Research Article

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The fine structure of the rotational periods of the solar mean magnetic field

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Abstract: The data of mean magnetic field (MMF) of the Sun obtained at the Wilcox Solar Observatory (Stanford, USA) in 1975–2020 are analysed. It was concluded that the MMF maximum occurs, on average, 1.5–2 years later relative to the maximum of Wolf numbers. To analyze the changes in the MMF, a new method for searching for periodicities has been developed, which takes into account the change in the sign of the magnetic field from cycle to cycle. This method made it possible to find the main synodic periods of rotation of the magnetic field with values of $27^{\text{d}}.021 \pm 0^{\text{d}}.008$, $26^{\text{d}}.796 \pm 0^{\text{d}}.008$ and $27^{\text{d}}.260 \pm 0^{\text{d}}.008$, each of which has two splitting components associated with a change in the polarity of the magnetic field during the transition from one cycle of solar activity to another. The stability of these periodicities for more than 45 years indicates that the Sun as a star looks like a horizontal magnetic dipole (in addition to the observed vertical one), which changes its sign every 11 years and rotates with different periods.

Keywords: Sun, mean magnetic field, periodicity, rotation

1 Introduction

The mean magnetic field (MMF) of the Sun is the average value of the magnetic flux from the entire visible solar hemisphere. Its measurements were started in 1968 by Severny (1969) at the Crimean Astrophysical Observatory, and then were continued by a few observatories. At different times, the MMF measurements were carried out by various ground-based instruments. Currently, there are data from seven observatories, as well as measurements from the SOHO and SDO satellites, where the MMF signal can be extracted. Of all the available observational series, the Wilcox Solar Observatory (Stanford, USA) data set is the longest, most numerous and homogeneous (Scherrer *et al.* 1977). The observations have been started here in 1975 and are continued to the present by the same instrument. Therefore, this series was chosen to study the variability of the MMF.

The study of the periodicity of the MMF has been previously performed by different authors, see, *e.g.*, Rivin and Obridko (1992); Haneychuk (1999); Chaplin *et al.* (2003); Haneychuk *et al.* (2003); Kotov (2020). As shown by these

and other authors, the main periodicity of the MMF is associated with the rotation of the Sun. This can be seen directly in the measurement data themselves, and, of course, on the power spectra or periodograms, where the peaks in the region of 26–27 days are dominated. In the present paper, we will consider the detailed structure of the peaks that appears when analyzing data of 4 solar activity cycles. For a detailed study, a new method of analysis was applied, which takes into account the change in the sign of the magnetic field in each cycle of solar activity.

2 Data of observations

Until 2009, WSO data were published in *Solar Geophysical Data*, and later (and at the present time) began to be published only in electronic form on the observatory website wso.stanford.edu. The data represents the MMF value obtained for each day of measurements—one point per day. During the period under study, from 1975 to 2020, the number of measurements was $N = 13666$, the total duration T of the series is about 46 years, the observation period covers a little more than four cycles of solar activity. The original values of MMF are shown in Figure 1a, where an increase in the amplitude of oscillations at the maximum of the cycle and a decrease in it at the minimum of activity are clearly visible. The values of the magnetic field usually do not exceed 1.5 Gs in absolute value, and at the minimum activity

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they do not outreach 0.5 Gs. Positive values correspond to the north polarity of the magnetic field, negative values correspond to the south one. In further analysis, the initial moment for calculating the phase curves was chosen as 00^h 00^m on January 1, 1975, the phases φ are given in fractions of a period, *i.e.* within the range 0–1.

Figure 1. a) Original data of WSO from 1975 to 2020. b) Standard deviation σ of WSO data (black, left scale); Wolf numbers averaged over the month (red dots) and their smoothed curve (blue line)—right scale.

