Research Article

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On the plausible origins of the spiral character of galaxies

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Abstract: We here-in demonstrate that the proposed *hitherto* unknown gravitomagnetic dark-force that hypothetically explains the *Flat Rotation Curves of Spiral Galaxies* — this same force, explains very well, the logarithmic and as-well, the barred spiral shapes of spiral galaxies. That is, much in line with Edward Arthur Milne (1896-1950)'s 1946 ideas — *albeit*, on a radically and asymptotically different philosophical train of thought, the galactic disk is here assumed to be in a state of free-fall around the central bulge with the hypothetical gravitomagnetic dark-force being the dominant force determining all gravity-related dynamics of the disk, thus leading to logarithmic and barred spiral orbits, hence the shape of spiral galaxies.

Keywords: darkmatter, gravitomagnetism, Tully-Fisher relation, galaxy rotation curves

1 Introduction

This reading is the fourth in our four part series where we demonstrate that the *Flat Rotation Curve Problem of Spiral Galaxies* — commonly known as the *Darkmatter Problem* — does have a solution within the frame of gravitomagnetism (presented in the reading Nyambuya 2015b). In the second part (*i.e.*, Nyambuya 2019b, and here-after Paper II) of the four part series — the proposed theory to explain the flat rotation curve problem of spiral galaxies was setup, and it was shown (demonstrated) therein (Nyambuya 2019b) how one can harness from this theory, the *Tully and Fisher (1977) Relation* ($M_{\text{gal}} \propto V_{\text{Disk}}^4$) relating the mass of the galaxy and the flat rotational speed in the material laying in the disk. We here-in demonstrate (show) how one can explain the shape of spiral galaxies from this same law that was able to explain the flat rotation curves of spiral galaxies, and as-well as the *Tully and Fisher (1977) Relation*.

When it comes to this issue of the flat rotation curve problem of spiral galaxies, nowhere is this mysterious phenomenon of ‘*dark-matter*’ more clearly evident than in spiral galaxies (see *e.g.*, Swaters et al. 2012; Broeils 1992a,b; Begeman et al. 1991; Begeman 1987; Rubin et al. 1985, 1978,

1970; Rubin and Ford 1970; Forman et al. 1970). It is in these spiral galaxies where this phenomenon manifests itself in the form of the flat rotation curves for test bodies lying in the galactic disk. Some of the finest and most important questions that naturally visit the seeking mind are:

1. Why is this flat rotation curve phenomenon more prominent in spiral galaxies and not any other type of galaxy?
2. Is there a relationship between the shape of spiral galaxies and the flat rotation curve phenomenon?

These are some of the questions that this reading makes the *temerarious endeavour* to provide some answers — or at the very least, set a base or basis from which these type of questions maybe answered.

For example, it is known that the shape of the spiral galaxies can be described by spiral equation of the form¹: $r = \mathcal{R}_D e^{\pm \lambda_{SL} \varphi}$ (see *e.g.*, Puerari et al. 2014; Savchenko and Reshetnikov 2013; Savchenko 2012; Savchenko and Reshetnikov 2011). If the stars in the galactic disk of spirals are on spiral trajectories, the question is ‘*What force law would give rise to such trajectories?*’ This is the key question that this work seeks a most perdurable answer.

To that end — *we here-in* — against prevalent and conventional wisdom, hold that stars, star clusters and molecular clouds (*etc*) resident in galactic disks around the galactic bulge of spiral galaxies — are in a state of free-fall on spiral trajectories in much the same way that planets in the Solar

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¹ This equation, $r = \mathcal{R}_D e^{\pm \lambda_{SL} \varphi}$, will be explained at the instance of Eq. (29).

system are in a state of free-fall around the Sun on elliptical trajectories. Consequently, the shape of these galaxies must be determined by the equation of motion that emerges from the consideration of the motion of test bodies orbiting the central bulge. Current wisdom holds that test bodies in the galactic disks of spiral galaxies are on elliptical trajectories with the shape of these galaxies being a result of a density wave sweeping the galaxy (Lin and Shu 1964).

Despite the general lack of agreement on the exact mechanism leading to the shape of spiral galaxies (e.g., Davis et al. 2015), the prevalent view is that the density waves (Lin and Shu 1964) propagating through the disk of the galaxy are the responsible agent. For example, according to Shu (2016), over the past six decades, Lin and Shu (1964)'s density wave theory has been the most cited as the main agent for the '*grand design spiral genesis in disk galaxies*'. Lin and Shu (1964)'s density wave theory introduces the idea of long-lived quasi-static density waves that sweep across the galactic disk in which *ensuring process* the spiral structure emerges. Apart from explaining the structure of spiral galaxies, this theory has been cited in the literature as having been very successful in explaining Saturn's rings (see e.g., Hedman and Nicholson 2016; Tiscareno et al. 2007, 2006; Phillipps 2005; Carroll and Ostlie 1995; Shu 1984; Goldreich and Tremaine 1982, 1978).

Now, in-closing this introduction, we shall give a synopsis of the reading — it is organised as follows: in §(2), we lay bare our (simplistic) working assumptions. In §(3), we define the dark-force. In §(4), we present our proposed theory as to why spiral galaxies have the shape that they have. In §(5), we give a general discussion and lastly, in §(6), the conclusion drawn thereof.

2 Working Assumptions

We herein make the following idealised assumptions about the mass and rotation curves of spiral galaxies:

2.1 Mass of a Spiral Galaxy

As depicted in Figure 1, the total mass of a spiral galaxy $\mathcal{M}_{\text{gal}}$, can be split into two parts, i.e.:

1. The mass of the galactic bulge, $\mathcal{M}_{\text{B}}$.
2. The mass of the galactic disk, $\mathcal{M}_{\text{Disk}}$.

That is to say: $\mathcal{M}_{\text{gal}} = \mathcal{M}_{\text{B}} + \mathcal{M}_{\text{Disk}}$. In our model, we assume that the bulk of the mass of the galaxy is contained in the bulge i.e.: $\mathcal{M}_{\text{B}} \gg \mathcal{M}_{\text{Disk}}$, so much so that: $\mathcal{M}_{\text{gal}} \sim \mathcal{M}_{\text{B}}$. That is to say, the bulk of the mass is concentrated in the

central region of the galaxy and in the case where there is a visible bulge, this mass can readily be assumed to be the mass of the central bulge itself. In the case where there is no appreciable bulge (see e.g., Fisher and Drory 2011; Kormendy et al. 2010), the mass should be concentrated in the assumed central massive black-hole of the galaxy in question. In both cases of bulge and bulge-less galaxy, the mass of the galaxy $\mathcal{M}_{\text{gal}}$ shall here be denoted by $\mathcal{M}_{\text{B}}$, and this is through the assumed relationship: $\mathcal{M}_{\text{gal}} \sim \mathcal{M}_{\text{B}}$.

2.2 Idealised Rotation Curve of Spiral Galaxies

As depicted in Figure 2, we assume a rotation curve that has two major components, the *Bulge Component* and the *Disk Component*, i.e.:

1. **Bulge Component:** In this region, the orbital speed of test bodies is assumed to increase in direct proportion with the radial distance from the galactic centre, i.e. ($V_{\text{B}} \propto r$). In the bulge region, the Newtonian gravitational component is the predominate gravitational force determining the dynamics of this region. This assumption that: $V_{\text{B}} \propto r$, coupled with the assumption that the Newtonian gravitational component ($\mathbf{F}_{\text{N}}$) is the predominate ($|\mathbf{F}_{\text{N}}| \gg |\mathbf{F}_{\text{D}}|$) gravitational force determining the dynamics of the bulge region directly implies that the density [$\rho_{\text{B}}(r)$] profile of the bulge must be a constant i.e.: $\rho_{\text{B}}(r) = \rho_{\text{B}} = \text{constant}$, for: $0 \leq r \leq \mathcal{R}_{\text{B}}$, and V_{B} is such that:

$$V_{\text{B}}(r) = \left(\frac{4\pi G \rho_{\text{B}}}{3} \right)^{1/2} r, \quad \text{for } [0 \leq r \leq \mathcal{R}_{\text{B}}]. \quad (1)$$

Figure 1. Assumed Model of Spiral Galaxies: The bulk of the gravitating material of a spiral galaxy is here assumed to be contained in the galactic bulge. The material contained in the disk is insignificant in comparison to the material in the bulge so that such that the stars, star clusters, molecular clouds etc, can be considered to be test bodies moving under the influence of the gravitational field setup by the gravitating material in the galactic bulge.

