Research Article

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Small-scale sectorial perturbation modes against the background of a pulsating model of disk-like self-gravitating systems

<https://doi.org/10.1515/astro-2022-0011>

received October 31, 2021; accepted January 30, 2022

Abstract: In this work, we consider small-scale sectorial perturbation modes in a disk-like model of a radially nonstationary spiral galaxy in order to study the gravitational instabilities of these modes. Calculations of horizontal sectorial small-scale perturbation modes, such as $(m; N) = (10; 10), (11; 11), (12; 12), (13; 13), (14; 14),$ and $(15; 15)$, against the background of a nonlinearly nonequilibrium anisotropic model of the self-gravitating disk have been carried out. For each of these perturbation modes, critical diagrams are plotted between the virial parameter and the degree of rotation. The growth rates of instability are calculated and compared for different values of the rotation parameter. The results of calculations and comparisons of instability regions show that with an increase in the degree of small scale, the instability region narrows as the wave numbers increase.

Keywords: nonlinear – non-stationary models, self-gravitating disk, gravitational instability, small-scale perturbation modes

1 Introduction

Gravitational instabilities of disk-like models of self-gravitating systems are of great interest not only for galactic disks but also for accretion ones studied by Lodato (2007), Forgan and Rice (2011a, b), Lodato (2012), Rice (2016), Kratter and Lodato (2016), Paneque-Carreño *et al.* (2021), and Bethune and Latter (2021).

Both large-scale and various small-scale formations are observed in disk-like galaxies. Small-scale formations of disk-like galaxies are mainly open star clusters (OSC) and molecular clouds (MC). Formation and origin of these structural formations in disk-like self-gravitating systems occur due to gravitational instabilities (Mayer *et al.* 2016, Sharma 2016, Inoue and Yoshida 2018, Roshan and Rahvar 2019). They determine the global structure of the disk of galaxies. However, there is still no analysis of the problems of their origin and no one has studied in detail small-scale perturbations against the background of nonlinearly non-stationary models of disk-like self-gravitating systems, in particular, for our Galaxy. This raises a number of new questions. Can their global distribution be explained by the corresponding formation theory? It is not clear under what physical conditions these objects can form in disk-like systems, and what are the characteristic times of these phenomena? In addition, it is not clear which objects or structural formations of disk-like subsystems of galaxies are directly related to small-scale disturbances. These questions arise primarily due to the lack of knowledge of small-scale high-order perturbations in specific stationary and non-stationary models.

In the disks under study, self-gravity plays the main role in the studies of Rice *et al.* (2011), Trova *et al.* (2014), Meru (2015), and Young and Clarke (2015). The gravitational interaction between different parts of the system compresses matter. This process is called gravitational or Jeans instability, and it was studied by Fridman and Khoperskov (2013). It leads to a redistribution of mass, *i.e.*, in one area of the system, the density increases, in another area, respectively, decreases. Clumps of MC are formed due to gravitational instabilities of gas disk-like galaxies in the studies of Forgan and Rice (2011a, b), Dipierro *et al.* (2014), and Tsukamoto *et al.* (2015).

Nowadays, many types of instabilities have already been identified for both equilibrium models (Binney and Tremaine 2008) and nonlinear nonstationary states of disk subsystems, which was considered by Nuritdinov (1993)

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and Mirtadjieva and Nuritdinov (2012), and the corresponding nonstationary dispersion equations (NDEs) were obtained for each perturbation mode.

2 Pulsating anisotropic model

This model is based on the well-known isotropic non-linear – pulsating model of Nuritdinov (1993)

$$\Psi(r, v_r, v_\perp, t) = \frac{\sigma_0}{2\pi\sqrt{1-\Omega^2}} \left[\frac{1-\Omega^2}{\Pi^2} \left(1 - \frac{r^2}{\Pi^2} \right) - (v_r - v_a)^2 - (v_\perp - v_b)^2 \right]^{-\frac{1}{2}} \cdot \chi(R-r). \quad (1)$$

Here $\sigma_0 = \sigma(0; 0)$, Ω is the dimensionless parameter characterizing the degree of rotation of the disk ($0 \leq \Omega \leq 1$). v_r and $v_\perp$ are radial and tangential particle velocities, respectively. The function $\Pi(t)$ has the meaning of the stretching coefficient and has the following form:

$$\begin{aligned} \Pi(t) &= (1 + \lambda \cos \psi) \cdot (1 - \lambda^2)^{-1}, \\ t &= \psi + \lambda \sin \psi \cdot (1 - \lambda^2)^{-3/2}. \end{aligned}$$

The model pulsates with an amplitude $\lambda = 1 - (2T/|U|)_0$, where $(2T/|U|)_0$ is the initial virial parameter.