In Figure 1b, black dots show the time variations of the standard deviation of this series. It was calculated for each point in time over an interval of 1 year (± 0.5 years from this point). This value, as can be seen, well characterizes the amplitude of any variability. It can be noted that the MMF was maximal in the cycle 22 of activity and the minimal one in the 24th cycle. It is also noticeable that there are two maxima in each cycle, and the second one is always higher than the first one. The red dots in Figure 1b show the Wolf monthly mean total sunspot numbers, and the blue curve shows the values of these numbers smoothed over 13 months (<https://www.bis.sidc.be/silso/datafiles>). It can be seen that the black curve is shifted to the right relative to the blue one, except for cycle 22. All in all it can be concluded that the MMF maximum is delayed (comes later) relative to the maximum of Wolf numbers by an average of 1.5–2 years. A possible explanation for this phenomenon may be that strong fields in groups decay with time, become smaller in magnitude, and no longer manifest themselves as sunspots, but are still visible in the MMF as weak fields occupying large areas on the solar surface. The role of these features with relatively small fields, but with large areas, as the main source of the MMF of the Sun, was noted at the beginning of studies of MMF (Severny *et al.* 1970; Severny 1971; Kotov *et al.* 1977).

3 Power Spectrum

Figure 2 shows the power spectrum (PS) of the MMF of the original series (Figure 1a) in the region of the solar rotation periods. The highest peak corresponds to the period of $26^{\text{d}}.879 \pm 0^{\text{d}}.008$. As one can see, this periodicity (like other high peaks) persists throughout all four cycles of the Sun's activity. This raises a natural question: how can such a periodicity be preserved while the magnetic field sign changes to the opposite one in each cycle? To obtain an answer to it, an additional data analysis was carried out.

Figure 2. Power spectrum of the MMF in the region of the solar rotation periods. The ordinate is the square of the harmonic oscillation amplitude in Gs^2 .

4 Variations of amplitudes in time

Figure 3 shows changes in the amplitude of oscillations with time for each of the four main periods indicated in Figure 2. The amplitude values were calculated for each point of the time series over an interval of 1 year, *i.e.* ± 0.5 years from current point. In this part of the time series, the least squares method was used to determine the amplitude and phase of the sinusoid for a given period, *i.e.* the time series was approximated by the function

$$y(t) = A_h \cos \left(\frac{2\pi t}{P} - \varphi \right). \quad (1)$$

As can be seen from Figure 3, the behavior of the amplitudes A_h of all oscillations looks almost the same—the deviations of the curves from each other are insignificant. Note that three of the four periods have the maximal power in Figure 2, while one of them, $27^{\text{d}}.055$, is much weaker than the

Figure 3. Time variation of the harmonic amplitude of oscillations for the periods indicated in Figure 2.

others. The latter one, having a small value in the PS, differs little from others in the behavior of the amplitudes in Figure 3. This effect can be partly explained by the fact that the spectral resolution for the 1 year section of the series does not allow to distinguish these periods from each other. Indeed, with a time series length of 46 years, the independent frequencies are at a distance of $\Delta\nu = 1/T \approx 0.69$ nHz, which corresponds to $\Delta P \approx 0^d.04$, while for a time series length of 1 year $\Delta\nu \approx 32$ nHz and $\Delta P \approx 2^d$. This resolution is evidently not enough to detect separation even for the maximal difference in periods $\Delta P = 0^d.48$. But, as we will see later, the matter is not only in the spectral resolution, but also in the variation with time of the amplitude and phase of the oscillation, which leads to the fact that the resulting power in the spectrum in Figure 2 is quite various for different periods.

5 Variation of amplitudes with fixed phase

Since we analyze the data of the magnetic field and know that it changes its sign in each cycle of solar activity, it is natural to expect that the oscillations can (and should) change their phase to the opposite one when the polarity of the cycle changes. This should lead to a decrease in the amplitude of oscillation, as well as to the appearance of new peaks in the PS. To test this assumption, a new method for calculating the oscillation amplitude was developed. For each period, the phase curve was first calculated for the entire time series, and then it was approximated by a sinusoid. Further, the phase of the maximum of this sinusoid φ_m was used to approximate a part of the time series

by the function

$$y^*(t) = A_h^* \cos\left(\frac{2\pi t}{P} - \varphi_m\right). \quad (2)$$

The difference from the previous method (1) is that the phase is fixed here for all parts of the time series, and the amplitude A_h^* can take negative values if the oscillation is in antiphase with φ_m .

Figure 4 shows, as an example, the phase curves for periods $26^d.879$ (a) and $27^d.055$ (b). They are calculated for 16 phase intervals. For each of the curves, the dashed line shows the sinusoidal fit. The phase of the maximum of this sinusoid φ_m is shown in each graph. Vertical bars indicate errors in each bin. The graphs are drawn at the same scale. Note that there are deviations from the sinusoid on both curves.

