Contributions

João Correia-da-Silva, Joana Pinho* and Hélder Vasconcelos How Should Cartels React to Entry Triggered by Demand Growth?

Abstract: We study the sustainability of collusion with optimal penal codes in markets where demand growth triggers the entry of a new firm. In contrast to grim trigger strategies, optimal penal codes make collusion easier to sustain before entry than after. This conclusion is robust to changes in the number of entrants and to the consideration of price-setting instead of quantity-setting. A comparison is given between different reactions of the incumbents to entry in terms of sustainability of collusion, incumbents' profits, entrant's profits, consumer surplus and social welfare. One of our findings is that the incumbent firms may prefer competition to collusion.

Keywords: collusion, demand growth, optimal penal codes, reactions to entry

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1 Introduction

Demand growth is usually understood as facilitating collusion, by reducing the short-term gains of deviating relative to the costs of future retaliation (Ivaldi et al. 2007). However, demand growth may attract new firms to the market, which hinders collusion.¹ Hence, the ultimate impact of demand growth on the likelihood of collusion is not straightforward.

¹ According to Levenstein and Suslow (2006, 45), “the most frequent causes of cartel failure are entry and bargaining problems.” Eckbo (1976) and Griffin (1989) concluded that, in their samples, the entry of a new firm caused 13% and 26% of the cartel breakdowns, respectively.

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The empirical evidence is mixed. For example, Dick (1996) found that Webb-Pomerene cartels are more frequent in growing industries. In contrast, Asch and Seneca (1975) concluded that firms whose growth is slow have more incentives to collude than those whose growth is fast. Symeonidis (2003) provided empirical evidence suggesting that the relationship between the sustainability of collusion and demand growth is not monotonic. More precisely, collusion is more sustainable in markets with moderate growth than in stagnant or declining markets, or in markets with rapid growth.²

Capuano (2002) and Vasconcelos (2008) studied this issue in a theoretical model where demand growth induces the entry of a new firm. Comparing the sustainability of collusion before and after entry, Vasconcelos (2008) concluded that collusion is more difficult to sustain before entry, i.e. when there are less firms in the market. The rationale for this result lies in the possibility that incumbents have of profitably delaying entry by disrupting the collusive agreement. As shown in this paper, however, this result crucially depends on the fact that Vasconcelos (2008) did not consider optimal penal codes (Abreu 1986, 1988).

In this paper, we investigate the case in which firms adopt a severe (Abreu-type) punishment strategy, which induces the lowest possible continuation payoff. The fact that a well-designed punishment is a threat that is not carried out (because it induces compliance with the collusive agreement) is an obstacle to its empirical observation. Still, there is some empirical support for the fact that firms engage in finite length price wars. In the words of Levenstein and Suslow (2006, 54): “one of the most clearly established stylized facts is that cartels form, endure for a period, appear to breakdown, and then re-form again.”³

² According to Symeonidis (2003, 46), rapid growth may be detrimental to the sustainability of collusion not only by attracting new firms to the market (which hinders coordination among firms) but also because capacity constraints may become binding and limit the scope for punishing defectors. For studies on the impact of capacity constraints, see Brock and Scheinkman (1985) and Compte, Jenny, and Rey (2002).

³ Bosch and Eckard (1991) concluded that, in a sample of 127 firms indicted of price-fixing in the United States during 1962–1980, around 14% were recidivists as much as four times. The American electrical manufacturing industry also provides an example of cartel re-establishment. Collusion was restored in 1955, after a fierce price war from late 1954 to September 1955 (Ayres 1987, f. 125). Another example is the parcel post cartel formed in the 1880s by railroads operating in the lines between Chicago and the Atlantic coast, where “there were several instances in which the cartel [...] cut prices for a time, and then returned to the collusive price” (Porter 1983, 302). Since March 1983, the OPEC cartel members have explicitly combined production quotas. Due to persistent cheating of the remaining oil producers, the market share of Saudi Arabia by August 1985 was 10% smaller than expected, inducing Saudi Arabia to reply by flooding the market, which sharply reduced the oil price. Nevertheless, in August 1986, the OPEC members agreed to restore the

The structure of our model is the same as in Vasconcelos (2008). The industry is composed of two incumbent firms and one potential entrant who set quantities in an infinite number of periods. Market demand is linear in price and grows at a constant rate. Production costs are null. To become active, the potential entrant has to incur a fixed setup cost. There is a trade-off in the choice of the entry period: on the one hand, postponing entry implies the loss of the current period's profits; on the other hand, it decreases the discounted value of the entry cost.