Using Eq. (1), it is possible to make an anisotropic model by averaging the phase density of the rotation parameter Ω (see Bisnovatij-Kogan and Zel'dovich 1970, Kalnajs 1972, Fridman and Polyachenko 1984).

$$\Psi_{\text{Aniz}} = \frac{\int_{-1}^{+1} \Psi_{\text{Izot}} \cdot \rho(\Omega) d\Omega}{\int_{-1}^{+1} \rho(\Omega) d\Omega}, \quad (2)$$

where $\rho(\Omega)$ is the weight function of Ω . We have constructed an anisotropic model, taking the weight function, for example, in the form $\rho(\Omega) = 2/\pi \cdot (1 - \Omega^2)^{1/2}$. After integration and some transformations, we obtain a nonlinearly pulsating model with the anisotropic velocity diagram as follows:

$$\begin{aligned} \Psi_{\text{Aniz}} &= \frac{\sigma_0}{\pi} [1 + \Omega \cdot (xv_y - yv_x)] \cdot \chi \\ &\cdot ((1 - r^2/\Pi^2)(1 - \Pi^2 v_\perp^2) - \Pi^2(v_r - v_a)^2). \end{aligned} \quad (3)$$

Next, we can find the NDE of model (3). It can be in the following form:

$$\begin{aligned} a_{\text{Aniz}}(\psi) &= \frac{4}{N(N^2 - 1)\Pi^3} \gamma_m^N (N-1)(N^2 + N - m^2) \\ &\cdot \overline{D}_N + i\Omega m(N^2 + N - m^2)d_N, \end{aligned} \quad (4)$$

where $\gamma_m^N = \frac{(N+m-1)!!(N-m-1)!!}{(N+m)!!(N-m)!!}$,

$$\begin{aligned} \overline{D}_N &= \int_{-\infty}^{\psi} E W^{N-1} \frac{dP_N}{d(\cosh)} d\psi_1, \quad W = \frac{1 + \lambda \cos \psi_1}{1 + \lambda \cos \psi}, \\ d_N &= \int_{-\infty}^{\psi} E I_N d\psi_1, \quad E = \Pi^3(\psi_1) S(\psi, \psi_1) a(\psi_1), \\ I_N &= W^{N-1} \left[\frac{1}{N+2} \frac{d^2 P_{N+1}}{d(\cosh)^2} - \frac{P_N}{d(\cosh)} \right] tgh, \\ \cosh &= \frac{(\cos \psi + \lambda)(\cos \psi_1 + \lambda) + (1 - \lambda^2) \sin \psi \sin \psi_1}{(1 + \lambda \cos \psi)(1 + \lambda \cos \psi_1)}, \end{aligned}$$

where $P_N(\cosh)$ is the Legendre polynomial, N, m are the radial and the azimuthal wave numbers, respectively.

3 Calculation results

In this section, we explore the results of calculations for six values of the sectorial small-scale oscillation modes taking $m = N = 10, 11, 12, 13, 14, 15$. The gravitational instabilities of these perturbation modes have been calculated using the NDE expression (4).

For example, NDE for the case $m = N = 10$ has the following form:

$$a(\psi)_{10;10} = \frac{1}{(1 + \lambda \cos \psi)^{21}} K_{10;10}(l_r), \quad \tau = \overline{0;9}. \quad (5)$$

Here

$$\begin{aligned} l_r(\psi) &= \int_{-\infty}^{\psi} \Pi^3(\psi_1) S(\psi, \psi_1) D(\psi_1) \\ &\cdot (\lambda + \cos \psi_1)^{m-1-\tau} \sin^\tau \psi_1 d\psi_1, \end{aligned}$$

Figure 1: Critical dependence of the virial ratio on the rotation parameter for $m = 10; N = 10$.

Figure 2: Critical dependence of the virial ratio on the rotation parameter for $m = 11$; $N = 11$.

where λ is the amplitude of the disk pulsation. The expression for $K_{10;10}$ is given in the Appendix.

Numerical calculation of the NDE shows that for the oscillation mode $m = N = 10$, the instability starts from the virial parameter's $(2T/|U|)_0 = 0.056$ at $\Omega = 0$ and reaches 0.325 when $\Omega = 1$ (Figure 1).

For the perturbation mode $m = N = 11$, the calculation of the NDE shows that the instability starts from the virial parameter's value $(2T/|U|)_0 = 0.049$ and reaches 0.282 (Figure 2).

It is shown in Figure 3 for the case $m = N = 12$, the instability starts from the virial parameter's $(2T/|U|)_0 = 0.044$ and reaches 0.248.

Instability in the fourth case $m = N = 13$ (Figure 4) starts from the value of the virial parameter's $(2T/|U|)_0 = 0.039$ and reaches 0.221, respectively.

