Letter to the Editor

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Bianchi type-*I* Model with Time Varying Λ and G : The Generalized Solution

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Abstract: In this paper, we have investigated the homogeneous and anisotropic Bianchi type-*I* cosmological model with a time-varying Newtonian and cosmological constant. We have analytically solved Einstein's field equations (EFEs) in the presence of a stiff-perfect fluid. We show that the analytical solution for the average scale factor for the generalized Friedman equation involves the hyper-geometric function. We have studied the physical and kinematical quantities of the model, and it is found that the universe becomes isotropic at late times.

Keywords: Bianchi Metric, Anisotropic, Varying Λ , Varying G “Newtonian Constant”

1 Introduction

Recently, cosmological models with time-varying vacuum energy density Λ and Newtonian gravitational Constant have attracted the attention of several authors (Beesham 1986; Kalligas et al. 1992; Carvalho et al. 1992; Singh and Tiwari 2008). Hence, a question of how the gravitational constant can be anchored with Λ without changing the framework of Einstein's Field Equations (EFEs) of the general theory of relativity (GR) is raised. The time dependent form of G was firstly suggested by Dirac (1937) and then followed by Beesham (1986). In addition to that, Bianchi types models with a dynamic form of Λ have extensively been studied by researchers (Kalligas et al. 1992; Carvalho et al. 1992; Tripathy et al. 2012; Behera et al. 2010; Arbab 1997).

In principle, Bianchi models are classified to be spatially homogeneous and in general anisotropic (Ellis 2006), and they are considered as a generalization of the Friedman Lemaître Robertson Walker (FLRW) model. In addition to that, the Bianchi type-*I*, *III*, *V*, *VII* are also anisotropic at late times even for ordinary matter, where the possible anisotropy of the Bianchi metrics necessarily dies away during the inflationary period. This isotropization of their metrics is due to the implicit assumption that the dark energy (DE) is isotropic. Moreover, and since, the present

observations indicate the existence of a few amounts of variation in the intensity of the temperature of the Cosmic Micro Wave Background Radiation (CMB) (Amirhashchi and Amirhashchi 2019); therefore, these models can provide a good framework to study the isotropy of CMB in the sky¹. Recently Bianchi type-*I* universe with a self-consistent nonlinear spinor field (NLSF) and a self-consistent system of interacting spinor and scalar fields and a general NLSF in presence of perfect fluid with time-dependent gravitational constant G and cosmological constant Λ in Einstein's equation have been used to study the initial singularity and asymptotic isotropization process by Saha (2001a,b, 2015) and Bronnikov et al. (2004). Saha (2001a) has found that the addition of time-dependent Λ term in Einstein's equation does not affect the initial singularity and asymptotic isotropization process which is dominated by the nonlinear spinor term in the Lagrangian. Moreover, it has been shown that the results remain the same even in the case when the Bianchi type-*I* universe is filled with a perfect fluid.

At this stage, it is worth mentioning that most of the previously published studies on Bianchi type *I*, *III*, *V* and *VI* (Pradhan and Kumar 2001; Mazumder 1994; Ram 1989; Singh and Tiwari 2008; Arbab 1998) with a time-varying G and cosmological constant $\Lambda = \frac{\alpha}{S^2} + \beta H^2$, the adjustable dimensionless parameters α and β , where S and H be meaning the average scale factor and H is the Hubble parameter, did not tackle the problem of solving the EFEs of GR directly, instead, they choose a constant value for α and β from the beginning (i.e. early stages of their solution process) and then solved the EFEs. In addition to that, they assume a

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¹ This issue is out of the scope of this paper at the mean time

coupling relationship for the metric variables A , B and C , which would be interesting if they let them smoothly come out of the solution process itself. Although, these coupling relations are thought to be reasonable by those authors, still addressing special cases.

In this paper, we will attempt to obtain the most general analytical solutions to EFEs for a stiff-fluid Bianchi type-I cosmological model without assuming any coupling relationship between the metric variables or making any prior choice in the numerical values α and β that appears on the Λ term.

This paper is structured as follows: We begin with a brief introduction in section one. In section two, we display the Bianchi type-I model and its solutions. Section three represents the generalized model and its behavior. We end the paper with a conclusion.