2. **Disk Component:** In this region: $\mathcal{R}_B < r \leq \mathcal{R}_{\text{gal}}$, the orbital speed V_{Disk} , of test bodies is assumed to be a constant throughout the disk. The galactic disk contains insignificant amounts of matter when compared to the galactic bulge, hence this region contains test particles under the gravitational influence and action of the material of the bulge.

Figure 2. Assumed Model of the Rotation Curve of Spiral Galaxies: In the bulge, the speed (V_B) is proportional to the radial distance, i.e.: $V_B \propto r$, and, the density (ρ_B) inside the bulge is assumed to be constant throughout. Inside the disk, the speed (V_{Disk}) is constant throughout the galactic disk.

2.3 Corollary

Bulge Secular Contraction: The assumption that the density (ρ_B) inside the bulge is a constant throughout, implies that:

$$\frac{\dot{\mathcal{R}}_B}{\mathcal{R}_B} = + \frac{1}{3} \frac{\dot{\mathcal{M}}_{\text{gal}}}{\mathcal{M}_{\text{gal}}} < 0. \quad (2)$$

This means that the galactic bulge and hence the galaxy (i.e., spiral galaxies) must — *in general* — be undergoing secular contraction if the bulge is undergoing mass loss via radiation with little — *if any* — accretion taking place. We here have assumed that the bulge emits matter-energy more than it accretes, i.e.: $\dot{\mathcal{M}}_{\text{gal}} < 0$.

Mass-Bulge-Radius Relation of Spirals: Continuity of the idealised rotation curve as given in Figure 2, at the point: $r = \mathcal{R}_B$, leads directly to the result that:

$$V_{\text{Disk}} = \left(\frac{4\pi G \rho_B}{3} \right)^{1/2} \mathcal{R}_D. \quad (3)$$

where $\mathcal{R}_D$ is the dark-matter radius. This dark-matter radius $\mathcal{R}_D$, has been defined in Nyambuya (2019b). It is the radius around a massive gravitating body where the dark-force is expected to dominate over the Newtonian component and

for a given galaxy, $\mathcal{R}_D$ is defined as follows:

$$\mathcal{R}_D = a_D^{\text{RM}} \left(\frac{\mathcal{M}_{\text{gal}}}{\mathcal{M}_{\odot}} \right)^{a_D} \mathcal{R}_{\text{kpc}}, \quad (4)$$

where² a_D is a constant to be determined by demanding that the dark-force [defined in Eq. (10)] stands up to the Tully and Fisher (1977) relation, a_D^{RM} is a dimensionless constant, and, *here-and-after*, $\mathcal{R}_{\text{kpc}} = 1.00 \text{ kpc}$. Beyond $\mathcal{R}_D$, the dark-force dominates.

This Eq. (3), can also be re-written as: $V_{\text{Disk}}^2 = G\mathcal{M}_B/\mathcal{R}_D$. Given that: $\mathcal{M}_{\text{gal}} \sim \mathcal{M}_B$, it follows that:

$$V_{\text{Disk}}^2 = \frac{G\mathcal{M}_{\text{gal}}}{\mathcal{R}_D}. \quad (5)$$

In Paper (II), we defined $\mathcal{R}_D$ (i.e., we set: $\mathcal{R}_D \propto \sqrt{\mathcal{M}_{\text{gal}}}$) so that one obtains from this relationship, the Tully and Fisher (1977) relation. With: $\mathcal{R}_D \propto \sqrt{\mathcal{M}_{\text{gal}}}$, as given, Eq. (5) suggests a *Mass-Radius Relation* for galactic bulges. Before we set sail to answer the question of why and how the spiral shape of spiral galaxies comes about, we shall immediately attend to this problem of the *Mass-Radius Relation* of galactic bulges.

Galaxies are known to exist in a variety of morphological types ranging from dwarf galaxies to disk galaxies. Based on their physical shapes, they can be grouped as irregular, elliptical and spheroidal galaxies. Disk galaxies have central bulges of different sizes (the origin of which is not yet clearly established), while elliptical galaxies have large mass and spatial dimensions. The mass and size among these different types may vary by orders of magnitudes. In all their various shapes and size, they seem to exhibit a strong correlation between their mass and radius (Kravtsov 2013). If $\mathcal{R}_{\text{gal}}$ is the radius of a galaxy and $\mathcal{M}_{\text{gal}}$ its mass, then, observations indicate that: $\mathcal{R}_{\text{gal}} \propto \mathcal{M}_{\text{gal}}^{\eta}$, where η varies from $\eta \sim 0.33$ (Woo et al. 2008) for dwarf galaxies to $\eta \sim 0.56$ (Chiosi and Carraro 2002) for Early Type Galaxies (ETGs). What we are interested here is the relationship between $\mathcal{R}_B$ and $\mathcal{M}_{\text{gal}}$ and not $\mathcal{R}_{\text{gal}}$ and $\mathcal{M}_{\text{gal}}$.

For non-barred spiral, we must have: $\mathcal{R}_B \simeq \mathcal{R}_D$. From this (i.e., $\mathcal{R}_B \simeq \mathcal{R}_D$), the Tully and Fisher (1977) relation and Eq. (5), it follows that:

$$\frac{\mathcal{R}_B}{\mathcal{R}_{\odot}^S} = \frac{\kappa_{\text{TF}}^{\alpha_{\text{TF}}}}{2} \left(\frac{\mathcal{M}_{\text{gal}}}{\mathcal{M}_{\odot}} \right)^{\frac{\alpha_{\text{TF}} - 2}{2\alpha_{\text{TF}}}}, \quad (6)$$

where:

$$\mathcal{R}_{\odot}^S = \frac{2G\mathcal{M}_{\odot}}{c^2}, \quad (7)$$

² We need to state here that the reader must take note that a_D and α_D are two different parameters. The parameter a_D is key in obtaining the TF-Relation, while ($\alpha_D \mapsto 0$) is key in obtaining an inverse distance law necessary to explain the flat rotation curves.

is the Solar Schwarzschild radius, κ_{TF} and α_{TF} are defined in Paper (II) where these are the constant coefficient and index of the exponent appearing in the Tully and Fisher (1977) relation, namely: $\mathcal{M}_{\text{gal}}/\mathcal{M}_{\odot} = \kappa_{\text{TF}}(V_{\text{disk}}/c)^{\alpha_{\text{TF}}}$, and these are such that: $\kappa_{\text{TF}} = (3.80 \pm 0.50) \times 10^{23}$, and: $\alpha_{\text{TF}} = 4$.

Now, by inserting the numbers, Eq. (6) can be re-written as:

$$\frac{\mathcal{R}_{\text{B}}}{\mathcal{R}_{\text{kpc}}} = a_{\text{B}}^{\text{RM}} \left(\frac{\mathcal{M}_{\text{gal}}}{\mathcal{M}_{\odot}} \right)^{\alpha_{\text{B}}^{\text{RM}}}, \quad (8)$$

where: $a_{\text{B}}^{\text{RM}} = (2.90 \pm 0.20) \times 10^{-5}$, and: $\alpha_{\text{B}}^{\text{RM}} = (\alpha_{\text{TF}} - 2)/2\alpha_{\text{TF}} = 1/4$. From this, it follows that, the mean density, ϱ_{B} , of the bulge is such that:

$$\frac{\varrho_{\text{B}}}{\varrho_{19}} = a_{\text{B}}^{\varrho} \left(\frac{\mathcal{M}_{\text{gal}}}{\mathcal{M}_{\odot}} \right)^{\alpha_{\text{B}}^{\varrho}}, \quad (9)$$

where: $\varrho_{19} = 10^{-19} \text{ g/cm}^3$, $a_{\text{B}}^{\varrho} = 6.00 \pm 1.00$, $\alpha_{\text{B}}^{\varrho} = 1/4$. In the next section, we shall briefly describe the gravitational darkforce and this we do for the benefit of the reader that has not read or failed to access Paper (I) and (II).