At the beginning of the game, the incumbent firms agree to produce quantities that maximize their joint profit in each period. Unilateral deviations originate a harsh punishment, which induces a null (or security level) continuation value after a deviation. A null continuation value can be imposed through reversion to an equilibrium in which firms have null profits forever. Alternatively, it can be imposed through stick-and-carrot strategies according to which firms have high losses in one or more punishment periods and have positive profits when collusion is restored.⁴

Assuming that the incumbents accommodate the entrant in their collusive agreement, we conclude, in contrast to Vasconcelos (2008), that collusion is more difficult to sustain after entry than before entry. The result of Vasconcelos (2008) does not hold with optimal penal codes because a deviation before entry ceases to be an effective way of delaying entry and enjoying additional pre-entry profits. In our setting, a deviation can still delay entry, as the entrant would avoid entering the market in the punishment phase. However, the fact that the penal code implies a null (or security level) continuation value means that the punishment absorbs all the gains from delaying entry.

The idea that firms benefit from establishing collusive agreements is widely accepted. Interestingly, however, we conclude that the incumbent firms may prefer competition to collusion. Two conditions are necessary for this to be the case: (i) entry occurs later under competition; and (ii) individual profits are greater under competition with two firms than under collusion with three

collusive agreement and decrease production levels to their initial values (for more details, see Griffin and Neilson (1994)).

⁴ Since there are no production costs, for losses to be feasible, firms must have access to some kind of dissipative costs that are observable (such as unproductive advertising, donations to charity or wanton destruction of equipment). These costs, which are sometimes designated as money burning, have been considered, in different contexts, by Milgrom and Roberts (1986), Bagwell and Ramey (1994), Lariviere and Padmanabhan (1997), Hertzendorf and Overgaard (2001) and Linnemer (2002), among others. If, for some reason, deviators can guarantee a strictly positive continuation value, then the security level continuation value can be imposed through stick-and-carrot strategies without requiring that firms incur in observable dissipative costs. Introducing strictly positive, but not too high, unit costs also allows dispensing with these observable dissipative costs.

firms. These conditions imply that, under competition, the incumbents enjoy a longer pre-entry period, in which, although competing, they obtain higher profits than if they were colluding after entry.

As explained by Harrington (1989), Stenbacka (1990), Friedman and Thisse (1994) and Vasconcelos (2004), immediate accommodation of the entrant in the collusive agreement is just one of the possible cartel reactions to the entry of a new firm. Therefore, we compare our base scenario of immediate accommodation, which is also the scenario that is considered by Vasconcelos (2008), with three alternative reactions to entry: discontinuance of collusion, entry deterrence and gradual accommodation.

Firstly, we consider that the collusive agreement is discontinued when entry occurs. In this scenario, relative to immediate accommodation, post-entry profits are lower and, as a result, entry occurs later. If the entry delay is sufficiently high, the incumbents prefer to discontinue collusion than to immediately accommodate the entrant. We also conclude that this reaction to entry makes the collusive agreement easier to sustain because then there is only collusion before entry. Notice, however, that collusion before entry becomes slightly more difficult to sustain as a result of the decrease of post-entry profits.

Alternatively, the cartel may respond to entry by implementing the following predatory strategy: if entry occurs, the incumbents start a punishment process, as if one of them had defected. As a result, if the new firm effectively entered the market, the discounted value of the flow of its profits would be null (or equal to the security level, which is assumed to be lower than the entry cost). Therefore, entry would not be profitable. This is the best scenario for the incumbents because the severity of this threat allows them to effectively deter entry. For the same reason, this is the worst scenario for the potential entrant.⁵ An interesting aspect of this scenario is that incumbents effectively deter entry at no cost and charge the monopoly price in all periods. Of course, it is necessary that the incumbents coordinate on this extreme form of punishment.