2 Bianchi Type-I Cosmology

The spatially homogeneous and anisotropic Bianchi type-I space-time line-element is given by:

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)dy^2 + C^2(t)dz^2, \quad (1)$$

where $A(t)$, $B(t)$ and $C(t)$ are the components of the fundamental metric tensor. We assume that the cosmic matter is a perfect fluid which is represented by the following energy-momentum tensor:

$$T_{ij} = (\rho + p)u_i u_j + p g_{ij}, \quad (2)$$

where ρ is matter density, $u^i = \delta_t^i = (-1, 0, 0, 0)$ is the normalized fluid four-velocity, which is a time-like quantity which stratifies $u^i u_i = -1$, and p is the fluid's isotropic pressure. ρ and p are related to each other through the barotropic equation of state:

$$p = w\rho, \quad (3)$$

and w is the equation of state (EoS) parameter. The EFEs with time-dependent Λ and G are given by

$$R_{ij} - \frac{1}{2}g_{ij}R = -8\pi G(t)T_{ij} + g_{ij}\Lambda(t), \quad (4)$$

where R_{ij} and g_{ij} are the Ricci and metric tensors respectively, and R is the Ricci scalar. Substituting Eqs. (1) and (2) into Eq. (4) the field equations are computed as follows:

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} = 8\pi G(t)\rho + \Lambda(t), \quad (5)$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -8\pi G(t)p + \Lambda(t), \quad (6)$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{C}\dot{A}}{CA} = -8\pi G(t)p + \Lambda(t), \quad (7)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -8\pi G(t)p + \Lambda(t), \quad (8)$$

with the dot over a letter representing differentiation with respect to the time coordinate. Moreover, the divergence of (4) produces auxiliary equation, in such way that

$$8\pi G \left[\dot{\rho} + (\rho + p) \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) \right] + 8\pi \dot{G} + \dot{\Lambda} = 0, \quad (9)$$

while the conservation of the usual energy-momentum tensor T^{ij} (i.e., $\nabla_j T^{ij} = 0$) yields

$$\dot{\rho} + (\rho + p) \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) = 0. \quad (10)$$

Substituting Eq. (10) into (9) we get

$$8\pi \rho \dot{G} + \dot{\Lambda} = 0. \quad (11)$$

This is an evolution equation for Λ and G , and it shows how they evolve with time depending on each other. In other words, if G increases then Λ will eventually decrease and vis versa. The average scale factor S for Bianchi-I models is defined to be

$$V = S^3 = \sqrt{-\det(g_{ij})} = ABC, \quad (12)$$

and the generalized Hubble parameter H is defined as in (Tiwari 2008)

$$H = \frac{\dot{S}}{S} = \frac{1}{3} \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) = \frac{1}{3}(H_x + H_y + H_z), \quad (13)$$

where H_x , H_y , and H_z are the directional Hubble's parameters along x , y and z directions respectively. The deceleration parameter q follows the usual definition

$$q = -\frac{\ddot{S}S}{\dot{S}^2} = -1 - \frac{\dot{H}}{H^2}. \quad (14)$$

The volume expansion parameter θ , the average anisotropy parameter A_p , and shear modules σ are defined as in (Alfedeel et al. 2018; Carvalho et al. 1992) by:

$$\theta = \nabla_i u^i = 3H, \quad (15)$$

$$A_p = \frac{1}{3} \sum_{i=1}^3 \left(\frac{H_i - H}{H} \right)^2, \quad (16)$$

$$\sigma^2 = \frac{1}{2} \sigma_{ij} \sigma^{ij} = \frac{1}{2} \left(\frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2} + \frac{\dot{C}^2}{C^2} \right) - \frac{\theta^2}{6}, \quad (17)$$

$$= \frac{1}{3} \left(\frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2} + \frac{\dot{C}^2}{C^2} \right) - \frac{1}{3} \left(\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} \right),$$

where the term σ^{ij} represents the shear tensor. For this model, its scalar quantity comes out to be

$$\sigma = \frac{K}{S^3}, \quad (18)$$

where K is a numerical constant that is related to the anisotropy of the model-for more details about the derivation see Amirhashchi and Amirhashchi (2019). Now, subtracting Eq. (6) from (7), (6) from (8), and (7) and integrating with respect to time coordinate we get

$$\frac{\dot{B}}{B} = \frac{\dot{A}}{A} + \frac{k_1}{S^3}, \quad (19)$$

$$\frac{\dot{C}}{C} = \frac{\dot{B}}{B} + \frac{k_3}{S^3}, \quad (20)$$

$$\frac{\dot{C}}{C} = \frac{\dot{A}}{A} + \frac{k_2}{S^3}, \quad (21)$$

where k_1, k_2 and k_3 are constants of integration. Integrating the above set of equations (19)-(20), we get