3 Gravitational Dark-Force

In the model under consideration [Paper (I) and (II)], the flat rotation curve problem common or prominent in spiral galaxies is here explained by means of a new force that we have coined the gravitational dark-force (and here symbolized as $\mathbf{F}_{\text{D}}$). This gravitational dark-force has been shown in the readings Nyambuya (2019b, 2015b) to arise from Nordström (1912)'s relativistic theory of gravitation. Further, the gravitational dark-force comes in as an addition to the Newtonian gravitational force: $\mathbf{F}_{\text{N}} = -G\mathcal{M}_{\text{gal}}m\hat{\mathbf{r}}/r^2$, that is to say, if $\mathbf{F}_{\text{res}}$ is the resultant gravitational force acting on a test particle orbiting the galactic bulge, then, in accordance with *Newton's Second Law of Motion*, we know that: $\mathbf{F}_{\text{res}} = \mathbf{F}_{\text{N}} + \mathbf{F}_{\text{D}}$, where here the dark-force $\mathbf{F}_{\text{D}}$ is defined as follows:

$$\mathbf{F}_{\text{D}} = -\frac{G\mathcal{M}m}{r\mathcal{R}_{\text{D}}}\hat{\mathbf{r}}. \quad (10)$$

In this Eq. (10), m is the mass of a test particle orbiting the central mass at the radial distance r and, $\mathcal{R}_{\text{D}}$ is the dark-force scale-length which according to Paper (II), is defined:

$$\mathcal{R}_{\text{D}} = a_{\text{D}}^{\text{RM}} \left(\frac{\mathcal{M}}{\mathcal{M}_{\odot}} \right)^{1/2} \mathcal{R}_{\text{kpc}}, \quad (11)$$

where:

$$a_{\text{D}}^{\text{RM}} = (3.00 \pm 0.20) \times 10^{-5}. \quad (12)$$

As stated in Paper (II), we have in Eq. (11) a mass dependent scale-length $\mathcal{R}_{\text{D}}$ which is similar to what Moffat et al. (2018)

have suggested in their MoG theory. It strongly appears that such a scale-length that is mass-dependent is a kind of *sine-quo-non* for any MoG theory to reproduce the Tully and Fisher (1977) relation. In the region: $r > \mathcal{R}_{\text{D}}$, the gravitational dark-force dominants and the Newtonian force is significantly smaller (i.e., $F_{\text{D}} \gg F_{\text{N}}$), hence we shall adopt that for test bodies orbiting in the region: $r > \mathcal{R}_{\text{D}}$, we have: $F_{\text{res}} \sim F_{\text{D}}$.

Before we close this section, it is important that we shade some light on why we refer to the present gravitational theory as a gravitomagnetic theory. Gravitomagnetism is usually understood as a phenomenon emerging from the first order approximation of Einstein's GTR commonly referred to as the linearised version of the GTR. Buiding on the *Azimuthally Symmetry Theory of Gravitation* (ASTG-model) presented in Nyambuya (2010), i.e., a theory based on the Laplace-Poisson equation ($\nabla^2 \Phi = 4\pi G\rho$), a natural extension of the Laplace-Poisson equation in the case of time-dependent gravitational potentials leads to Nordström (1912)'s relativistic theory of gravitation ($\square \Phi = 4\pi G\rho$) whose solutions we explored (in the reading, Nyambuya 2015b) and latter justified in Paper (I). In a latter reading (Nyambuya 2015c), we argued that the equation: $\square \Phi = 4\pi G\rho$, can be thought of as originating from the soils of gravitomagnetism — *albeit*, an exact gravitomagnetic phenomenon different from the linearised version that emerges from Einstein's GTR. More recently, working within the framework of Einstein (1905)'s Special Theory of Relativity (STR), Vieira and Brentan (2018), where able to show that if one imposes covariance on the gravitational force with respect to the Lorentz transformations, a magnetic-like force associated with the gravitational force will emerge. Vieira and Brentan (2018) demonstrated that the emergent gravitomagnetic fields satisfy a system of differential equations similar to the Maxwell equations of electrodynamics. In this way, the idea of spacetime is not lost in the new gravitomagnetic theory, it has the same meaning as it always has in Einstein (1905)'s STR.

4 Why the Spiral Shape?

According (e.g.) to Davis et al. (2017), logarithmic spirals are ubiquitous throughout *Nature* — manifesting themselves in the polar coordinate system setup as optimum rates of radial growth for azimuthal winding in numerous structures such as mollusc shells, tropical cyclones and the arms of spiral galaxies (amongst others). In the case of galaxies, do these spirals occur as a result of Lin and Shu (1964)'s density waves or a result of tidal interactions as (e.g.) Sage

(1993) and Vader et al. (1993) have argued? Is there an alternative genesis to these spiral grand structures? We are of the view that — *yes, there is an alternative view* and this is the view that these beautiful structures may arise as a result of the very force that results in the flat rotation curves. In this new proposed view, stars and molecular gas (and any other material therein) in the arms of these majestic structures may very well be on spiral courses just as planets are on elliptical courses about the Sun.

In-order to appreciate the above stated view — perhaps — one needs to reflect on the words of the pre-eminent American theoretical physicist, John Archibald Wheeler (1911-2008), who once said: “*Spacetime tells matter how to move; matter tells spacetime how to curve.*” This is Wheeler’s succinct summary of Einstein (1915a,b, 1916a,b)’s General Theory of Relativity, in the book *Geons, Black Holes, and Quantum Foam* (Wheeler and Ford 2010). By this, Wheeler meant that the distribution of matter determines the metric of spacetime and conversely, a given metric implies a particular distribution of mass — in a nutshell, one implies the other.

In the same vein, a given gravitational potential implies a certain trajectory (or family of trajectories) for test bodies in free-fall around a given gravitating mass; and conversely, a given trajectory (or family of trajectories) implies a certain gravitational potential. On the basis of this understanding, if indeed the test bodies lieing along the disk of spiral galaxies are in free-fall, then, the majestic spiral character of spiral galaxies ought to arise from the test body’s trajectories around the galactic bulge and this would invariably imply a gravitational potential different from the usual Newtonian gravitational. Thus, assuming no exotic invisible dark-matter, certainly, these spiral trajectories arise from a particular *hitherto* unknown gravitational force governing the gravito-dynamics of the disk. In the reading Nyambuya (2019b), this *hitherto* unknown gravitational force governing the gravito-dynamics of the disk has been termed — the dark-force and it has been demonstrated therein, that this dark-force can, *in-principle*, explain the flat rotation curves of spiral galaxies without the need to invoke exogenous and exotic phenomenon such as dark-matter, extra-dimensions *etc.*

Moving forward, in §(4.2), we shall derive the general equations of motion for a test particle moving on a place under the action of a general gravitational potential. Before deriving the said equations of motion, we shall in §(4.1) start by redefining the way gravitational force is harnessed from a given gravitational potential and this necessary exercise, we conduct in-order that the resulting equations of motion yield the desired spiral orbits. Having done this, we shall insert the *hitherto* unknown gravitational force law

presented in the reading Nyambuya (2019b), as-well as the observation of the constancy of the speed of test particles (this afore-stated observation implies $J_\varphi \propto r$) in the galactic disk of spiral galaxies: from this, we shall demonstrate in §(4.3.1) and (4.3.2), that spiral orbits emerge as the most natural outcome of a dominant inverse distance force.