Finally, we analyze the sustainability of a collusive agreement that gradually accommodates the entrant (Friedman and Thisse 1994). In this scenario, firms maximize their joint profits in all periods, but the entrant is not treated as a full partner immediately after entering the market. In the first period, the profit of the entrant can be as low as the Cournot profit. Then, as time passes, the entrant is gradually accommodated in the collusive agreement, in the sense that

⁵ There is some evidence of this kind of predatory practice in the real world. For example, Morton (1997) reported that when a new firm tried to enter the British shipping market in the late nineteenth century, the colluding incumbents often started a fierce price war, typically when the entrant had low financial resources, little experience and no customer base.

its share of cartel profits increases until reaching a fraction not higher than one third of the monopoly profit. When compared to immediate accommodation, gradual accommodation is preferred by the incumbents because: (i) each incumbent receives a greater share of the monopoly profit during the accommodation phase; and (ii) entry is delayed. For the same reasons, the entrant prefers immediate accommodation. As a result, the entrant has more incentives to disrupt the collusive agreement than the incumbents. This imbalance implies that collusion is less likely to be sustained when accommodation is gradual than when it is immediate.

In terms of social welfare, collusion is detrimental relative to competition, independent of the reaction of the cartel to entry. Naturally, it is less so if the agreement is discontinued following entry. The remaining collusive scenarios are equivalent in terms of output and prices, as firms produce the monopoly output in every period. Still, entry deterrence is slightly preferred to accommodation because the entry cost is not incurred. Gradual accommodation is socially preferred to immediate accommodation because the entry cost is incurred later in time.

The finding that collusion is more difficult to sustain after entry than before entry extends to a scenario in which firms set prices instead of quantities. In such a scenario, collusion is even more difficult to sustain both before and after entry because the one-shot gain from deviating is higher, while the continuation value after a deviation is the same. We remark that, in the case of price-setting firms, permanent reversion to competition is an optimal penal code, in the sense that it also implies a null continuation value.

We also demonstrate that the same conclusion (that collusion is more difficult to sustain after entry than before entry) is robust to the consideration of any number of incumbents and any number of entrants. However, in our environment, the greater the total number of firms (independent of whether they are incumbents or entrants), the harder it is to sustain collusion.

The contribution of Capuano (2002) is closely related to our study, since he also examined the sustainability of collusion when firms use stick-and-carrot strategies to punish deviations. There are, however, important differences worth mentioning. Firstly, Capuano (2002) considered that the incumbents immediately accommodate the entrant in their collusive agreement, while we also study alternative cartel reactions to entry and compare them in terms of surplus for firms and consumers. Secondly, we analyze the willingness of the incumbents to establish the collusive agreement by comparing their discounted flow of profits under collusion and competition. Thirdly, Capuano (2002) assumed that entry occurs when the discounted value of the profits of the entrant becomes positive, while we consider that entry occurs when the discounted value of the entrant's

profits is maximal.⁶ Fourthly, Capuano (2002) did not consider that, above a certain threshold level of output, the price is zero.⁷ This apparently insignificant technical point is a key issue in the construction of the penal code. Finally, we study the sustainability of collusion with several entrants, while Capuano (2002) restricted his analysis to the case of one single entrant.

The remainder of the paper is organized as follows. Section 2 presents the assumptions of the model and derives the optimal entry period. Section 3 describes the collusive agreement in the scenario of immediate accommodation and analyzes its sustainability before and after entry. Section 4 studies alternative reactions to entry that may delay or even deter the entry of a new firm. Section 5 studies two extensions of the baseline model: the case in which firms set prices instead of quantities; and the case in which there is an arbitrary number of incumbents and entrants. Finally, Section 6 summarizes the main conclusions of the paper. Appendix A extends the description of the punishment strategy to the case in which the security level is strictly positive. Appendix B contains most of the proofs.