$$\frac{B}{A} = d_1 \exp\left(k_1 \int \frac{dt}{S^3}\right), \quad (22)$$

$$\frac{C}{B} = d_2 \exp\left(k_2 \int \frac{dt}{S^3}\right), \quad (23)$$

$$\frac{C}{A} = d_3 \exp\left(k_3 \int \frac{dt}{S^3}\right), \quad (24)$$

Using Eq. (12) with a bit of algebraic manipulation, Eq. (22)-(24) can be combined to give

$$A = \mathcal{A}_0 S \exp\left(m_1 \int \frac{dt}{S^3}\right), \quad (25)$$

$$B = \mathcal{B}_0 S \exp\left(m_2 \int \frac{dt}{S^3}\right), \quad (26)$$

$$C = \mathcal{C}_0 S \exp\left(m_3 \int \frac{dt}{S^3}\right), \quad (27)$$

where

$$\mathcal{A}_0 = \frac{\sqrt[3]{d_2 d_3}}{d_3}, \quad \mathcal{B}_0 = \frac{\sqrt[3]{d_2 d_3}}{d_2}, \quad \mathcal{C}_0 = \sqrt[3]{d_2 d_3},$$

$$m_1 = \frac{k_2 - 2k_3}{3}, \quad m_2 = \frac{k_3 - 2k_2}{3}, \quad m_3 = \frac{k_2 + k_3}{3},$$

are constants values that satisfying the following relation:

$$\mathcal{A}_0 \mathcal{B}_0 \mathcal{C}_0 = 1, \quad m_1 + m_2 + m_3 = 0. \quad (28)$$

Up to this stage, we see that the Bianchi type-*I* model is fully characterized by a set of several parameters $\theta, \sigma, G, \Lambda, q$, the metric variables A, B , & C and the energy density ρ as shown in Eqs. (10), (11), (14), (15), (18), (32) (25)-(27). Unfortunately, this system is not complete, because of the unknown value of the average scale factor $S(t)$. To find its value, we can re-express the field equations (5)-(8) and (10) in terms of S, H, q and σ as

$$8\pi G\rho - \Lambda = (2q - 1)H^2 - \sigma^2, \quad (29)$$

$$8\pi G\rho + \Lambda = 3H^2 - \sigma^2, \quad (30)$$

so that when subtracting Eq. (29) from Eq. (30), we eliminate the σ term, obtaining

$$\frac{\ddot{S}}{S} + 2\frac{\dot{S}^2}{S^2} = 4\pi G(t)\rho(1 - w) + \Lambda(t). \quad (31)$$

This equation cannot be integrated as it is because of the unknown functions Λ, G, ρ and the parameter w . Hence, we use the same form of Λ as in Alfedeel et al. (2018) and Tiwari (2008)

$$\Lambda(t) = \frac{\alpha}{S^2} + \beta H^2. \quad (32)$$

Eq. (31) becomes

$$\frac{\ddot{S}}{S} + (2 - \beta)\frac{\dot{S}^2}{S^2} - \frac{\alpha}{S^2} = 4\pi G(t)\rho(1 - w), \quad (33)$$

This equation is called the generalized Friedmann equation, which is a non-linear second order differential equation, and it is similar to the equation of Bianchi type *V* model that we have obtained in our previous paper (Alfedeel et al. 2018), except the third term on the left-hand side here is different by a factor of 2 (i.e. $\alpha_{type_1} = \alpha_{type_V} + 2$). Now, integrating Eq. (33) twice with respect to time through the intermediate variable $P = \dot{S}$ for $w = 1$ "a stiff perfect fluid" gives

$$\int \frac{dS}{\sqrt{\frac{c}{\beta-2} S^{2(\beta-2)} + \frac{\alpha}{2-\beta}}} = \quad (34)$$

$$\frac{S}{\sqrt{\frac{\alpha}{2-\beta}}} {}_2F_1\left(\frac{1}{2}, \frac{1}{2(\beta-2)}, 1 + \frac{1}{2(\beta-2)}; \frac{c}{\alpha} S^{2(\beta-2)}\right)$$

$$= \pm(t - t_0),$$

which can be written as

$$S {}_2F_1\left(\frac{1}{2}, \frac{1}{2(\beta-2)}, 1 + \frac{1}{2(\beta-2)}; \frac{c}{\alpha} S^{2(\beta-2)}\right) = \quad (35)$$