4.1 Redefining How the Gravitational Force is Harnessed

In-order to obtain the desired equations of motion, that is, the equations of motion that yield the observed logarithmic and barred spiral shapes observed in galaxies, we need to redefine how the gravitational force is harnessed from the gravitational potential. Traditionally, the gravitational force ($\mathbf{F}_g$) is harnessed from the gravitation potential (Φ_g) as follows:

$$\mathbf{F}_g = -m \nabla \Phi_g = -m \left[\frac{\partial \Phi_g}{\partial r} \hat{\mathbf{r}} + \frac{1}{r} \frac{\partial \Phi_g}{\partial \varphi} \hat{\boldsymbol{\phi}} + \frac{1}{r} \frac{\partial \Phi_g}{\partial \theta} \hat{\boldsymbol{\theta}} \right]. \quad (13)$$

That is to say — in the new definition, we have to give-up the partial derivatives and replace them with the straight-derivative, as follows:

$$\mathbf{F}_g = -m \left[\frac{d\Phi_g}{dr} \hat{\mathbf{r}} + \frac{1}{r} \frac{d\Phi_g}{d\varphi} \hat{\boldsymbol{\phi}} + \frac{1}{r} \frac{d\Phi_g}{d\theta} \hat{\boldsymbol{\theta}} \right]. \quad (14)$$

In this way, the chain rule of calculus:

$$\frac{d\Phi_g}{d\varphi} = \frac{d\Phi_g}{du} \frac{du}{d\varphi}, \quad (15)$$

applies.

Now, since: $du/d\varphi \neq 0$, for the $\hat{\boldsymbol{\phi}}$ -component of the gravitational force, we will have: $F^\varphi \neq 0$, hence, a *non-conserved angular momentum* as desired. With this having been said, we need to state this clearly, that, the non-conservation of orbital angular momentum may lead one to think that the general law of the conservation of angular momentum is here violated. *No, it is not violated* as the lost ($\Delta \mathbf{J}$) orbital angular momentum ($\mathbf{J}$) can always to compensated by a change ($\Delta \mathbf{S}$) in the spin angular momentum ($\mathbf{S}$). In a nutshell what is conserved is: $\mathbf{J} + \mathbf{S}$, i.e.: $\Delta \mathbf{J} = -\Delta \mathbf{S}$.

Apart from the above attempt at trying to justify why: $F^\varphi \neq 0$, we must realise that in the usual definition of the gravitational force in-terms of the partial derivatives as given in Eq. (14), the gravitational potential is traditionally assumed to be dependent on the radial distance r alone, i.e.: $\Phi_g = \Phi_g(r)$. This assumption can be dropped as the gravitational potential is not according to the Poission-Laplace equation [$\nabla^2 \Phi_g = 4\pi G \rho$] restricted to radial solution. In an attempt at an alternative solution to the existing gravitational anomalies, as has been done in e.g. Nyambuya (2010),

2015a,b); Nyambuya et al. (2015), the Poisson-Laplace equation: $\nabla^2 \Phi_g = 4\pi G \rho$, can be solved to give an angular dependent gravitational potential, i.e.: $\Phi_g = \Phi_g(r, \theta)$ or $\Phi_g = \Phi_g(r, \theta, \varphi)$. The angular dependence has been assumed to arise from the rotation on the central gravitating body in question and the generated angular dependent terms are then used to try to explain the observed anomalies. In the same vein, if we assumed that: $\Phi_g = \Phi_g(r, \theta, \varphi)$, we would have: $F^\varphi \neq 0$, in the normal definition of the gravitational force in-terms of the partial derivatives as given in Eq. (14). We must say that, the attainment of: $F^\varphi \neq 0$, is crucial in-order to attain the solutions that we here obtain — i.e., solutions that try and explain the shape of the rotation curves of spiral galaxies. The angular dependence is not assumed to be strong but only comes in as a small order perturbation to the main Newtonian potential.

4.2 Equations of Motion

For instructive purposes, we shall derive the equations of motion. For a test body that is undergoing a steady secular mass loss under the action of a resultant force $\mathbf{F}_{\text{res}}$ and whose (test body) mass and velocity are m and $\mathbf{v}$ respectively — it follows that in spherical coordinates, for two dimensional motion on a plane (galactic disk in the present case), resultant force $\mathbf{F}_{\text{res}}$, velocity $\mathbf{v}$, acceleration: $\mathbf{a} = d\mathbf{v}/dt$, and mass loss rate: $\dot{m} = dm/dt$, are, according to Newton's second law of motion, related as follows:

$$\mathbf{F}_{\text{res}} = \frac{d(m\mathbf{v})}{dt} = m \frac{d\mathbf{v}}{dt} + \mathbf{v} \frac{dm}{dt}, \quad (16)$$

hence:

$$\mathbf{a} = \frac{\mathbf{F}_{\text{res}}}{m} - \left(\frac{\dot{m}}{m} \right) \mathbf{v}, \quad (17)$$

where in two dimensional polar coordinates (r, φ) : $\mathbf{v} = \dot{r}\hat{\mathbf{r}} + r\dot{\varphi}\hat{\boldsymbol{\phi}}$, and, $\mathbf{a}$ is given by:

$$\mathbf{a} = (\ddot{r} - r\dot{\varphi}^2)\hat{\mathbf{r}} + (r\ddot{\varphi} + 2\dot{r}\dot{\varphi})\hat{\boldsymbol{\phi}}. \quad (18)$$

The resultant force acting on a test particle in the galactic disk is the dark-force $\mathbf{F}_D$: this implies that: $\mathbf{F}_{\text{res}} = \mathbf{F}_D = F_D^r \hat{\mathbf{r}} + F_D^\varphi \hat{\boldsymbol{\phi}}$, where F_D^r and F_D^φ are the components of the darkforce along the $\hat{\mathbf{r}}$ and $\hat{\boldsymbol{\phi}}$, hence:

$$\mathbf{a} = \frac{\mathbf{F}_D}{m} - \left(\frac{\dot{m}}{m} \right) \mathbf{v}, \quad (19)$$

where: $\mathbf{v} = \dot{r}\hat{\mathbf{r}} + r\dot{\varphi}\hat{\boldsymbol{\phi}}$. From this, it follows that if one makes the substitution: $u = 1/r$, and, thereafter solve the resulting system of equations, then, for the $\hat{\mathbf{r}}$ and the $\hat{\boldsymbol{\phi}}$ -components, they will obtain:

$$\frac{d^2 u}{d\varphi^2} + \left(\frac{\dot{J}_\varphi}{u^2 J_\varphi^2} + \frac{\dot{m}/m}{u^2 J_\varphi} \right) \frac{du}{d\varphi} + u = -\frac{F_D^r(u)/m}{u^2 J_\varphi^2}, \quad (20)$$

and:

$$\frac{\dot{J}_\varphi}{J_\varphi} = -\frac{1}{J_\varphi} \frac{d\Phi_D}{d\varphi} - \frac{\dot{m}}{m}, \quad (21)$$

respectively, where: $J_\varphi = r^2 \dot{\varphi}$, is the specific angular momentum, and, Φ_D is the gravitational dark-potential that gives rise to the dark-force ($\mathbf{F}_D$) and this gravitational dark-potential is defined in Eq. (47) in Paper (II). The term in the under-brace in Eq. (20) is the radial acceleration, $\ddot{r}$. Combining Eq. (20) and (21), we have:

$$\frac{d^2 u}{d\varphi^2} - \left(\frac{1}{u^2 J_\varphi^2} \frac{d\Phi_D}{d\varphi} \right) \frac{du}{d\varphi} + u = -\frac{F_D^r(u)/m}{u^2 J_\varphi^2}. \quad (22)$$

Now, according Eq. (15): $d\Phi_D/d\varphi = (d\Phi_D/du)(du/d\varphi)$, from this it follows that Eq. (22), can be re-written as:

$$\frac{d^2 u}{d\varphi^2} - \frac{1}{u^2 J_\varphi^2} \frac{d\Phi_D}{du} \left(\frac{du}{d\varphi} \right)^2 + u = -\frac{F_D^r(u)/m}{u^2 J_\varphi^2}. \quad (23)$$

Given that: $F_D^r(u)/mu^2 = -d\Phi_D/du$, we can write Eq. (23), as:

$$\frac{d^2 u}{d\varphi^2} + \frac{F_D^r(u)/m}{u^4 J_\varphi^2} \left(\frac{du}{d\varphi} \right)^2 + u = -\frac{F_D^r(u)/m}{u^2 J_\varphi^2}. \quad (24)$$

With Eq. (24), in place, we are ready to derive the equation of motion for test bodies about the galactic bulge.