2 Model

Consider an industry with two incumbents (firms 1 and 2) and a potential entrant (firm 3) that sell a homogeneous product in an infinitely repeated setting. In each period, the active firms simultaneously choose the quantities to produce. Production costs are null. Market demand in period $t \in \{0, 1, 2, \dots\}$ is given by⁸:

6 Our assumption is appropriate to study markets in which there is a single potential entrant in the market, while the assumption made by Capuano (2002) is appropriate to study markets in which there are several potential entrants, but only one can effectively enter the market. Our assumption is more realistic if entry requires ownership of specific assets (e.g. a source of fresh water is necessary to enter the bottled water industry), knowledge of a specific technology (e.g. knowing how to produce high-resolution screens is necessary to enter the tablet industry), or when there are important economies of scope with related industries (e.g. between providing internet, cable tv and telephone services to households).

7 If, for example, the inverse demand function is $P(Q) = 1 - Q$, we consider that $P(Q) = 0$ for $Q \geq 1$.

8 Consideration of a more general demand function, $Q_t = (a - bp_t)\mu^t$, would have no impact on our results. Single-period profits in all market regimes would be multiplied by the same constant (a^2/b). Therefore, the conditions for collusion to be sustainable would remain unaltered. In a recent contribution, Correia-da-Silva, Pinho, and Vasconcelos (2014) consider a general demand function and show that the conditions for the sustainability of collusion are

$$Q_t = (1 - p_t)\mu^t,$$

where $\mu > 1$ is the demand growth rate and p_t is the price in period t . We assume “free disposal”. Therefore, the inverse demand function is

$$p_t = \begin{cases} 1 - Q_t\mu^{-t} & \text{if } Q_t \leq \mu^t \\ 0 & \text{if } Q_t > \mu^t. \end{cases}$$

To enter the market, firm 3 has to incur a setup cost, $K > 0$. Before entry, the timing of each period’s interaction is the following⁹:

- 1st: firm 3 decides whether or not to enter the market in the current period;
- 2nd: the active firms simultaneously choose their output levels.

After entry, the single-period game is reduced to the second stage. The objective of each firm is to maximize the discounted value of its flow of profits: $\sum_{t=0}^{+\infty} \delta^t \pi_t$, where $\delta \in (0, 1)$ is the common discount factor. For this sum to be finite, we make the following assumption.

Assumption 1. Demand does not grow too fast: $\mu\delta < 1$.¹⁰

When there are $n \in \{2, 3\}$ firms engaging in Cournot competition, the output and profit of each firm, in period t , are, respectively:

$$q_t^{cn} = \frac{1}{1+n}\mu^t \quad \text{and} \quad \pi_t^{cn} = \frac{1}{(1+n)^2}\mu^t.$$

If firms collude to maximize their joint profit, the individual output and profit are

$$q_t^{mn} = \frac{1}{2n}\mu^t \quad \text{and} \quad \pi_t^{mn} = \frac{1}{4n}\mu^t. \quad [1]$$

similar, as long as market growth increases all profits by the same proportion (collusive and deviation profits, with two and three firms).

9 Our framework has some elements in common with the literature on contestable markets (small number of firms, possibility of entry, symmetric production costs), first described by Baumol (1982). However, there is a crucial difference in the timing of the interaction. In the theory of contestable markets: the incumbents set quantities, and then the entrant decides whether to enter and what quantity to produce. In our model, however, the entrant decides whether to enter or not, and then the active firms set quantities. Here, the fact that the incumbents can immediately react to entry by adjusting their output prevents the hit-and-run strategy that usually characterizes contestable markets.

10 Expressing the discount factor as $\delta = \frac{1}{1+r}$, with r being the interest rate, Assumption 1 can be written as $\mu - 1 < r$. As we shall see, in this model, the long-term growth rate of profits is equal to $\mu - 1$. Hence, the value of the firm would be infinite unless $\mu - 1 < r$.

In case of a unilateral deviation, the output and the profit of the defector are

$$q_t^{dn} = \frac{n+1}{4n} \mu^t \quad \text{and} \quad \pi_t^{dn} = \left(\frac{n+1}{4n} \right)^2 \mu^t. \quad [2]$$

If firm 3 enters the market in period T and obtains a profit $\pi_t = \Pi \mu^t$ in all the following periods, $t \geq T$, the discounted value of its profits is

$$V_3(T) = \sum_{t=T}^{+\infty} \delta^t \Pi \mu^t - \delta^T K = \frac{(\mu\delta)^T}{1-\mu\delta} \Pi - \delta^T K.$$

When deciding whether to enter the market or not, firm 3 faces the following trade-off: on the one hand, entering early allows the firm to start receiving profits sooner; on the other hand, by delaying entry, the firm decreases the (discounted) entry cost.¹¹ Entry occurs when $V_3(T)$ is maximal.

