64

Nikiforov, I.I. 2004, in *Order and Chaos in Stellar and Planetary Systems*, eds. G. G. Byrd et al., ASP Conf. Ser., 316, 199

Nikiforov I. I., Shekhovtsova T. V. 2001, in *Stellar Dynamics: from Classic to Modern*, eds. L. P. Ossipkov & I. I. Nikiforov, St. Petersburg Univ. Press, 28

Reid M. J., Menten K. M., Brunthaler A. et al. 2014, ApJ, 783, 130

Xu Y., Li J. J., Reid M. J. et al. 2013, ApJ, 769, 15