4.3 Spiral Orbits

Now, we shall consider Eq. (24) for a test particle in the gravitational field of the central galactic bulge where the dominant force is not the Newtonian gravitational field ($\mathbf{F}_N$), but the gravitational dark-force ($\mathbf{F}_D$). We shall start off by making $F_D^r(u)/m$ the subject of the formula in Eq. (24). So doing, we obtain that:

$$\begin{aligned} \frac{F_D^r(u)}{m} &= - \left[\frac{1 + \frac{1}{u} \frac{d^2 u}{d\varphi^2}}{1 + \left(\frac{1}{u} \frac{du}{d\varphi} \right)^2} \right] u^3 J_\varphi^2 \\ &= - \left[\frac{1 + \frac{1}{u} \frac{d^2 u}{d\varphi^2}}{1 + \left(\frac{1}{u} \frac{du}{d\varphi} \right)^2} \right] u V_{\text{Disk}}^2 = -\frac{u V_{\text{Disk}}^2}{\xi^2}, \end{aligned} \quad (25)$$

where:

$$\xi^2 = \frac{1 + \left(\frac{1}{u} \frac{du}{d\varphi} \right)^2}{1 + \frac{1}{u} \frac{d^2 u}{d\varphi^2}} > 0, \quad (26)$$

Now, at this point — we must realise that: $F_D^r(u) \propto -u$, and if this is the case, it follows that the quantity: ξ^2 , must be

independent of, u , and ultimately, V_{Disk}^2 , must be a constant. These constraints that, ξ^2 , must be independent of, u , and that, V_{Disk}^2 , must be a constant are a direct consequence of the fact that: $F_D^r(u) \propto -u$.

From the foregoing, it follows that Eq. (26) is the appropriate equation that determines the possible orbits under the action of the darkforce F_D . We shall re-write this Eq. (26), as follows:

$$\frac{1}{u} \frac{d^2 u}{d\varphi^2} = \frac{1}{\xi^2} \left(\frac{1}{u} \frac{du}{d\varphi} \right)^2. \quad (27)$$

In the subsequent subsections — as is expected of any second order differential equation — we shall in §(4.3.1) and (4.3.2) below, demonstrate that Eq. (27) has two (*albeit*, non-linear) solutions and these solutions exist only for the case: $\xi^2 = 1$. One of the solutions leads to logarithmic spiral orbits and the other leads to a family of barred spiral orbits. As to what determines whether a not the trajectory of the disk material will follow a logarithmic or barred spiral, the theory is *annoyingly* silent on this matter. Our intuitive feeling on this matter is that, whether or not a forming spiral galaxy becomes a logarithmic or barred spiral — this may depend on the initial conditions. This is a matter that we will explore in a separate reading in the very near future.

4.3.1 Logarithmic Spiral Solution

Our first family of solutions to Eq. (27), is:

$$u = u_{\text{SL}} e^{\pm \lambda_{\text{SL}} \varphi}, \quad (28)$$

where: $\lambda_{\text{SL}} = \tan(P_{\text{SL}}) > 0$, and, $u_{\text{SL}} = 1/\mathcal{R}_D$, are constants, and: P_{SL} , is the pitch angle of the (logarithmic) spiral galaxy. Substituting this solution Eq. (28) into Eq. (26), leads to: $\xi^2 = 1$. Written otherwise in-terms of r and not u , Eq. (28) is:

$$r = \mathcal{R}_D e^{\pm \lambda_{\text{SL}} \varphi}. \quad (29)$$

This Eq. (28) [or Eq. (29)], is the equation of a spiral orbit and for this equation. The curve or trajectory: $r = \mathcal{R}_D e^{+\lambda_{\text{SL}} \varphi}$, represents one arm of the spiral for the case: $\varphi > 0$, while the curve or trajectory: $r = \mathcal{R}_D e^{-\lambda_{\text{SL}} \varphi}$, represents the other arm for the case: $\varphi < 0$.

4.3.2 Barred Spiral Solution

Our second family of solutions to Eq. (27), is:

$$u = u_{\text{SB}} \sqrt{1 - \lambda_{\text{SB}}^{\pm} \varphi}, \quad (30)$$

where: $\lambda_{\text{SB}}^{\pm}$, and, $u_{\text{SB}} = 1/\mathcal{R}_{\text{SB}}$, are constants, with, $\mathcal{R}_{\text{SB}}$, being the size of the bar of the given barred spiral galaxy and:

$\lambda_{\text{SB}}^+ = +\lambda_{\text{SB}}$, while: $\lambda_{\text{SB}}^- = -\lambda_{\text{SB}}$, for: $\lambda_{\text{SB}} > 0$. Substituting this solution Eq. (30) into Eq. (26), leads to: $\xi^2 = 1$.

Now, written otherwise in-terms of r and not u , Eq. (30) is:

$$r = \frac{\mathcal{R}_{\text{SB}}}{\sqrt{1 - \lambda_{\text{SB}}^{\pm} \varphi}}. \quad (31)$$

This Eq. (30) [or Eq. (31)], is the equation of a barred-spiral orbit. The curve or trajectory:

$$r = \frac{\mathcal{R}_{\text{SB}}}{\sqrt{1 - \lambda_{\text{SB}}^+ \varphi}}, \quad (32)$$

represents one arm of the spiral for the case: $\varphi > 0$, while the curve or trajectory:

$$r = \frac{\mathcal{R}_{\text{SB}}}{\sqrt{1 - \lambda_{\text{SB}}^- \varphi}}, \quad (33)$$

represents the other arm for the case: $\varphi < 0$. Summarily, Eq. (32) and (33), can be written more compactly as:

$$r = \frac{\mathcal{R}_{\text{SB}}}{\sqrt{1 - \lambda_{\text{SB}} |\varphi|}}. \quad (34)$$

Just as in the case of logarithmic spirals, we shall define the ‘pitch angle’ of barred spirals, P_{SB} , by the same equation as: $\lambda_{\text{SB}} = \tan(P_{\text{SB}})$. In the next sub-section, we shall make an appeal to observations, that is, what is it that they (observations) have to say regarding the above theoretical findings. To ourself, it can hardly be a coincidence that an inverse distance law of gravitation under the assumption of a variable mass: $m = m(\Phi)$, and specific orbital angular momentum J_{φ} , will lead to equations of motion whose two solutions describe the two types of spirals observed in *Nature* (i.e., logarithmic and barred spiral galaxies).

4.4 Observations

As stated in the introductory section, the *Density-Wave Theory* (DW-theory) — of the spiral structure in disk galaxies proposed in the mid 1960s by Lin and Shu (1964); is what is believed to explain not only the shapes of spiral galaxies (Shu 2016; Bertin and Lin 1995), but the trigger for star formation activity (Kennicutt and Evans 2012). The DW-theory assumes that:

1. Matter (i.e., molecular gas and stars, etc) is in elliptical orbits around the galactic bulge.
2. Long-lived quasi-stationary density waves (also called heavy sound), impose a semi-permanent spiral pattern on the face of the galactic disk.

