Research Article

Salakhutdin Nuritdinov*, Alexsey Rastorguev, and Ikram Tadjibaev

Determination of the degree of star concentration in globular clusters based on space observation data

<https://doi.org/10.1515/astro-2022-0001>

Received Nov 05, 2021; accepted Dec 20, 2021

Abstract: We have proposed a generalized King's model for the observed density of stars in globular clusters (GC). Using two methods of minimizing the functional, composed in the form of the square of the difference between the theoretical and observed density values, we obtained the behavior of the degree of concentration for 26 GC. We have taken the observational data for these clusters from the work of Miocchi *et al.* (2013), which is a combination of ground-based and space-based observations. The minimized functional depends on three free parameters and one of them clearly characterizes the concentration of stars towards the center of the cluster. The two research methods used here give almost the same results with reasonably good accuracy. In contrast to the work of Nuritdinov *et al.* (2021), the χ^2 method is used to determine the degree of concentration. In addition, we analyzed data from de Boer *et al.* (2019). However, these data do not contain information for the central regions of the clusters. The lack of these data does not allow finding the degree of concentration with the required accuracy.

Keywords: classification problems, globular star clusters, concentration of stars toward the center, observed densities of stars in clusters

1 Introduction

Currently, there are quite a lot of observational data of globular clusters (GC). But despite this, there is still no satisfactory solution to the classification problem of GCs, but during the last about half century there have been many attempts to solve this problem, which are given in detail in the work of Kukarkin (1971). Here we are obliged to note the works of Hartwick (1968); Mironov *et al.* (1976); Straizys (1982); Eigenson and Yatsyk (1995); Tadjibaev and Nuritdinov (2019); Nuritdinov *et al.* (2021), and others.

It is important to note that the classification of GC should, first of all, be easily applicable in practice. In addition, it should contain the main distinctive features, for example, specific parameters associated with shape of GCs, concentration to the center, size, etc. But most importantly, any proposed classification should have a physical correlation with the main observational characteristics of GCs.

Obviously, if it is in no way possible to develop a classification, using a specific property, then it is necessary to search for an auxiliary parameter. As an example, we should note King's concentration parameter c , introduced by him in 1962. Later it became clear that this parameter does not have the necessary correlation with the main physical characteristics of GCs obtained from observations.

Generally speaking, classification problems were first analyzed by Shapley and Sawyer (1927). It were they who proposed for the first time to classify clusters according to the degree of concentration of stars to their center. So these authors were able to divide photographic images of 95 GC into 12 classes based on a comparison of the apparent concentration to the center in them. Later it was found that this division of 12 classes is very subjective.

The results of Mowbray (1946); Hartwick (1968); Kukarkin (1971); Mironov (1973); Mironov and Samus (1974); Eigenson and Yatsyk (1995) and Dotter *et al.* (2010) are also interesting from a standpoint. Unfortunately, these results do not in any way solve the problem of GC classification, since they do not solve the main problem related to the correlation between the classification and the observed characteristics of clusters. To solve the problem of GC classification, as will be seen below, the observational data in the central regions of the GC and correct modeling of the

Salakhutdin Nuritdinov: National University of Uzbekistan, Tashkent, 100174, Uzbekistan; Email: nur200848@mail.ru

Alexsey Rastorguev: Moscow State University, Moscow, 119991, Russia; alex.rastorguev@gmail.com

Ikram Tadjibaev: Chirchik State Pedagogical Institute in Tashkent region, Chirchik, 111700, Uzbekistan; tadj_ikram@mail.ru

apparent density in them turned out to be necessary. That is why in this work we are forced to consider the GC model with three free parameters.

