Research Article

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Nonlinear cosmogony of the spiral galaxy bulges

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Abstract: In this paper, we develop an early idea of one of the authors (Nuritdinov 1992a,b), who was the first to propose the mechanism of instability of the warp perturbation mode on the background of a nonstationary disk. For this aim, we have studied a model of a nonlinearly non-stationary self-gravitating disk with an anisotropic velocity diagram. The model has a composite nature, or rather, it is a superposition of isotropic and anisotropic states of the disk. In the general case, it is obtained a nonstationary analogue of the dispersion equation of this composite model. We have also investigated the behavior of the domed perturbation mode, the instability of which leads to the formation of a classical bulge in the central region of the disk. In addition, we considered the critical diagrams of the dependence of the virial ratio on the rotation rate of the system for various values of the superposition parameter and the corresponding diagrams for the increments of instability.

Keywords: nonlinear model; non-stationary self-gravitating disk; spiral galaxy bulges; dome perturbation mode

1 Introduction

The central bulges of disk galaxies are very different from each other. We observe large classical bulges of galaxies like the galaxy NGC4594 and vice versa, clearly smaller boxy bulges of later-type disk galaxies, similar to our own Galaxy. However, the origin of bulges is still not well understood. Especially, it seems likely now that there are galaxies that have very small bulges or no bulges (Freeman 2008).

The origin of galaxy bulges is a very interesting task. Most authors (see, e.g., Laurikainen *et al.* 2016; Mercedes *et al.* 2000; Zinn 1985) believe that bulges are well described within the framework of galaxy mergers. Detailed consideration of the bulges indicates that they have various external shapes. Nowadays, basically, three types of bulges can be distinguished: classical bulges; box/peanut bulges; pseudo (e.g., disk-like) bulges.

Obviously, the bulge cannot form on the background of a stationary model. In reality, its formation can occur at the nonlinearly non-stationary evolution stage of galaxies, for example, due to the gravitational instability of vertical oscillations of the central region of the disk. This idea was proposed by one of the authors of this work (Nuritdinov 1992a,b). Below we develop this idea by first constructing a non-stationary disk model with an anisotropic velocity diagram.

2 Non-stationary disk model

Among the various possible non-stationarities of the disk subsystem of galaxies, a special place is occupied by its radial oscillations. In reality, they may have not very large amplitudes, but theoretically it is easier to construct radially pulsating disks, generalizing some well-known equilibrium models to this case. Earlier, in the work of Nuritdinov (1992a,b) the pulsating model of self-gravitating disk with the phase density as

$$\Psi(r, v_r, v_\perp, t) = \frac{\sigma_0}{2\pi\Pi\sqrt{1-\Omega^2}} \left[\frac{1-\Omega^2}{\Pi^2} \left(1 - \frac{r^2}{\Pi^2} \right) - (v_r - v_a)^2 - (v_\perp - v_b)^2 \right]^{-\frac{1}{2}} \cdot \chi(R-r), \quad (1)$$

which is a generalization of the equilibrium model of Bisnovatyi-Kogan and Zel'dovich (1970) for the case of radial pulsation. Where Ω is a dimensionless parameter which characterizing the degree of solid rotation of the disk and range of $0 \leq \Omega \leq 1$. R is the radius of disk and it is equal to $R(t) = R_0\Pi(t)$,

$$\Pi(t) = \frac{1 + \lambda \cos \psi}{1 - \lambda^2}, \quad t = \frac{\Psi + \lambda \sin \Psi}{(1 - \lambda^2)^{3/2}}. \quad (2)$$

Where $\Pi(t)$ is the variable function of time, and λ is the amplitude of the radial pulsation, which is related to the

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virial parameter and one can write in the following form as

$$\lambda = 1 - (2T/|U|)_0, \quad (3)$$

with $(2T/|U|)_0$ is the know virial parameter. We can write v_a and v_b as follows

$$v_a = -\lambda \frac{r \sin \psi}{\sqrt{1 - \lambda^2 \Pi^2}}, \quad v_b = \frac{\Omega r}{\Pi^2}. \quad (4)$$

We have one more parameter which χ is the Heaviside function in the Eq. (1), and σ_0 is the value of the surface density of the disk in its center and it is in the form below

$$\sigma(\vec{r}, t) = \sigma_0 \sqrt{1 - (r/\Pi)^2}, \quad \pi^2 G \sigma_0 = 2R_0, \quad (5)$$

and for the convenience of calculations, we assume that $R_0 = 1$. Therefore, for $\lambda = 0$ in Eq. (1) we are dealing with the case of the authors' model (Bisnovatyi-Kogan and Zel'dovich 1970).