Further, all subsequent versions (see e.g., Phillipps 2005; Carroll and Ostlie 1995) and variants of the DW-theory, agree

that the density wave causes star formation to occur by compressing clouds of gas as they pass through the spiral arm. In the foregoing suggestion, material around the galactic bulge is assumed to be in free-fall and on spiral trajectories. The constancy of the rotational speed in the galactic disk, V_{Disk} , is as a result of the inverse distance law of the gravitational darkforce F_D under the assumption of a variable mass: $m = m(\Phi)$, and specific orbital angular momentum J_φ .

That said — to the best of our knowledge, it most probably was the renowned British astrophysicist and mathematician — Edward Arthur Milne (1896–1950), who — perhaps, made the first attempt to derive the origins of the spiral character of galaxies from pure theory (Milne 1946). This theory of Milne (1946) resulted in test bodies around the central bulge that travel on spiral trajectories as happens in the present suggestion. Against this result of Milne (1946), contemporary astronomers and astrophysicists — in their majority, somewhat agree that stellar orbits around bulges of spirals are essentially circular and that the majestic spiral structure seen in spiral galaxies is a result of an evolving pattern, much like a Moiré pattern (Pour-Imani et al. 2016; Bertin et al. 1989a,b; Lin and Shu 1964) or a dynamic modal structure (Bertin et al. 1989a,b; Bertin 1993).

Like Edward Milne (1946), our theory not only predicts spiral orbits — *i.e.*, stars in free-fall on logarithmic-spiral courses about the galactic bulges of these spiral galaxies, but, that their speeds will be a constant throughout the disk. Amongst many other reasons, one contribution to the disagreement on the logarithmic-spiral courses for stars is that, logarithmic-spirals must have a constant pitch angle P_{SL} (Ringermacher and Mead 2009), where the pitch angle³ P , is formally and theoretically defined as (Binney and Tremaine 1988):

$$\cot(P) = r(\varphi) \frac{d\varphi}{dr} = -u(\varphi) \frac{d\varphi}{du}. \quad (35)$$

The pitch angle is key parameter that characterizes the nature of spiral arms in disk galaxies. It measures the inclination of a spiral arm to the direction of galactic rotation and this angle differs from galaxy to galaxy (see *e.g.*, Hart et al. 2017; Puerari et al. 2014; Michikoshi and Kokubo 2014; Savchenko and Reshetnikov 2013), suggesting that the differential rotation law of galactic disks determines it somehow. If what we have uncovered in the present work is to be believed, it [pitch angle] is because of the fact that this material is under free-fall under the dominant action of an inverse distance law.

Practically, the pitch angle [$P_{\text{SL}} = \tan^{-1}(\lambda_{\text{SL}})$] of a spiral galaxy is the angle between the tangent lines to the spiral arm and to the circle centred on the galactic nucleus and running through the given point (*e.g.*, Savchenko and Reshetnikov 2013; Savchenko 2012; Savchenko and Reshetnikov 2011). There are several methods of measuring the pitch angle (see *e.g.*, Shields et al. 2015; Davis et al. 2014; Puerari et al. 2014; Michikoshi and Kokubo 2014; Savchenko and Reshetnikov 2013). The common feature of all these methods, however, is that they all require the determination of the orientation of galactic disks in space, which is described by the inclination angle of the disk plane relative to the sky plane and the position angle of the major axis. The pitch angle characterizes the degree of twist of the spiral arms: galaxies with tightly wound spiral arms have small pitch angles and those with open arms have large pitch angles (*e.g.*, Savchenko and Reshetnikov 2013; Savchenko 2012; Savchenko and Reshetnikov 2011). The pitch angles for most spiral galaxies lie in the range: $5^\circ \lesssim P_{\text{SL}} \lesssim 30^\circ$. In addition, the pitch angle has been shown to have a strong correlation with the mass of the supposed super massive black-hole resident at the centre of the galaxy (see *e.g.*, Davis et al. 2017, 2014; Seigar et al. 2014; Berrier et al. 2013; Kennicutt 1981).

True logarithmic spirals, that is, spirals described by Eq. (28) have a constant pitch angle (see *e.g.*, Ma 2001). As stated in the opening paragraph of this section, not every astronomer and astrophysicist agrees that spiral galaxies have a constant pitch angle. For example, Ringermacher and Mead (2009) proposes a more general law to Eq. (28), namely:

$$r(\varphi) = \frac{A}{\log [B \tan (\varphi/2N)]}, \quad (36)$$

where A is simply a scale parameter for the entire structure while B , together with a new parameter N , determines the spiral pitch. The ‘winding number’, N , need not be an integer. Unlike the logarithmic spiral, this spiral does not have constant pitch but has precisely the pitch variation found in galaxies. The use of this formula assumes that all galaxies have ‘bars’ *albeit* hidden within a bulge consistent with recent findings. Roughly speaking, the greater the value of N , the tighter the winding and on the same pedestal, the greater the value of B , the greater the arm sweep and smaller bar/bulge, while smaller B fits larger bar/bulge with a sharper bar/arm junction. Thus, B controls the ‘bulge-to-arm’ size, while N controls the tightness much like the *Hubble Classification Scheme*.

In the present model, we have two types of spirals, the logarithmic and the barred spirals, and, it is clear from the proposed model that the pitch angle [as defined in Eq. (35)] will be constant only for logarithmic spirals. In the case of

³ Here the pitch angle has been written in a general form without the subscript ‘SL’ denoting the pitch angle of logarithmic spirals.

barred spirals, the same definition yields:

$$\begin{aligned} \cot(P) &= \frac{2}{\lambda_{SB}} - |\varphi| = \frac{2}{\lambda_{SB}} \left(\frac{\mathcal{R}_{SB}}{r} \right)^2 \\ &= 2 \left(\frac{\mathcal{R}_{SB}}{r} \right)^2 \cot(P_{SB}), \end{aligned} \quad (37)$$

where-in, P , in this Eq. (37) is actually the pitch angle of an assumed logarithmic spiral galaxy. It is clear from this that if one were to use the same method used to measure the pitch angle of logarithmic spirals to the measure the pitch angle of barred spirals, they would obtain a variable pitch angle. If one wanted a constant pitch angle for barred spirals, it is clear that this pitch angle will have to be defined as:

$$\begin{aligned} P_{SB} &= \tan^{-1} \left[\frac{2}{\cot(P_{SL}) + |\varphi|} \right] \\ &= \tan^{-1} \left[2 \left(\frac{\mathcal{R}_{SB}}{r} \right)^2 \tan(P_{SL}) \right]. \end{aligned} \quad (38)$$

In simple terms, if one found that [as defined in Eq. (35)] the pitch angle of a spiral galaxy is variable — then — that spiral must be a barred spiral and the definition Eq. (38), should be used to measure this galaxy's pitch angle and, if the present models is a good description of physical and natural reality (as we are strongly inclined to believe), one should obtain a constant pitch angle for such a galaxy. We thus are proposing Eq. (38) as a new way to measure the pitch angle of barred spirals.

Of variable pitch angles of spiral galaxies, it must be said that, in the past ten years or so, there have been several studies in the literature (e.g., Pour-Imani et al. 2016; Davis et al. 2015, 2014; Martínez-García et al. 2014; Michikoshi and Kokubo 2014) concerning the variances in pitch angle measurements caused by numerous factors such as the wavelength of Light (Pour-Imani et al. 2016; Martínez-García et al. 2014) and galactocentric radius (Davis et al. 2015, 2014). For example, from a study of five galaxies across the optical spectrum — Martínez-García et al. (2014), find that the absolute value of pitch angle gradually increases at longer wavelengths for three galaxies. This result can be contrasted with the larger study of Pour-Imani et al. (2016), who use a sample of 41 galaxies imaged from FUV to $8.0 \mu\text{m}$ wavelengths of Light. Pour-Imani et al. (2016) find that the absolute pitch angle of a galaxy is statistically smaller (tighter winding) when measured using Light that highlights old stellar populations, and larger (looser winding) when using Light that highlights young stars.