Lemma 1. *If firm 3 expects its profit in each period t to be given by $\pi_t = \Pi \mu^t$, it enters the market at:*

$$T = \left\lceil \frac{\ln \left[\frac{K(1-\delta)}{\Pi} \right]}{\ln(\mu)} \right\rceil, \quad [3]$$

where $\lceil x \rceil$ denotes the smallest integer that is greater than or equal to x .

Proof. See Appendix B.

To calculate the optimal entry period for the case in which the entrant expects to collude with the incumbents, T^m , simply set $\Pi = \frac{1}{12}$ in eq. [3]. Similarly, to calculate the optimal entry period for the scenario in which the entrant expects to engage in Cournot competition, T^c , set $\Pi = \frac{1}{16}$ in eq. [3].

The following assumption guarantees that $T^m \geq 1$, which means that the incumbents are alone in the market at $t = 0$ (even if the entrant expects to collude after entry).¹²

Assumption 2. *The entry cost is not too small: $K > \frac{1}{12(1-\delta)}$.*

¹¹ Alternatively, we can examine this trade-off with the following current value reasoning. While the entry cost remains constant in time, the value of the prospective flow of profits increases. Due to market growth, the later the entry occurs, the faster the break-even is. The current value of the investment (difference between the value of the prospective flow of profits and the entry cost) is increasing over time, but at a decreasing rate. Entry occurs when this increase becomes insufficient to compensate the time-discount.

¹² It is straightforward that $V_3(1) > V_3(0)$ if and only if $\frac{K(1-\delta)}{\Pi} > 1$. Since $\pi_t^{m3} = \frac{1}{12} \mu^t$, replace $\Pi = \frac{1}{12}$ to obtain $K > \frac{1}{12(1-\delta)}$.

3 Collusion with entry accommodation

In this section, we investigate the sustainability of a collusive agreement with an optimal penal code, assuming that the entrant is included in the cartel immediately after entering the market. Firms are assumed to maximize their joint profits and inflict the harshest possible punishment on deviators.

3.1 Collusive agreement

At the beginning of period $t = 0$, the incumbents establish the following collusive agreement, which extends to firm 3 after it enters the market. In each period:

- Produce the quantities that maximize their joint profit, if all firms adhered to the agreement in the previous period.
- Start a severe punishment, which reduces the continuation value to zero, if the agreement was violated in the previous period.

Incentives to comply with a collusive agreement naturally depend on the strength of the punishment. We focus on the harshest possible punishment, i. e. the one that reduces to zero the continuation value of a firm that deviates (firms can always guarantee a payoff of zero by producing zero in every period). If firms had the option of exiting the market and receiving a residual value $E > 0$, the security level would be E . To avoid unnecessary complications, we restrict the exposition to the case in which the security level is zero.¹³

3.2 Punishment strategies

In this section, we describe some punishment processes that induce a zero continuation value after a deviation.¹⁴ In Section 3.3, we show that, despite

13 It is straightforward to adapt the penal code for the case in which the security level is strictly positive (see Appendix A).

14 In our model, the only way to punish a deviator is by increasing output in order to decrease the market price. In the real world, there are other forms of punishing deviations from the collusive agreement. According to Ayres (1987), cartels usually adopt one of the following punishment strategies: (i) sharp price cuts; (ii) reduction of the defectors' demand, namely through the offer of higher quality products, changes in the advertising level or investment in capacity; (iii) increase in the production costs of the deviator, namely by establishing exclusive dealing contracts or promoting legislation that harms the rival (Salop and Scheffman 1983); (iv) invasion of the territory of the defector, in cartels that divide the market geographically; (v) refusal to sell to the deviator, if it has insufficient capacity to meet its demand, or refusal to buy the excess capacity of the deviator. In future research, it would be interesting to analyze these alternative punishment mechanisms.

being very harsh, these punishments are credible as firms do not gain by unilaterally deviating.