2 Generalization of King's model

According to observations, GCs have a number of remarkable properties, among which the phenomenon of the concentration of stars towards their center is the most important from the point of view of the classification of these objects. However, the division of GCs by Shapley and Sawyer into 12 classes turned out to be very subjective, which is not difficult to see if we consider the dependence of these classes on the value of the King parameter 1962

$$c = \lg(R_t/R_c), \quad R_t = R_{gc} \left(\frac{M_{gc}}{2M} \right)^{1/3}. \quad (1)$$

In (1) R_c is the radius of the GC core, R_t is its tidal radius, R_{gc} is the distance from the GC to the center of the Galaxy, M_{gc} is the GC mass, and M is the mass of our Galaxy. Analysis of the dependences of the King parameter with the characteristics noted here shows that these dependences do not have the necessary physical correlation. That is why, instead of the King concentration parameter c , it is necessary to find another quantity that characterizes the phenomenon of the concentration of stars towards the center of the clusters. If we compare the apparent density profiles of GCs and take into account the well-known King's model, where the theoretical density σ behaves in the form $[1 + (r/R_c)^2]^{-1}$, then it can be shown that this model needs generalization as follows:

$$\sigma(r, \sigma_0, \alpha, R_0) = \sigma_0 \left(1 + r^2/R_0^2 \right)^{-\alpha}, \quad (2)$$

where α , σ_0 and R_0 have the meaning of free parameters of the model. Here α describes the degree of concentration of stars to the center of the cluster, R_0 characterizes the radius of the GC core R_c , and σ_0 is the surface density of the model at $r = 0$.

As we can see, the accuracy of determining the parameters of function (2) from observations, in particular, the value of α clearly depends on the accuracy of the visible distribution in the central region, since we determine them below by minimizing the square of the difference between the theoretical and observed densities. The observed density is determined by dividing the GC into rings around the center and counting the number of stars in these rings. It is very difficult to determine the number of stars in the central region of the GC by ground-based observations due to the

high density there. In this regard, in the work of Miocchi *et al.* (2013), the apparent densities for the central region 26 GC were found on the basis of CCD observations from the Hubble Space Telescope, and for the rest of the clusters, data from ground-based telescopes were used.

3 Finding the values of the degree of concentration

As mentioned above, to calculate the free parameters of the generalized model (2), we minimized the following sum:

$$f = \sum_n \left[\sigma(r_n, \sigma_0, \alpha, R_0) - \sigma_{obs}^{(n)} \right]^2, \quad (3)$$

where $\sigma_{obs}^{(k)}$ is the observed density value inside the n -ring around the GC center. The sum (3) was minimized by two methods: the symplectic method (see, for example, Ashurov and Nuritdinov (2001)) and the χ^2 method. We performed calculations using these two methods for the observational data of Miocchi *et al.* (2013). It is seen that both methods give very close results for the values of α . The results of calculating free parameters by the first method are given in detail in the work of Nuritdinov *et al.* (2021). Therefore, here we will focus on the results of the calculation by the χ^2 method:

$$\chi^2 = \sum_n \frac{\left| \sigma_{obs}^{(n)} - \sigma(r_n, \sigma_0, \alpha, R_0) \right|^2}{\sigma(r_n, \sigma_0, \alpha, R_0)}.$$

The results of the χ^2 method and the errors of the three free parameters are given by us in Table 1. Thus, we have obtained the values of the free parameters, which almost coincide with the results found by us using the symplectic method.

Among the three parameters of the model, almost complete agreement with the results of Nuritdinov *et al.* (2021) is well observed particularly for the concentration α . We can confidently assert that the values of the degree of concentration α lie in the interval (0.68; 2.16). In the work of Nuritdinov *et al.* (2021), a preliminary classification of GC into four classes was carried out: 1) densest ($\alpha \leq 0.90$), 2) moderately dense ($0.90 \leq \alpha \leq 1.15$), 3) moderately sparse ($1.15 \leq \alpha \leq 1.40$), and 4) sparse ($\alpha > 1.40$). This classification of GC is much simpler than the Shapley-Sawyer classification of 12 classes.