Figure 4. The phase curves for the periods: a) $P = 26^d.879$ and b) $P = 27^d.055$. Number of the phase intervals is 16. The dashed line shows the approximation of the data by a sinusoid, the phase of the maximum of which φ_m is indicated in each figure.

Figure 5 shows a comparison of the oscillation amplitudes A_h and A_h^* for the period $P = 26^d.879$, and in Figure 6—for the period $P = 27^d.055$. The first period, $P = 26^d.879$, has a maximum amplitude in the PS (Figure 2). The amplitude A_h^* for it in Figure 5 is almost always positive, although there are short parts in time when it is negative. Thus, this oscillation is almost always in phase with φ_m .

Quite another picture is obtained for the period $P = 27^d.055$ (Figure 6). Here the difference between the amplitudes A_h and A_h^* from each other is stronger. On the whole, in total, the amplitude of A_h^* for the entire time interval has a positive value, but there are quite a few negative values of A_h^* , i.e. oscillation is often in antiphase with φ_m . And as a result of this, we have a small power of this peak in the PS in Figure 2. Note that the values of the A_h^* amplitude in Figures 5 and 6 differ little from each other by order

Figure 5. Time variation of the harmonic amplitude of oscillations for the period $P = 26^d.879$: black dots are the amplitude with a phase that gives the maximum value of A_h ; red dots are the amplitudes for fixed phase $\varphi_m = 0.132$.

Figure 6. Time variation of the harmonic amplitude of oscillations for the period $P = 27^d.055$: black dots are the amplitude with a phase that gives the maximum value of A_h ; red dots are the amplitudes for fixed phase $\varphi_m = 0.612$.

of magnitude and get a maximum of about 0.8 G for both oscillations.

As can be seen from Figures 5 and 6, the behavior of the amplitudes A_h^* can differ greatly from the values of A_h , despite the fact that they were determined on the same time interval of 1 year as the values of A_h in Figure 3 for different periods. The phase analysis method turns out to be very sensitive for detecting subtle effects.

Thus, we come to the conclusion that the magnitudes of the peaks in the PS are determined by how long in time the oscillation is in the same phase with φ_m . Those oscillations that retain their phase as much as possible will have more power. And for those periodicities that change the phase to the opposite, the power will be small or even close to zero. In other words, in the classical PS, we may not find the

main period of rotation due to the fact that this oscillation changes its phase to the opposite, which should be the case when analyzing the magnetic field, which changes its sign. To identify such periodicities, it is necessary to apply other methods of analysis, which take into account the possible sign change in the source data.

6 New method for calculating power spectrum

Usually, the power spectrum of a time series is calculated as the square of the Fourier transform, and for our data it can be written as

$$A_h^2(\nu) = \left(\frac{2}{N} \sum_{i=1}^N H_i \cos(2\pi\nu t_i) \right)^2 + \left(\frac{2}{N} \sum_{i=1}^N H_i \sin(2\pi\nu t_i) \right)^2, \quad (3)$$

where H_i is the value of the magnetic field at time t_i , ν is the frequency, N is the number of data in the time series, $A_h^2(\nu)$ is the square of the harmonic amplitude at the frequency ν . Note that the factor 2 before the sum is necessary to take into account negative frequencies and allows us to right away obtain the harmonic amplitude A_h . The original time series $H(t_i)$ can be represented as the sum of sinusoidal components with amplitudes $A_h(\nu)$ and phases $\varphi_m(\nu)$:

$$H(t_i) = \sum_{k=1}^M A_h(\nu_k) \cos(2\pi\nu_k t_i - \varphi_m(\nu_k)), \quad (4)$$

where M is the number of independent frequencies. The quantities $A_h(\nu_k)$ and $\varphi_m(\nu_k)$ characterize the amplitude and phase of the sinusoid at a given frequency for the entire time series.

If we consider the change in the amplitude of oscillations with time, an example of which is shown in Figures 5 and 6, then we can assume that for a given frequency the average value of the amplitudes A_h^* , calculated for each point of the time series, should be equal to the value A_h :

$$A_h(\nu) = \frac{1}{N} \sum_{i=1}^N A_h^*(\nu, t_i). \quad (5)$$

This formula was written from general considerations and has not yet been proved strictly mathematically, but it finds its confirmation in practice.