3.2.1 Stick-and-carrot punishment with observable dissipative costs

If firms have access to some kind of unproductive activities that are observable (e.g. dissipative advertising, donations to charity or destruction of equipment),¹⁵ it is possible to construct an optimal penal code with a single punishment period, or with any desired number of punishment periods.

In the case of a single punishment period, the penal code could be as follows:

- If a deviation occurs in period t , then, in period $t + 1$, the $n \in \{2, 3\}$ active firms produce quantities that completely satiate the market (implying that the price is zero), even if one of the firms deviates and produces nothing:

$$q_{t+1}^{pn} = \frac{\mu^{t+1}}{n-1};$$

and spend an amount in dissipative advertising, A_{t+1} (calculated in Appendix A), which implies a zero continuation value.

- If no firm deviates from the punishment at $t + 1$, firms return to collusion in period $t + 2$. Otherwise, the punishment is restarted.

3.2.2 Stick-and-carrot punishment with side-payments

In a recent empirical study on the duration of international cartels, Levenstein and Suslow (2011) found that “one-third of the cartels compensate members when realised sales differ from proposed allocations; [...] cartels that retaliate in response to deviations are significantly more likely to break up.”¹⁶

¹⁵ The possibility of incurring in observable dissipative costs is frequently considered in signaling models (Milgrom and Roberts 1986; Bagwell and Ramey 1994; Lariviere and Padmanabhan 1997; Bernheim and Redding 2001; Hertzendorf and Overgaard 2001; and Linnemer 2002). Authors sometimes refer to these activities as “money burning”.

¹⁶ Levenstein (1997) also concluded that defectors from the international price-fixing agreement in the bromine industry were usually punished through side-payments, instead of price wars. Along these lines, Harrington (2006, 43) highlighted that: “According to the theory of collusive pricing, punishment not compensation is critical. However, these case studies reveal an importance attached to compensation.”

In the context of our model, instead of requiring that the deviator makes observable and unproductive expenses, the other cartel members could demand a side-payment.¹⁷ The penal code would be exactly as stated above, except for the fact that the deviator would transfer the amount A_{t+1} to the remaining firms that were active in period t (divided in equal parts), instead of spending it in unproductive activities.

In our environment, the incentives to comply with the collusive agreement would remain the same because firms are indifferent between “burning money” and transferring it to rivals. However, since side-payments are usually perceived as a “smoking gun”, the expected cost of cartel indictment (fines, or a bad image) should be taken into account.

3.2.3 Stick-and-carrot punishment with production costs

Suppose that the marginal production cost is strictly positive and constant in output, $c > 0$. In this case, the following penal code would induce a zero continuation value after a deviation:¹⁸

- In period $t + 1$, the firms that complied with the collusive agreement in period t produce quantities that completely satiate the market (driving the price to zero), even if the deviator produces nothing:¹⁹

$$q_{t+1}^{pn_{t+1}} = \frac{1}{n_{t+1} - 1} \mu^{t+1}.$$

- In period $t + 1$, the firm that deviated in period t produces the quantity that originates a loss of the same magnitude as the discounted value of future profits (so that its value of being in the market becomes null):

17 Monetary transfers are far from being the only way to implement side-payments among firms. As pointed out by Levenstein and Suslow (2006), in the citric acid cartel that operated from July 1991 to June 1995 in the U.S.A.: “A company selling more than its quota was required the next year to purchase citric acid from a cartel member that was under quota.” Alternatively, the firm could remain inactive for a certain period of time and allow the rivals to enjoy a privileged position. See also Harrington (2006).

18 If production costs are null, firms may still induce a null continuation value after a deviation by permanently reverting to a zero profit equilibrium in which it is also the case that firms produce quantities that completely satiate the market (implying that the price is zero), even if one of them deviates and produces nothing. This would be a grim trigger punishment strategy, with the particularity that firms permanently revert to the worst equilibrium of the stage game. The robustness of such an equilibrium is, however, questionable, since a small fixed cost would be sufficient to eliminate it.