Figure 3: Critical dependence of the virial ratio on the rotation parameter for $m = 12$; $N = 12$.

Figure 5: Critical dependence of the virial ratio on the rotation parameter for $m = 14$; $N = 14$.

Figure 4: Critical dependence of the virial ratio on the rotation parameter for $m = 13$; $N = 13$.

Figure 6: Critical dependence of the virial ratio on the rotation parameter for $m = 15$; $N = 15$.

Figure 7: (a to c) Comparison of the increments of instabilities of all small-scale oscillation modes for different values of the rotation parameter.

Figure 8: (a to c) Comparison of the increments of instabilities of all small-scale oscillation modes for different values of the rotation parameter.

For the oscillation mode $m = N = 14$, the calculation of the NDE shows that the instability starts from the value of the virial parameter's $(2T/|U|)_0 = 0.035$ and reaches 0.199 (Figure 5).

The instability in the sixth case, $m = N = 15$, starts from the virial parameter's value $(2T/|U|)_0 = 0.032$ at $\Omega = 0$ and reaches 0.180 when $\Omega = 1$.

The shaded area defines the instability regions for the indicated disturbance modes in the figures (Figures 1–6). It can be seen from the figures that as the speed of rotation of the model grows, the region of instabilities always grows, and with increasing values of m and N , the region of instability, correspondingly, narrows.

We also calculated and compared the increments of instabilities of small-scale oscillation modes for different values of the parameters (Figures 7 and 8). In Figures 7 and 8 show the increments of instabilities for all small-scale oscillation modes at different values of the virial ratio and degree of rotation. It can be seen from the figures that the curves of the growth rates of instabilities of the studied perturbation modes are arranged in descending order and it should be noted that with an increase in the value of the wavenumbers, the increments of instabilities also increase. The curved lines intersect each other, merging at some values of the increment of instabilities, except for the value of the rotation parameter $\Omega = 0.2$, without changing their position as the wave numbers increase.

4 Conclusion

The gravitational instabilities are deeply studied for oscillation modes $m = N = 10, 11, 12, 13, 14, 15$ in this work. Diagrams of the critical dependence of the virial ratio on the rotation parameter are plotted and the increments of instabilities of all small-scale oscillation modes for various values of the model parameters are compared. Figures 1–6 show that with an increase in the value of the model rotation speed, an increase in the region of instabilities is observed.

From a comparison of the graphs (Figures 7 and 8), we can see that the intersection of the curves at some values of the instability increments except for the case $\Omega = 0.2$, while the location of the curves does not change. Note that an increase in the value of wave numbers leads to an increase in the value of the increments of instabilities.

Acknowledgements: This work was partly supported by the Ministry of Innovative Development of the Republic of Uzbekistan.

Funding information: The authors state no funding involved.

Author contributions: All authors have accepted responsibility for the entire content of this manuscript and approved its submission.

Conflict of interest: The authors state no conflict of interest.

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Appendix

Substituting all the expressions of the functions in (4) and calculating each of them step by step, we find that

$$\begin{aligned}
 K_{10;10} = & \frac{4199}{65536} \left(\left(55c^9 - 495c^7e^2b^2 + \frac{3465}{4}c^5e^4b^4 \right. \right. \\
 & - \frac{5775}{16}c^3e^6b^6 + \frac{3465}{128}ce^8b^8 \Big) l_0 \\
 & + \left(1485c^8e^2b - 6930c^6e^4b^3 + \frac{51975}{8}e^6b^5c^4 \right. \\
 & - \frac{10395}{8}e^8b^7c^2 + \frac{3465}{128}e^{10}b^9 \Big) l_1 \\
 & + \left(\frac{28215}{2}e^4b^2c^7 - \frac{530145}{16}e^6b^4c^5 - 495c^9e^2 \right. \\
 & + \frac{509355}{32}e^8b^6c^3 - \frac{10395}{8}e^{10}b^8c \Big) l_2 \\
 & + \left(\frac{234465}{4}e^6b^3c^6 - \frac{557865}{8}e^8b^5c^4 - 6930e^4bc^8 \right. \\
 & + \frac{509355}{32}e^{10}b^7c^2 - \frac{5775}{16}e^{12}b^9 \Big) l_3 \\
 & + \left(\frac{7494795}{64}e^8b^4c^5 - \frac{557865}{8}e^{10}b^6c^3 \right. \\
 & - \frac{530145}{16}e^6b^2c^7 + \frac{51975}{8}e^{12}b^8c + \frac{3465}{4}c^9e^4 \Big) l_4 \\
 & + \left(\frac{7494795}{64}e^{10}b^5c^4 - \frac{530145}{16}e^{12}b^7c^2 \right. \\
 & - \frac{557865}{8}e^8b^3c^6 + \frac{3465}{4}e^{14}b^9 + \frac{51975}{8}e^6bc^8 \Big) l_5 \\
 & + \left(\frac{234465}{4}e^{12}b^6c^3 - 6930e^{14}b^8c \right. \\
 & - \frac{557865}{8}e^{10}b^4c^5 + \frac{509355}{32}e^8b^2c^7 - \frac{5775}{16}c^9e^6 \Big) l_6 \\
 & + \left(\frac{28215}{2}e^{14}b^7c^2 - 495e^{16}b^9 - \frac{530145}{16}e^{12}b^5c^4 \right. \\
 & + \frac{509355}{32}e^{10}b^3c^6 - \frac{10395}{8}e^8bc^8 \Big) l_7 \\
 & + \left(1485e^{16}b^8c - 6930e^{14}b^6c^3 + \frac{51975}{8}e^{12}b^4c^5 \right. \\
 & - \frac{10395}{8}e^{10}b^2c^7 + \frac{3465}{128}c^9e^8 \Big) l_8 \\
 & + \left(55e^{18}b^9 - 495e^{16}b^7c^2 + \frac{3465}{4}e^{14}b^5c^4 \right. \\
 & \left. \left. - \frac{5775}{16}e^{12}b^3c^6 + \frac{3465}{128}e^{10}bc^8 \right) l_9 \right)
 \end{aligned}$$