For the case of variable pitch angle with galactocentric radius, Savchenko and Reshetnikov (2013) find that most galaxies cannot be described by a single pitch angle.

In those cases, the absolute value of pitch angle decreases with increasing galactocentric radius (i.e., the arms become more tightly wound). This is in agreement with Davis et al. (2015), who predict that there should be a natural tendency for pitch angle to decrease with increasing galactocentric radius due to conditions inherent in the density wave theory. While we have not made any exploration of this very interesting matter, the increasing pitch angle with increasing galactocentric radius is in agreement with the present findings. What needs to be done is to check if the predictions of the present model as given by Eq. (37) is what is obtaining in reality or there is a meaningful correlation.

5 General Discussion

In summary and in a nutshell, we have demonstrated herein that one of the five solutions to the *Four Poisson-Laplace Theory of Gravitation* (FPLTG-model) presented in the reading Nyambuya (2015b), can — *in-principle* — explain the spiral character of galaxies provided that the proposed gravitational dark-force is part and parcel of the natural forces. This dark-force will have to dominate the Newtonian gravitational force so that, beyond the dark-force scale-length $\mathcal{R}_D$, the dark-force, F_D , dominates the scene.

Perhaps, we must — *at this juncture before we present our conclusion* — hasten and say that, this FPLTG-model presented in the reading Nyambuya (2015b), is actually a reincarnation of the Finnish theoretical physicist — Gunnar Nordström (1912)'s relativistic theory of gravitation which is believed to be the first failed relativistic theory of gravitation (see e.g., Weinstein 2015; Norton 2007, 1992). In the reading Nyambuya (2019a), we have argued that Nordström (1912)'s relativistic theory of gravitation can be reconsidered back into the fold of variable gravitation theories.

6 Conclusion

1. Stars and molecular gas in spiral galaxies may very well be in a state of free-fall under the action of an inverse distance law that we have coined the *gravitational dark-force*.
2. Given that the *gravitational dark-force* is an inverse distance law which on the scale of galactic bulges dominates the Newtonian gravitation force, and as well the fact that these stars and the molecular gas have a net zero radial acceleration, the resulting gravitational dynamics require that they [stars and the

molecular gas] follow spiral trajectories, hence the shape of these galaxies is spiral in nature.

Dedications: As is the case with Paper (II), this paper is dedicated to the memory, illustrious life and enduring works the brilliant *Nobel Laureat* that was not afforded to the world — Professor Dr. Vera Cooper Rubin (July 23, 1928 – December 25, 2016). May Her Dear Soul Rest In Peace. Despite being submitted more than a year latter, this work was completed just before the passing on of Professor Dr. Vera Cooper Rubin and prior to her passing on, this was already dedicated to her, hence, we felt it proper to maintain this dedication as our own lasting tribute to her unparalleled devotion on this issue of dark-matter. We have kept this work unpublished because, related work — that would support the main theory out of which the present dark-matter model has been build, was still ongoing.

References

- Begeman KG. 1987. HI Rotation Curves of Spiral Galaxies. Ph.D. thesis, Kapteyn Institute, The Netherlands.
- Begeman KG, Broeils AH, Sanders RH. 1991. Extended rotation curves of spiral galaxies: Dark haloes and modified dynamics. *MNRAS*. 249:523–537.
- Berrier JC, Davis BL, Kennefick D, Kennefick JD, Seigar MS, Barrows RS, et al. 2013. Further evidence for a supermassive black hole – pitch angle relation. *ApJ*. 769:132.
- Bertin G. 1993. The role of resonances in the modal theory of spiral structure in galaxies. *Astron. Soc. Pacific*. 105(688):640–643.
- Bertin G, Lin CC. 1995. Spiral structure in galaxies a density wave theory. Cambridge, MA: MIT Press.
- Bertin G, Lin CC, Lowe SA, Thurstans RP. 1989a. Modal approach to the morphology of spiral galaxies. I – Basic structure and astrophysical viability. *ApJ* 338:78–120.
- Bertin G, Lin CC, Lowe SA, Thurstans RP. 1989b. Modal approach to the morphology of spiral galaxies. II. Dynamical mechanisms. *ApJ*. 338:104–120.
- Binney J, Tremaine S. 1988. Galactic dynamics. Princeton Series in Astrophysics, Princeton University Press.
- Broeils, A.H., 1992a. Dark and Visible Matter in Spiral Galaxies. Ph.D. thesis. University of Groningen, The Netherlands.
- Broeils AH. 1992b. The Mass distribution of the dwarf spiral NGC 1560. *A&A*. 256:19–32.
- Carroll BW, Ostlie DA. 1995. An introduction to modern astrophysics. Benjamin Cummings.
- Chiosi C, Carraro G. 2002. Formation and evolution of elliptical galaxies. *MNRAS*. 335:335–357.
- Davis BL, Berrier JC, Johns L, Shields DW, Hartley MT, Kennefick D, et al. 2014. The black hole mass function derived from Local Spiral Galaxies. *ApJ*. 789:124.
- Davis BL, Graham AW, Seigar MS. 2017. Updating the (supermassive black hole mass)-(spiral arm pitch angle) relation: A strong correlation for galaxies with pseudobulges. *MNRAS*. 471:2187–2203.
- Davis BL, Kennefick D, Kennefick J, Westfall KB, Shields DW, Flatman R, et al. 2015. A fundamental plane of spiral structure in disk galaxies. *ApJL*. 802:L13.
- Einstein A. 1905. Zur Elektrodynamik bewegter Körper. *Annalen der Physik*. 322:891–921 (In German).
- Einstein A. 1915a. Grundgedanken der allgemeinen Relativitätstheorie und Anwendung dieser Theorie in der Astronomie (Fundamental Ideas of the General Theory of Relativity and the Application of this Theory in Astronomy). *Preussische Akademie der Wissenschaften, Sitzungsberichte (Part 1)*, 315. DOI:10.1002/3527608958.ch14.
- Einstein A. 1915b. Zur allgemeinen Relativitätstheorie (On the General Theory of Relativity). *Preussische Akademie der Wissenschaften, Sitzungsberichte (Part 2)*, p. 778–786, p. 799–801. DOI:10.1002/3527608958.ch23.
- Einstein A. 1916a. Grundlage der allgemeinen Relativitätstheorie (The Foundation of the General Theory of Relativity). *Ann. Phys. (Leipzig)*. 49:769–822.
- Einstein, A., 1916b. Näherungsweise Integration der Feldgleichungen der Gravitation (Approximative Integration of the Field Equations of Gravitation). *Preussische Akademie der Wissenschaften, Sitzungsberichte (Part 1)*, 688–696. DOI:10.1002/3527608958.ch7.
- Fisher DB, Drory N. 2011. Demographics of bulge types within 11 Mpc and implications for galaxy evolution. *ApJ*. 733:L47.
- Forman W, Jones C, Tucker W. 1970. Hot coronae around early-type galaxies. *ApJ*. 293:102–119.
- Goldreich P, Tremaine S. 1978. The formation of the Cassini Division in Saturn's rings. *Icarus*. 34:240–253.
- Goldreich P, Tremaine S. 1982. The Dynamics of Planetary Rings. *ARA&A*. 20:249–283.
- Hart RE, Bamford SP, Hayes WB, Cardamone CN, Keel WC, Kruk SJ, et al. 2017. Galaxy zoo and SpArcFIRE: Constraints on spiral arm formation mechanisms from spiral arm number and pitch angles. *MNRAS*. 472:2263–2279.
- Hedman MM, Nicholson PD. 2016. The B-Ring's surface mass density from hidden density waves: Less than meets the eye? *Icarus*. 279:109–124.
- Kennicutt RC, Evans NJ. 2012. Star formation in the Milky Way and nearby galaxies. *ARA&A* 50:531–608.
- Kennicutt RC. 1981. The shapes of spiral arms along the Hubble sequence. *AJ*. 86:1847–1859.
- Kormendy J, Drory N, Bender R, Cornell ME. 2010. Bulgeless giant galaxies challenge our picture of galaxy formation by hierarchical clustering. *ApJ*. 723:54–80.
- Kravtsov AV. 2013. The size-virial radius relation of galaxies. *ApJL*. 764:L31.
- Lin CC, Shu FH. 1964. On the spiral structure of disk galaxies. *ApJ*. 140:655.
- Ma J. 2001. A method of obtaining the pitch angle of spiral arms and the inclination of galactic discs. *ChJAA*. 1:395–405.
- Martínez-García EE, Puerari I, Rosales-Ortega FF, González-Lópezlira RA, Fuentes-Carrera I, Luna A. 2014. The behavior of the pitch angle of spiral arms depending on optical wavelength. *ApJ*. 793:L19.
- Michikoshi S, Kokubo E. 2014. Pitch angle of galactic spiral arms. *ApJ*. 787:174.