Now it is necessary to dwell on the most important question: to check whether there is a correlation between α and the observed characteristics of the 26 GCs under consideration. For this purpose, using the observational data

Table 1. The results of calculating the free parameters of the model by the χ^2 method for observational data Miocchi *et al.* (2013)

Name GC	α	$\Delta\alpha$	R_0	ΔR_0	σ_0	$\Delta\sigma_0$
NGC 104	0.73	0.24	18.68	8.52	7.69	0.96
NGC 288	1.38	0.96	93.75	63.18	0.17	0.16
AM 1	1.72	1.19	14.63	9.71	0.47	0.45
Eridanus	1.51	0.60	19.87	7.86	0.86	0.15
NGC 1851	0.99	0.36	4.78	2.05	5.62	1.17
NGC 1904	0.91	0.27	7.42	3.32	5.70	1.34
NGC 2419	1.18	0.33	20.06	6.39	3.60	0.43
Palomar 3	2.11	0.91	48.45	18.21	0.04	0.01
Palomar 4	2.16	1.01	39.31	15.01	0.04	0.01
NGC 5024	1.00	0.24	22.08	7.41	2.96	0.40
NGC 5272	0.81	0.23	17.21	7.72	3.10	0.51
NGC 5466	1.48	0.60	102.88	38.95	0.10	0.01
NGC 5824	0.73	0.17	2.71	1.12	7.56	1.34
NGC 5904	0.68	0.14	14.64	5.31	4.07	0.53
Palomar 14	0.99	0.47	38.37	21.65	0.25	0.06
NGC 6121	0.81	0.35	47.43	30.39	0.26	0.09
NGC 6205	1.05	0.32	48.22	15.18	3.58	0.31
NGC 6229	1.02	0.34	8.27	3.28	5.26	0.78
NGC 6254	1.02	0.28	38.33	14.38	0.50	0.08
NGC 6266	0.86	0.21	12.44	4.17	10.4	1.21
NGC 6341	1.03	0.46	14.54	8.28	2.54	0.81
Terzan 5	1.31	0.45	10.28	3.87	7.17	1.06
NGC 6626	0.71	0.18	6.85	3.18	2.36	0.58
NGC 6809	0.85	0.35	69.07	40.75	0.20	0.06
NGC 6864	1.18	0.46	5.71	2.5	16.82	3.01
NGC 7089	0.89	0.29	12.43	5.8	3.13	0.71

mainly from the Harris catalog 2010, as well as from the works of Borkova and Marsakov (2000) and Kukarkin (1971), we found the statistical dependences of the value of α with the following characteristics of clusters: galactocentric distance, absolute magnitude, mass, Kukarkin's index of richness, King parameter and age.

1. The empirical formula between α and galactocentric distance is:

$$\alpha = (0.008 \pm 0.001) + (0.088 \pm 0.007)R_{gc} \quad (4)$$

with a correlation coefficient of +0.76.

2. The empirical dependence between α and absolute magnitude:

$$\alpha = (2.52 \pm 0.26) + (0.19 \pm 0.03)M_V \quad (5)$$

with a correlation coefficient of +0.76.

3. The empirical formula between α and the mass of cluster is:

$$\alpha = (3.65 \pm 0.56) - (0.46 \pm 0.10) \lg \frac{M_{gc}}{M_\odot} \quad (6)$$

with a correlation coefficient of -0.68.

4. The empirical dependence between α and Kukarkin's index of richness:

$$\alpha = (1.88 \pm 0.17) - (1.41 \pm 0.28)IR \quad (7)$$

with a correlation coefficient of -0.73.

5. The empirical formula between α and King parameter:

$$\alpha = (2.16 \pm 0.26) - (0.70 \pm 0.17)c. \quad (8)$$

with a correlation coefficient of -0.63.

6. Analysis of the relationship between α and age (τ) of clusters shows a rather weak correlation of -0.43. Thanks to the proposal of the reviewer of our paper, we examined relatively new observational data published by Dotter *et al.* (2010) and Vandenberg *et al.* (2013). Using these data, we found that the correlation of these two values for data of Vandenberg *et al.* (2013) is very low (0.19), and for data of Dotter *et al.* (2010) is -0.57. For the last case, for the sake of interest, we decided to find corresponding empirical dependence. So it is obtained that

$$\alpha = (4.36 \pm 1.15) - (0.26 \pm 0.09)\tau. \quad (9)$$

From here it can be seen that the cluster becomes more denser if the age grows.

The obvious question is: does the degree of concentration α take values in the interval (0.68; 2.16) for all GCs of our Galaxy? Of course, the values of this parameter for some GC can go beyond this interval. For the final solution of this issue, we carried out a literature search for data on the apparent density profiles. This is how we found an interesting work by de Boer *et al.* (2019), where the apparent densities were found for 81 GCs within the Gaia DR2 space program. However, in this work, the data on the central regions for literally all 81 GCs are clearly insufficient for the purpose of processing, modeling and finding the values of α .