$$\begin{aligned}
 & + 4i\Omega e \left(\left(-\frac{495}{4}bc^8 + \frac{1155}{2}e^2b^3c^6 - \frac{17325}{32}e^4b^5c^4 \right. \right. \\
 & + \frac{3465}{32}e^6b^7c^2 - \frac{1155}{512}e^8b^9 \Big) l_0 \\
 & + \left(-\frac{5445}{2}c^7e^2b^2 + \frac{495}{4}c^9 + \frac{197505}{32}c^5e^4b^4 \right. \\
 & - \frac{93555}{32}c^3e^6b^6 + \frac{121275}{512}ce^8b^8 \Big) l_1 \\
 & + \left(-\frac{5445}{2}c^7e^2b^2 + \frac{495}{4}c^9 + \frac{197505}{32}c^5e^4b^4 \right. \\
 & - \frac{93555}{32}c^3e^6b^6 + \frac{121275}{512}ce^8b^8 \Big) l_2 \\
 & + \left(-\frac{1867635}{32}e^6b^4c^5 + \frac{308385}{16}e^4b^2c^7 + \frac{4163775}{128}e^8b^6c^3 \right. \\
 & - \frac{1155}{2}c^9e^2 - \frac{93555}{32}e^{10}b^8c \Big) l_3 \\
 & + \left(-\frac{21417165}{256}e^8b^5c^4 + \frac{1867635}{32}e^6b^3c^6 \right. \\
 & + \frac{696465}{32}e^{10}b^7c^2 - \frac{197505}{32}e^4bc^8 - \frac{17325}{32}e^{12}b^9 \Big) l_4 \\
 & + \left(-\frac{1867635}{32}e^{10}b^6c^3 + \frac{21417165}{256}e^8b^4c^5 \right. \\
 & + \frac{197505}{32}e^{12}b^8c - \frac{696465}{32}e^6b^2c^7 + \frac{17325}{32}c^9e^4 \Big) l_5 \\
 & + \left(-\frac{308385}{16}e^{12}b^7c^2 + 186763532e^{10}b^5c^4 + \frac{1155}{2}e^{14}b^9 \right. \\
 & - \frac{4163775}{128}e^8b^3c^6 + \frac{93555}{32}e^6bc^8 \Big) l_6 \\
 & + \left(-\frac{5445}{2}e^{14}b^8c + \frac{308385}{16}e^{12}b^6c^3 - \frac{696465}{32}e^{10}b^4c^5 \right. \\
 & + \frac{620235}{128}e^8b^2c^7 - \frac{3465}{32}c^9e^6 \Big) l_7 \\
 & + \left(-\frac{495}{4}e^{16}b^9 + \frac{5445}{2}e^{14}b^7c^2 - 19750532e^{12}b^5c^4 \right. \\
 & + \frac{93555}{32}e^{10}b^3c^6 - \frac{121275}{512}e^8bc^8 \Big) l_8 \\
 & + \left(\frac{495}{4}e^{16}b^8c - \frac{1155}{2}e^{14}b^6c^3 + \frac{17325}{32}e^{12}b^4c^5 \right. \\
 & \left. \left. - \frac{3465}{32}e^{10}b^2c^7 + \frac{1155}{512}c^9e^8 \right) l_9 \right),
 \end{aligned}$$

where $c = \lambda + \cos\psi$, $b = \sin\psi$, $e = \sqrt{1 - \lambda^2}$.