Figure 7 shows a comparison of the power spectra calculated by two methods, according to formulas (3) and (5).

Figure 7. The power spectra of the time series, calculated in two ways: 1) the Fourier transform according to the formula (3) (red line) and 2) the sum of the amplitudes of the sinusoids according to the formula (5) (black line).

It can be seen that the spectra are practically the same, the difference between them is very small. Notice that the classical spectrum by formula (3) is calculated much faster than the other by equation (5).

Since we are analyzing a series of data for a magnetic field that changes its sign, we can try to take into account the effect of changing the sign by taking in equation (5) not the quantities A_h^* themselves, but their absolute values:

$$A_h^a(\nu) = \frac{1}{N} \sum_{i=1}^N |A_h^*(\nu, t_i)|. \quad (6)$$

Then the oscillations will not compensate each other if they are in antiphase, but, on the contrary, will amplify. But in this case, an increase in the constant component in the spectrum should also be expected, since all noise will also be amplified, not suppressed.

Figure 8. PS of the MMF, calculated by formula (6), in the region of the periods of the Sun's rotation.

Figure 8 shows the PS calculated by the equation (6). As expected, it shows a noticeable constant component associated with an overall strong periodicity in this area. It is also noticeable that the peak heights are, on average, two times higher than for the usual spectra in Figures 2 and 7. It can be seen that there are quite a few different peaks in the spectrum, which are not present in the usual spectra.

7 Comparison of two spectra

Figure 9 shows in detail the part of the spectrum with the maximum power in the region of rotation periods. In the upper Figure 9a, the spectrum is calculated by the formula (6) as the sum of the absolute values of the oscillation amplitudes, in the lower Figure 9b—by the Fourier transform formula (3). It should be noted right away that the spectral resolution in Figure 9a is higher. The reason for this is not yet clear; this is the subject of further investigation.

Figure 9. Power spectra of the MMF in a narrow range of the solar rotation periods: a) the spectrum is calculated by the formula (6), b) the usual spectrum is calculated by the formula (3).

Among the many peaks in the spectrum in Figure 9a, three groups can be distinguished, which are marked by different colors. They are notable for the fact that the distance between neighboring peaks in each triplet is close to the period of the solar magnetic cycle of 22 years. The magnitudes of these splittings are shown in the graph. Periods $26^d.796$, $27^d.021$ and $27^d.260$ correspond to the central components of each triplet. One can also see that most of the peaks in Figure 9a do not coincide with the peaks in Figure 9b, with the exception of two with periods of $27^d.170$ and $26^d.884$. They are, however, not central to each of the triplets, but lateral components. The central components

in each triplet in Figure 9a have very small amplitudes in Figure 9b.

8 Variation of oscillation amplitudes in time

To test the validity of the assumption about the relationship between the peaks that form triplets in Figure 9a, we analyzed the behavior of the amplitudes A_n^* for each of these periods with time.

Figure 10 shows the variation of the oscillation amplitudes for the periods from the central triplet of peaks: $27^d.112$ (left component, red), $27^d.021$ (central, black), $26^d.931$ (right, blue). Figure 10a shows the variation in oscillation amplitudes for the central and left splitting components. It can be seen that they are alternately in phase, then in antiphase to each other, depending on the cycle of solar activity. In the 21st and 23rd cycles, the components are in antiphase, and in the 22nd and 24th they are in phase. The lateral components (Figure 10b) are almost always in phase, with the exception of some time intervals in 1984–1985, 1994–1995 and 1999–2000. The correlation coefficient for these two lateral components $r = 0.77 \pm 0.01$. For the central and one of the lateral components, the correlation coefficient is about zero, since half of the data is positively correlated, and the other half is negatively correlated.

Figure 10. Changes in oscillation amplitudes with time for the central triplet of peaks from Figure 9a: a) for the central (black) and left (red) components, b) for the left (red) and right (blue) components.

This behavior of the component is not accidental and is evidence in favor of the fact that there is a connection between these components, and we actually observe a splitting associated with the solar magnetic cycle of 22 years.