4 Conclusions

In this work, the chi-square method is used to find the values of free parameters of model (2) and their errors for 26 GCs. These results are in good agreement with our early determination of the degree of GC concentration using the symplectic method in the work of Nuritdinov *et al.* (2021). Based on the found values of the degree of concentration α , we obtained empirical formulas for the relationship between α and the main observed characteristics of the clusters studied here.

Unfortunately, Gaia DR2 analysis by de Boer *et al.* (2019) does not provide the necessary information for the central regions of 81 clusters, which requires additional analysis of the corresponding data and the introduction of some correction in the model (2).

Acknowledgment: We are very grateful to the referee of the paper for the useful remarks.

Funding information: The authors state no funding involved.

Author contributions: All authors have accepted responsibility for the entire content of this manuscript and approved its submission.

Conflict of interest: The authors state no conflict of interest.

References

- Ashurov AE, Nuritdinov SN. 2001. Finding out the velocity anisotropy parameter for some globular clusters: the case of stationary model. *ASP Conf Ser.* 228:371-375.
- Borkova TV, Marsakov VA. 2000. Globular cluster subsystems in the Galaxy. *Astron Rep.* 44:665-684.
- de Boer TJL, Gieles M, Balbinot E, Hénault-Brunet V, Sollima A, Watkins LL, et al. 2019. Globular cluster number density profiles using Gaia DR2. *MNRAS.* 485:4906-4935.
- Dotter A, Sarajedini A, Anderson J, Aparicio A, Bedin LR, Chaboyer B, et al. 2010. The ACS Survey of galactic globular clusters. IX. Horizontal branch morphology and the second parameter phenomenon. *ApJ.* 708:698-716.
- Eigenson AM, Yatsyk OS. 1995. Comparative classification of globular clusters in the Galaxy and M31 using cluster analysis. *Astron Rep.* 39:569-577.
- Harris WE. 2010. A New Catalog of Globular Clusters in the Milky Way. *arXiv:1012.3224H.*
- Hartwick FD. 1968. Two-Dimensional Classification for Galactic Globular Clusters. *ApJ.* 154:475-479.
- King IR. 1962. The structure of star clusters. I. An empirical density law. *AJ.* 67:471-482.
- Kukarkin BV. 1971. The Concentration Classes of Globular Clusters. *Sov Astron.* 15:89-94.
- Miocchi P., Lanzoni B. et al. 2013. Star count density profiles and structural parameters of 26 Galactic globular clusters. *ApJ.* 774:151-166.
- Mironov AV. 1973. Chemical Composition of Globular-Cluster Stars and the Form of the Horizontal Branch. *Sov Astron.* 17:16-23.
- Mironov AV, Samus NN. 1974. On the chemical composition and the origin of the globular clusters of our Galaxy Perem. *Zvezdy, Byull.* 19:337-356.
- Mironov AV et al. 1976. Kinematic and dynamical differences between two groups of globular clusters. *Sov Astron.* 20:656-659.
- Mowbray AG. 1946. The Diameters of Globular Clusters. *Sov Astron.* 104:47-53.
- Shapley H, Sawyer HB. 1927. A classification of globular clusters *Harvard Observ Bull.* 849:11-14.
- Straizys V. 1982. Two-Dimensional Classification of Globular Clusters. *Astrophys Space Sci.* 81:179-197.
- Tadjibaev IU, Nuritdinov SN. 2019. Can globular clusters in the Galaxy be classified by the velocity anisotropy parameter? *Ukr J Phys.* 64:271-275.
- Nuritdinov SN, Tadjibaev IU, Rastorguev AS. 2021. To the Globular Cluster Classification Problem. Calculating the Concentration of Stars for 26 Clusters. *Astron Letters.* 43:163-169.
- VandenBerg DA, Brogaard K, Leaman R, Casagrande L. 2013. The ages of 55 globular clusters as determined using an improved ΔV HB to method along with color-magnitude diagram constraints, and their implications for broader issues. *ApJ.* 775:134-182.