$$\pm \sqrt{\frac{\alpha}{2-\beta}}(t - t_0),$$

where $S = S(t)$, $\beta \neq 2$ and $\alpha \neq 0$, c is constant of integration, and ${}_2F_1$ is the hyper-geometric function. Eq. (35) represents the generalized solution of the Friedman equation in (33) for the average scale factor S . Moreover, throughout the solution process, we did not assume any coupling relation between the metric variables or a prior value on the parameters β and α in Λ term. So, this is the most general solution which was not previously obtained by any authors work in the Bianchi type-*I* models. Thus, to have an explicit function formula for S as a function of time, we must fix the value of β by making appropriate gauge choices.

3 The Generalized Model

Throughout this section, we will present our generalized model that emerged from a suitable choice of β value. Setting $\beta = 1$ such that Eq.(35) gives a power solution of the average scale factor S as

$$S = \sqrt{\alpha} \left[\tau^2 + \frac{c}{\alpha^2} \right]^{\frac{1}{2}}. \quad (36)$$

For this known the value of S , then the kinematical and dynamical parameters H , θ , σ , q , and ρ , the Newtonian gravitational “constant” G , and the metric variables A , B , & C are calculated as

$$H = \frac{\dot{S}}{S} = \frac{\tau}{\tau^2 + \frac{c}{\alpha^2}}, \quad (37)$$

$$\theta = 3H = \frac{3\tau}{\tau^2 + \frac{c}{\alpha^2}}, \quad (38)$$

$$\sigma = \frac{K}{S^3} = \frac{K}{\left[\sqrt{\alpha} \sqrt{\tau^2 + \frac{c}{\alpha^2}} \right]^3}, \quad (39)$$

$$\rho = \frac{\rho_0}{S^6} = \frac{\rho_0}{\left[\alpha \tau^2 + \frac{c}{\alpha} \right]^3}, \quad (40)$$

$$G = - \int \frac{\dot{\Lambda}}{8\pi\rho} dt = \frac{\alpha^3}{4\pi\rho_0} \tau^4 - \frac{c}{4\pi\rho_0} \tau^2 + G_0, \quad (41)$$

$$q = -\frac{\ddot{S}S}{\dot{S}^2} = -\frac{c}{\alpha^2\tau^2}, \quad (42)$$

$$A = \mathcal{A}_0 \sqrt{\alpha} \sqrt{\tau^2 + \frac{c}{\alpha^2}} \exp \left[\frac{m_1}{\sqrt{\alpha}} \frac{\tau}{\sqrt{\tau^2 + \frac{c}{\alpha^2}}} \right], \quad (43)$$

$$B = \mathcal{B}_0 \sqrt{\alpha} \sqrt{\tau^2 + \frac{c}{\alpha^2}} \exp \left[\frac{m_2}{\sqrt{\alpha}} \frac{\tau}{\sqrt{\tau^2 + \frac{c}{\alpha^2}}} \right], \quad (44)$$

$$C = \mathcal{C}_0 \sqrt{\alpha} \sqrt{\tau^2 + \frac{c}{\alpha^2}} \exp \left[\frac{m_3}{\sqrt{\alpha}} \frac{\tau}{\sqrt{\tau^2 + \frac{c}{\alpha^2}}} \right], \quad (45)$$

where $\tau = t - t_0$, and G_0 are constants of integration. This model has an initial point of singularity at $t = t_0$ and $c = 0$. If $\beta = 3$, Eq. (35) can produce a Hyperbolic model with average scale factor $S(t) = -\sqrt{\alpha/c} \sinh(\sqrt{c}\tau)$, but this model is not realistic and it describes the evolution of the universe in negative time. However, Eq. (35) put a limitation on the choice of β , and that it should not be greater than $\beta > 3$ or $\beta = 2$.