The stability tasks of a pulsating model with an isotropic velocity diagram were studied in detail by several authors (see, e.g., Nuritdinov 2008, 2009) and these researches studied critical diagrams of the connection between Ω and $(2T/|U|)_0$. However, the isotropy of the velocity diagram is not realistic, so anisotropic models are required. To construct an anisotropic model, we use the well-known method of averaging model (1) through the parameter Ω as

$$\Psi_A = \int_{-1}^{+1} \rho(\Omega) \cdot \Psi d\Omega, \quad \int_{-1}^{+1} \rho(\Omega) d\Omega = 1, \quad (6)$$

where $\rho(\Omega)$ is the weight function.

According to Eq. (2), function $\rho(\Omega)$ must be even. We consider in detail various cases for $\rho(\Omega)$. Now we work with one of the interesting model for $\rho(\Omega)$ in the form as

$$\rho(\Omega) = \frac{2}{\pi} \frac{\Omega^2}{\sqrt{1 - \Omega^2}}. \quad (7)$$

Substituting Eq. (7) into Eq. (6), taking into account Eq. (1) and after some transformations, we easily get new form of phase density for the anisotropic case

$$\Psi_A = \frac{\sigma_0}{\pi^2} \chi(D) \int_{-1}^{+1} \frac{\Omega^2}{(1 - \Omega^2) \sqrt{D - (\Omega - rv_\perp)^2}} d\Omega, \quad (8)$$

where D is the discriminant of the equation square with respect to Ω under the square root in Eq. (8).

$$D = \left(1 - \frac{r^2}{\Pi^2}\right) \left(1 - \Pi^2 v_\perp^2\right) - \Pi^2 (v_r - v_a)^2 \geq 0. \quad (9)$$

Where we introduced $\sin(\theta) = (\Omega - rv_\perp)/\sqrt{D}$ for the simplification and we get a convenient form of phase density

$$\Psi_A = \frac{\sigma_0}{\pi^2} \chi(D) \int_{-\pi/2}^{+\pi/2} \frac{(\sqrt{D} \sin \theta + rv_\perp)^2}{(1 - (\sqrt{D} \sin \theta + rv_\perp)^2)} d\theta. \quad (10)$$

After calculating expression (10), it is easy to obtain the following result of Eq. (10) below

$$\Psi_A = \frac{3\sigma_0}{2\pi} \left[\left(\left(\frac{r}{\Pi} + v_\perp \Pi \right)^2 + \Pi^2 (v_r - v_a)^2 \right)^{-1/2} - \left(\left(\frac{r}{\Pi} - v_\perp \Pi \right)^2 + \Pi^2 (v_r - v_a)^2 \right)^{-1/2} - \frac{2}{3} \right] \chi(D). \quad (11)$$

The derivation of Eq. (11) is the first stage in the construction of an anisotropic pulsating model, and the second stage is the creation of a composite model by superposition of Eq. (1) and Eq. (11) in the form

$$\Psi_{new}(r, v_r, v_\perp, t, \mu) = (1 - \mu) \cdot \Psi + \mu \cdot \Psi_A, \quad (12)$$

where μ is the superposition parameter, which takes values from the interval $[0; 1]$. Consequently, at $\mu = 0$ we have a purely isotropic case, and at $\mu = 1$ is anisotropic. Using of the superposition principle in Eq. (12), it is possible to cover all models between the two states Ψ and Ψ_A . We note that earlier this method was successfully studied in the work (see Mirtadjieva 2012) for a clearly other weight function $\rho(\Omega)$.

In the theory of gravitational instability of purely equilibrium systems, the dependence on the perturbation frequency on the wave number is usually called the dispersion equation, and in the case when a non-stationary model is considered in the initial state, it seems to us that it is better to call this equation as a non-stationary dispersion equation (NDE).

Consequently, the NDE of the composite model Eq. (12), according to the principle of superposition, can be as

$$\begin{aligned} & (1 + \lambda \cos \psi) \frac{d^2 B}{d\psi^2} + \left[\lambda \sin \psi + 2im\Omega \sqrt{1 - \lambda^2} (1 - \mu) \right] \frac{dB}{d\psi} + 2 \left\{ Y_{mN} - 1 - \frac{\mu(1 - \lambda^2)(N^2 + N + 8m^2 - 2)}{24(1 + \lambda \cos \psi)} \right. \\ & + \frac{(1 - \mu)(1 - \lambda^2)}{6(1 + \lambda \cos \psi)} \left[\frac{6im\Omega \lambda \sin \psi}{\sqrt{1 - \lambda^2}} - 3m^2 \Omega^2 - (1 - \Omega^2)(N^2 - m^2 + N - 2) \right] \Big\} B(\psi) = 0, \end{aligned} \quad (13)$$

With specifying the values of N and m , using the NDE Eq. (13) of the composite model (12), we can investigate specific vertical perturbation modes, both large-scale and small-scale perturbation.