19 The expressions for $q_{t+1}^{pn_{t+1}}$ and $q_{t+1}^{pdn_{t+1}}$ are derived in Appendix B, after the proof of Lemma 1.

$$q_{t+1}^{pdn_{t+1}} = \frac{1}{c} \sum_{s=t+2}^{+\infty} \delta^{s-(t+1)} \pi_s^{mn_s}(c),$$

where $\pi_s^{mn_s}(c)$ is the collusive profit if firms have unit costs equal to c , and n_s denotes the number of active firms in period s .

- If no firm deviates from the punishment at $t + 1$, the collusive agreement is restored in period $t + 2$. Otherwise, the punishment is restarted.

This penal code is credible as long as unit costs are sufficiently low for the deviator to produce more than the other firms ($q_{t+1}^{pd} \geq q_{t+1}^{pn_{t+1}}$). Otherwise, in the punishment period, the firms that had complied with the collusive agreement would have a greater loss than the deviator (and would not, therefore, be able to recover these losses in the future periods). In that case, these firms would prefer to produce zero forever instead of punishing the deviator.

3.3 Credibility of the punishment

We must guarantee that firms do not deviate in the punishment phase. Firms adhere to the punishment if and only if the following incentive compatibility constraint (henceforth, ICC) is satisfied:

$$V_{t+1}^{pn} \geq \pi_{t+1}^{dpn} + \delta V_{t+2}^{pn}, \quad \forall t \geq 0, \quad [4]$$

where π_{t+1}^{dpn} is the maximum profit that a firm can earn by deviating in the punishment period, while the remainder $n - 1$ firms play the punishment strategy; and V_{t+1}^{pn} is the discounted value of profits along the punishment path.

By construction, the optimal penal code gives a zero continuation value after a deviation ($V_{t+1}^{pn} = V_{t+2}^{pn} = 0$). This allows us to write the ICC [4] as:

$$\pi_{t+1}^{dpn} \leq 0, \quad \forall t \geq 0.$$

This condition is satisfied in all forms of punishment that we have described because if $n - 1$ firms adhere to the punishment in period $t + 1$, they produce an output which ensures that the resulting price is zero, regardless of the output of the deviator. Therefore, it is impossible for a deviator to attain a strictly positive profit ($\pi_{t+1}^{dpn} = 0$).

Suppose that firms could exit the market and receive a strictly positive residual value, $E > 0$ (which may result, for example, from the sale of machinery, plants, copyrights or other kinds of licenses). In this case, their security level would coincide with this residual value, i.e. $V_{t+1}^{pn} = V_{t+2}^{pn} = E$, and the ICC

would become $\pi_{t+1}^{dpn} \leq (1 - \delta)E$, which is even more easily satisfied. In the opposite case, in which exit is costly (e.g. due to legal requirements such as compensation to employees, clients or suppliers, or from broken contracts), for the security level to become negative, it would also be necessary to introduce fixed production costs (otherwise, instead of incurring the cost of exiting, firms would produce zero forever at no cost).

We will now study whether firms, knowing that the penal code is credible, have incentives to deviate from the collusive agreement.

3.4 Sustainability of collusion

The collusive agreement is sustainable if firms have no incentives to deviate either before or after entry (which occurs at T^m).

Collusion is sustainable after entry if and only if the following incentive compatibility constraint holds for all $t \geq T^m$:

$$\sum_{s=t}^{+\infty} \delta^{s-t} \pi_s^{m3} \geq \pi_t^{d3}, \quad [5]$$

where π_t^{d3} is the deviation profit. The continuation value after a deviation is omitted from the ICC because, under the optimal penal code, it is null.

Proposition 1. *Collusion is sustainable after entry if and only if: $\mu\delta \geq \frac{1}{4}$.*

Proof. See Appendix B.

Before entry, the collusive agreement is sustainable if and only if the following incentive compatibility constraint holds for all $t \leq T^m - 1$:

$$\sum_{s=t}^{T^m-1} \delta^{s-t} \pi_s^{m2} + \sum_{s=T^m}^{+\infty} \delta^{s-t} \pi_s^{m3} \geq \pi_t^{d2}, \quad [6]$$

where π_t^{d2} is the deviation profit before entry (the continuation value after a deviation is, again, omitted because it is null).