- Milne EA. 1946. On the spiral character of external galaxies. *MNRAS*. 106:180–199.
- Moffat, J.M., Rahvar, S., Toth, V., 2018. Applying MOG to lensing: Einstein Rings, Abell 520 and the Bullet Cluster. *Galaxies*. 6:43.
- Nordström G. 1912. Relativitätsprinzip und Gravitation. *Physikalische Zeitschrift*. 13:1126–1129 (In German). DOI:10.1002/andp.19133471303.
- Norton JD. 1992. Einstein, Nordström and the early demise of scalar, Lorentz-Covariant theories of gravitation. *Arch. Hist. Exact Sci*. 45:17–94.
- Norton JD. 2007. Einstein, Nordström, and the early demise of scalar, Lorentz Covariant theories of gravitation. Springer Netherlands, Dordrecht. p. 1337–1411.
- Nyambuya GG. 2010. Azimuthally symmetric theory of gravitation (I) – On the perihelion precession of planetary orbits. *MNRAS*. 403:1381–1391.
- Nyambuya GG. 2015a. Azimuthally symmetric theory of gravitation (II). On the perihelion recession of solar planetary orbits. *MNRAS*. 451:3034–3043.
- Nyambuya GG. 2015b. Four Poission-Laplace theory of gravitation (I). *Journal of Modern Physics*. 6:1195–1206.
- Nyambuya GG. 2015c. Fundamental physical basis for Maxwell-Heaviside gravitomagnetism. *Journal of Modern Physics*. 6:1207–1219.
- Nyambuya GG. 2019a. An attempt at a revival of Nordström's first theory of gravitation. *New Astron*. 67:16–21.
- Nyambuya GG. 2019b. On the gravitomagnetic origins of the anomalous flat rotation curves of spiral galaxies. *New Astron*. 67:1–15.
- Nyambuya GG, Makwanya T, Tuturu BA, Tsoka W. 2015. On the secular recession of the Earth-Moon system as an azimuthal gravitational phenomenon. *Astrophys. and Space Sci*. 358:1–12.
- Phillipps S. 2005. The structure and evolution of galaxies. Wiley, 316 p.
- Pour-Imani H, Kennefick D, Kennefick J, Davis BL, Shields DW, Abdeen MS. 2016. Strong evidence for the densitywave theory of spiral structure in disk galaxies. *ApJ*. 827:L2.
- Puerari I, Elmegreen BG, Block DL. 2014. A new method to estimate local pitch angles in spiral galaxies: Application to spiral arms and feathers in M81 and M51. *AJ*. 148:133.
- Ringermacher, H.I., Mead, L.R., 2009. A new formula describing the scaffold structure of spiral galaxies. *MNRAS*. 397:164–171.
- Rubin VC, Burstein D, Ford WK Jr., Thonnard N. 1985. Rotation velocities of 16 SA galaxies and a comparison of Sa, SB, and SC rotation properties. *ApJ* 289:81–98, 101–104.
- Rubin VC, Ford WK Jr. 1970. Rotation of the Andromeda Nebula from a spectroscopic survey of emission regions. *AJ*. 159:379.
- Rubin VC, Roberts MS, Graham JA, Ford WK Jr., Thonnard N. 1970. Motion of the Galaxy and the Local Group determined from the velocity anisotropy of distant SC I galaxies. I - The data. *AJ*. 81:687–718.
- Rubin VC, Ford WK Jr, Thonnard N. 1978. Extended Rotation Curves of High-Luminosity Spiral Galaxies. IV – Systematic Dynamical Properties, *Sa -> Sc*. *ApJL*. 225:L107–L111.
- Sage, L.J., 1993. Molecular Gas in Nearby Galaxies - Part One – Co/Observations of a Distance-Limited Sample. *A&A* 272, 123–136. 85
- Savchenko SS. 2012. On estimation of the pitch angle for spiral arms of galaxies. *Astrophys. Bull*. 67:310–317. Original text in Russian has been published in: Savchenko SS. 2012. *Astrofizicheskii Byulleten*. 67(3):324–331.
- Savchenko SS, Reshetnikov VP. 2011. Pitch angles of distant spiral galaxies. *Astron. Lett*. 37:817–825.
- Savchenko SS, Reshetnikov VP. 2013. Pitch angle variations in spiral galaxies. *MNRAS*. 436:1074–1083.
- Seigar MS, Davis BL, Berrier J, Kennefick D. 2014. Constraining dark matter halo profiles and galaxy formation models using spiral arm morphology. II. Dark and stellar mass concentrations for 13 nearby face-on galaxies. *ApJ*. 795:90.
- Shields DW, Boe B, Pfountz C, Davis BL, Hartley M, Pour-Imani H, et al. 2015. Spirality: A novel way to measure spiral arm pitch angle. *ArXiv e-prints*: arXiv:1511.06365v2. Available from: <http://arxiv.org/abs/1511.06365>.
- Shu, F.H., 1984. Waves in Planetary Rings, in: Greenberg, R., Brahic, A. (Eds.), *IAU Colloq. 75: Planetary Rings*, pp. 513–561.
- Shu, F.H., 2016. Six Decades of Spiral Density Wave Theory. *Annual Review of Astronomy and Astrophysics* 54, 667–724. 110 doi:10.1146/annurev-astro-081915-023426.
- Swaters, R.A., Sancisi, R., van der Hulst, J.M., van Albada, T.S., 2012. The Link Between the Baryonic Mass Distribution and the Rotation Curve Shape. *MNRAS* 425, 2299–2308. doi:10.1111/j.1365-2966.2012.21599.x. 115
- Tiscareno MS, Burns JA, Nicholson PD, Hedman MM, Porco CC. 2007. Cassini imaging of Saturn's rings: II. A wavelet technique for analysis of density waves and other radial structure in the rings. *Icarus*. 189:14–34.
- Tiscareno MS, Nicholson PD, Burns JA, Hedman MM, Porco CC. 2006. Unravelling temporal variability in Saturn's spiral density waves: Results and predictions. *ApJL*. 651:L65–L68.
- Tully RB, Fisher JR. 1977. A new method of determining distances to galaxies. *A&A*. 54:661–673.
- Vader JP, Frogel JA, Terndrup DM, Heisler CA. 1993. Galaxies with spectral energy distributions peaking near 60 micrometers. 1: Optical spectroscopy, infrared photometry, and radio continuum data. *A&A*. 106:1743.
- Vieira RS, Brentan HB. 2018. Covariant Theory of Gravitation in the framework of Special Relativity. *Eur. Phys. J. Plus*. 133:165.
- Weinstein G. 2015. Einstein's pathway to the Special Theory of Relativity. Cambridge Scholars Publishing.
- Wheeler JA, Ford K. 2010. Geons, black holes, and quantum foam: A life in physics. W. W. Norton & Company, Volume 413, p. 235.
- Woo J, Courteau S, Dekel A. 2008. Scaling relations and the fundamental line of the Local Group Dwarf Galaxies. *MNRAS*. 390:1453–1469.