3 Warp perturbations on the background of a pulsating disk

It is known that a flat disk-shaped galaxy subsystem was formed during the nonlinearly non-stationary evolution stage of this galaxy by Fridman and Polyachenko (1984). This period is characterized by a collapse not only along the axis of rotation, but also in the direction perpendicular to it, along which, according to observations (Eggen *et al.* 1962; Marochnik and Suchkov 1988), the gravitational compression has non-stationary or oscillation character. This is on the one hand. On the other hand, it should be taken into account that the disk-shaped subsystem is the most highly unstable configuration. Vertical perturbation modes can also be gravitationally unstable. Among these modes we are most interested in this work in the case when the radial wavenumber $N = 3$, and the azimuthal wave number $m = 0$. As will be seen below, the perturbation of a nonstationary background leads to a very interesting, more real picture of instability than a stationary background.

In the general case, the perturbation in the form of a vertical warp has the form (Hunter and Toomre 1969)

$$H(\vec{r}, t) = a(t) \frac{1}{\xi} P_N^m(\xi) e^{im\varphi}, \quad \xi = \sqrt{1 - \frac{r^2}{R^2}}, \quad (14)$$

where the value of the function $a(t)$ characterizes the amplitude of the perturbation, $P_N^m(\xi)$ is the associated Legendre polynomial, but $(N - m)$ must be odd.

For example, for $N = 3$, $m = 0$:

$$H = a(t) \left(1 - \frac{5r^2}{2R^2} \right), \quad (15)$$

for this mode, the azimuthal wavenumber $m = 0$, which means the symmetry of the perturbation relative to the axis of rotation of the disk, and the values of the radial harmonic $N = 3$ are associated with the shape of the dome, which is oscillating vertically relative to the plane of the disk. Such a picture follows from the expression for the perturbation Eq. (14). Substituting $m = 0$, $N = 3$ in Eq. (13), we find the corresponding NDE of dome perturbations for model (12)

$$(1 + \lambda \cos \psi) \frac{d^2 B}{d\psi^2} + \lambda \sin \psi \frac{dB}{d\psi} + \left[\frac{5}{2} - \frac{10 \nu (1 - \Omega^2)(1 - \lambda^2)}{3(1 + \lambda \cos \psi)} - \frac{5(1 - \nu)(1 - \lambda^2)}{6(1 + \lambda \cos \psi)} \right] B(\psi) = 0 \quad (16)$$

As seen, NDE Eq. (16) depends on three parameters: λ , μ and Ω . Obviously, a purely analytical study of it is impossible here, and therefore we perform numerical integration

Figure 1. Critical dependence of the virial ratio on the rotation parameter for the perturbation mode $N = 3$, $m = 0$ at different values of μ . The dark areas correspond to the unstable ones.

Eq. (16) for specific values of physical parameters. To construct the critical dependence of $\lambda = 1 - (2T/|U|)_0$ on the rotation parameter Ω for given values of μ , we apply the well-known method of stability of parametric resonance (Babakov 1968; Malkov 1986). For this purpose, we considered cases where the superposition parameter μ is 0; 0.25; 0.5 and 0.75. The calculation results are clearly shown in Figure 1.

Figure 1 shows that at $\mu = 0$, when we consider only an unsteady disk with an isotropic velocity diagram, more than half of the entire region is unstable, and as we approach the state of purely radial motions with $(2T/|U|)_0 \rightarrow 0$, there is a countable set of small “petals” of instability. The area of “petals” gradually increases at $(2T/|U|)_0 \rightarrow 1$. Now let’s turn our attention to the physics of instability at $(2T/|U|)_0 \rightarrow 1$. With an increase in the value of $\mu = 0$, the region of instability as a whole decreases, i.e. the inclusion of the anisotropic part (11) in the composite model leads to a decrease in the degree of instability of the composite model (12).

Consequently, the anisotropy of the velocity diagram plays a stabilizing role. We also note that in the absence of radial pulsations, i.e. when $\lambda = 0$, $(2T/|U|)_0 = 1$, the composite model is completely unstable at $\Omega < 0, 47$, if $\mu = 0$. Starting from $\mu \approx 0, 2$ on axis $(2T/|U|)_0$ we have narrow “islands” of stability. With an increase in μ , model (12) becomes clearly stable.

4 Conclusions

In the present paper, the nonlinear cosmology of the spiral galaxy bulges has been studied in detail. Based on the results obtained, we may summarize the following statements and findings:

- A new nonlinear non-stationary model of a self-gravitating disk with the anisotropic velocity diagram is constructed.
- Non-stationary dispersion equations for two warp perturbation modes are derived.
- The expression found for the phase density of the new anisotropic non-stationary model and it depends on the main three parameters: the virial parameter for the moment of beginning of the collapse, the superposition parameter and the degree of disk rotation.
- The NDEs of warp perturbation modes for the constructed disk model in general form are obtained for arbitrary values of the indices of perturbation N and m .
- The instability of dome-shaped perturbation modes (0.3) has been studied.
- Critical diagrams of the relationship between the value of the virial ratio and the degree of disk rotation for a given value of the superposition parameter are investigated and we found the growth rates of instabilities of the models.
- It is shown that with an increase in the value of the superposition parameter, the composite model becomes more stable.
- The average rotation of the disk plays a stabilizing role for the domed disturbance mode on the background of our model.

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