3.1 The behavior at late times

In this subsection, the basic concept of the limit will be used to study the behavior of the model kinematical and dynamical parameters at late times (when $t \rightarrow \infty$).

$$\lim_{t \rightarrow \infty} H = \lim_{t \rightarrow \infty} \left[\frac{\tau}{\tau^2 + \frac{c}{\alpha^2}} \right] = 0, \quad (46)$$

$$\lim_{t \rightarrow \infty} \theta = \lim_{t \rightarrow \infty} \left[\frac{3\tau}{\tau^2 + \frac{c}{\alpha^2}} \right] = 0, \quad (47)$$

$$\lim_{t \rightarrow \infty} \sigma = \lim_{t \rightarrow \infty} \left[\frac{K}{\left[\sqrt{\alpha} \sqrt{\tau^2 + \frac{c}{\alpha^2}} \right]^3} \right] = 0, \quad (48)$$

$$\lim_{t \rightarrow \infty} \sigma/\theta = \lim_{t \rightarrow \infty} \left[\frac{K \left(\tau^2 + \frac{c}{\alpha^2} \right)}{3\tau \left[\sqrt{\alpha} \sqrt{\tau^2 + \frac{c}{\alpha^2}} \right]^3} \right] = 0, \quad (49)$$

$$\lim_{t \rightarrow \infty} \rho = \lim_{t \rightarrow \infty} \left[\frac{\rho_0}{\left[\alpha \tau^2 + \frac{c}{\alpha} \right]^3} \right] = 0, \quad (50)$$

$$\lim_{t \rightarrow \infty} G = \lim_{t \rightarrow \infty} \left[\frac{\alpha^3}{4\pi\rho_0} \tau^4 - \frac{c}{4\pi\rho_0} \tau^2 + G_0 \right] = \infty, \quad (51)$$

$$\lim_{t \rightarrow \infty} q = \lim_{t \rightarrow \infty} \left[-\frac{c}{\alpha^2\tau^2} \right] = 0, \quad (52)$$

$$\lim_{t \rightarrow \infty} \Lambda = \lim_{t \rightarrow \infty} \left[\frac{1}{\left(\tau^2 + \frac{c}{\alpha^2} \right)} + \frac{3\tau^2}{\left(\tau^2 + \frac{c}{\alpha^2} \right)^2} \right] = 0, \quad (53)$$

$$\lim_{t \rightarrow \infty} A = \lim_{t \rightarrow \infty} \left\{ \mathcal{A}_0 \sqrt{\alpha} \sqrt{\tau^2 + \frac{c}{\alpha^2}} \right. \quad (54)$$

$$\left. \exp \left[\frac{m_1}{\sqrt{\alpha}} \frac{\tau}{\sqrt{\tau^2 + \frac{c}{\alpha^2}}} \right] \right\} = \infty,$$

$$\lim_{t \rightarrow \infty} B = \lim_{t \rightarrow \infty} \left\{ \mathcal{B}_0 \sqrt{\alpha} \sqrt{\tau^2 + \frac{c}{\alpha^2}} \right. \quad (55)$$

$$\left. \exp \left[\frac{m_2}{\sqrt{\alpha}} \frac{\tau}{\sqrt{\tau^2 + \frac{c}{\alpha^2}}} \right] \right\} = \infty,$$

$$\lim_{t \rightarrow \infty} C = \lim_{t \rightarrow \infty} \left\{ \mathcal{C}_0 \sqrt{\alpha} \sqrt{\tau^2 + \frac{c}{\alpha^2}} \right. \quad (56)$$

$$\left. \exp \left[\frac{m_3}{\sqrt{\alpha}} \frac{\tau}{\sqrt{\tau^2 + \frac{c}{\alpha^2}}} \right] \right\} = \infty,$$

Our analysis shows that H , θ , σ , Λ , the magnitude of q and ρ all decrease as time goes on, whereas G , the metric vari-

ables A , B and C and the volume V are increasing functions of time. The dieing-off of θ and σ or the fundamental ratio $|\sigma/\theta|$ indicates that the model approaches isotropy for large value of t .

4 Conclusion

The main objective of this paper was to find a generalized solution for Bianchi type-I cosmological model with time-varying Newtonian and Cosmological constant for a stiff perfect fluid without making any constraints on the adjustable constants of quantum field theory on curved and expanding background α and β that appears in Λ term.

The EFEs are analytically solved for a stiff perfect fluid whose equation of state parameter (EoS) $w = 1$. We show that the solution of the time-dependent average scale factor $S(t)$ involved the hyper-geometric function. In principle, this is a generalized solution of all previous work on the Bianchi type-I model. Finally, the behavior of the physical and kinematical quantities of the model is discussed in §3.1. As it is clear from Eqs. (36), (43), (44) and (45) at late time the space-time becomes isotropic. However, in this model, the behavior of the initial anisotropic which asymptotically evolves to an isotropic as time increases is in a good agreement with the result that obtained by Bijan Saha (Saha 2015) for the role of nonlinear spinor field in the scope of Bianchi type-I universe.

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