We observe a similar picture for the left triplet of peaks in Figure 11. Shown here are the components $27^d.355$ (left, red), $27^d.260$ (central, black), $27^d.170$ (right, blue). The upper Figure 11a shows the central component with a period of $27^d.260$ and the left one with a period of $27^d.355$. In the 21st and 23rd cycles they are in antiphase, and in the 22nd and 24th they are in phase. The lateral components in Figure 11b behave almost in phase except for some time intervals. The correlation coefficient between them is $r = 0.77 \pm 0.01$.

Figure 11. Changes in oscillation amplitudes with time for the left triplet of peaks from Figure 9a: a) for the central (black) and left (red) components, b) for the left (red) and right (blue) components.

Figure 12 shows the time course of the components for the right triplet of peaks from Figure 9a for periods $26^d.884$ (left component, red), $26^d.796$ (central, black) and $26^d.715$ (right, blue). Note that the behavior of the central and left components in Figure 12a is opposite to the behavior of the analogous components in Figures 10 and 11. Namely: here these components are in phase in cycles 21 and 23 and in antiphase in cycles 22 and 24, while in Figures 10 and 11 the situation is opposite. The lateral components in Figure 12b are predominantly in phase, but there are intervals where they differ greatly from each other. The correlation coefficient between them is $r = 0.62 \pm 0.01$. This is slightly less than for the two previous triples of peaks. It should be noted that the magnitude of the splitting between the central peak $26^d.796$ and the right component $26^d.715$ is 24.4 years, which is somewhat longer than the cycle duration. The right-hand component in this splitting, as can be seen, is determined less confidently, and the correlation with it is somewhat less.

In general, the presented Figures 10–12 indicate the mutual relationship between the splitting components, and we can conclude that these splittings are indeed associated with the solar magnetic cycle of 22 years.

Figure 12. Changes in oscillation amplitudes with time for the right triplet of peaks from Figure 9a: a) for the central (black) and left (red) components, b) for the left (red) and right (blue) components.

9 Discussion and conclusions

The analysis made it possible to identify those periods that are not visible in the usual power spectrum. The main periodicities, as can be seen from Figure 9, have small values in the PS due to the fact that with a change in the sign of the magnetic field, the oscillation goes into antiphase and compensates for itself. This should be the case when the polarity of the magnetic field is changed: the central component should disappear (or be hardly noticeable), while the lateral components of the splitting in this case can still manifest themselves in some way. This is what we observe in the ordinary PS in Figure 2, where the peaks reflect the random predominance of one component over the other due to the different power of oscillations in different cycles of activity, as well as different durations of the cycles. For the analysis of a long-term sign-alternating series of a magnetic field, a conventional PS is not sufficiently informative.

A new method of searching for periodicities, which takes into account the change in the sign of the magnetic field, makes it possible to detect hidden oscillations and see their splitting and change in time. Analysis of the MMF time series by the proposed method made it possible to draw a conclusion about the main periods of rotation of the magnetic field: $27^{\text{d}}021 \pm 0^{\text{d}}008$, $26^{\text{d}}796 \pm 0^{\text{d}}008$ and $27^{\text{d}}260 \pm 0^{\text{d}}008$, each of which it splits into two components due to a change in the polarity of the magnetic field.

The change in the amplitudes of the main components, shown by the black line in Figures 10a–12a, indicates that each oscillation changes its sign from cycle to cycle and this change of sign occurs during the minimum solar activity. Obviously, this is due to a change in the polarity of the field in the cycle. As is known, herewith this changes

the polarity of the field in the leading and following spots, and also changes the leader of the groups in the northern and southern hemispheres of the Sun. Due to the north-south asymmetry, which is almost always observed on the Sun, one of the hemispheres is always more active than the other. In a more active hemisphere, sunspots appear earlier, magnetic fields are usually stronger, more flares and active phenomena are observed, and background magnetic fields, which make the main contribution to the MMF, occupy more large areas. Only over time, the same phenomena are entirely observed also in the other hemisphere. In each cycle of activity, the leadership of either the northern or the southern hemisphere can be noted. Most likely, this is precisely the reason for the change in the sign of the MMF components from cycle to cycle, which is observed in Figures 10–12.

The stability of discovered periodicities over more than four cycles of solar activity suggests that the Sun as a whole looks like a horizontal magnetic dipole (in addition to the observed vertical one), which rotates with different periods and changes its sign every 11 years.

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