Lemma 2. *Let $\mu\delta \geq \frac{1}{9}$. If the ICC [6] is satisfied in the period that immediately precedes entry, $t = T^m - 1$, it is satisfied in all previous periods, $t \in \{0, 1, \dots, T^m - 2\}$.*

Proof. See Appendix B.

Considering the ICC [6] at $t = T^m - 1$, we obtain the critical discount factor above which collusion is sustainable before entry (under the hypothesis that it is sustainable after entry).

Proposition 2. *The ICC [6] is satisfied for all $t \leq T^m - 1$ if and only if: $\mu\delta \geq \frac{3}{19}$.*

Proof. See Appendix B.

Combining Propositions 1 and 2, we conclude that collusion is less sustainable after entry than before. The reason relates to the standard result that collusion is less sustainable if the number of firms is higher (Ivaldi et al. 2007).

Our conclusion is the opposite of the one obtained by Vasconcelos (2008).²⁰ In his model, after a deviation from the collusive agreement, firms permanently revert to Cournot equilibrium (grim trigger strategies). In that setting, a deviation effectively delays entry, and this delay is profitable because Cournot profits with two firms are greater than collusion profits with three firms. As a result, the incumbents have an additional incentive to deviate before entry. In our model, however, the incumbents cannot benefit from delaying entry through a deviation from the collusive agreement because the ensuing punishment absorbs all the continuation value. Firms deviate if and only if the single-period deviation profit exceeds the value of colluding forever.

Corollary 1. *Collusion is sustainable (before and after entry) if and only if: $\mu\delta \geq \frac{1}{4}$.*

4 Can the incumbents profitably delay entry?

In this section, we consider four alternative cartel reactions to entry and compare them in terms of incumbents' profits, entrant's profits, consumer surplus and total surplus.²¹

²⁰ Brandão, Pinho, and Vasconcelos (2014) also studied the sustainability of collusion in markets with growing demand, assuming grim trigger strategies but allowing for cost asymmetries between firms. In their model, collusion may be more sustainable before entry than after, depending on the entry delay that would result from a deviation before entry.

²¹ It would be interesting to study a more comprehensive game, in which the cartel reaction to entry is determined endogenously. One difficulty of such an analysis would be dealing with the possible existence of multiple equilibria. Harrington (1989, section 7) suggested a possible way out: to eliminate the accommodation strategy, on the one hand; and to assume that incumbents punish entry (external defection) at least as severely as deviations from the collusive agreement (internal defection), on the other hand.

- (i) no collusion;
- (ii) collusion before entry and competition after entry;
- (iii) collusion before entry and punishment if and when entry occurs;
- (iv) gradual accommodation of the entrant in the collusive agreement.

The motivation for studying the case in which there is no collusion is obvious, as it allows us to understand the implications of collusion. It is also the scenario that arises if the discount factor is not sufficiently high for collusion to be sustainable.

The remaining alternative scenarios correspond to plausible cartel reactions to entry, previously considered and compared by Harrington (1989), Friedman and Thisse (1994) and Vasconcelos (2004). See also the discussion by Harrington (1991b).

We could have also considered a scenario of competition before entry and collusion after entry (possibly in response to fiercer competition). The condition for sustainability of collusion would be the same as in the baseline scenario of immediate accommodation. However, the incumbents would be worse off than in the baseline scenario because they would have lower profits before entry and the same level of profits after entry.

4.1 No collusion

If there is no collusion, the present value of the profits of each incumbent is

$$V_i^c = \sum_{s=0}^{T^c-1} \delta^s \pi_s^{c2} + \sum_{s=T^c}^{+\infty} \delta^s \pi_s^{c3},$$

where T^c denotes the entry period of firm 3.

Under collusion with immediate accommodation, this value is given by:

$$V_i^m = \sum_{s=0}^{T^m-1} \delta^s \pi_s^{m2} + \sum_{s=T^m}^{+\infty} \delta^s \pi_s^{m3}.$$

The comparison between the present value of profits under competition, V_i^c , and under collusion, V_i^m , is not immediate because entry under competition may occur later than under collusion (i.e. $T^c \geq T^m$).

Proposition 3. *The incumbents are better off under competition than under collusion with entry accommodation if and only if: $2 - 6(\mu\delta)^{T^m} + 7(\mu\delta)^{T^c} < 0$.*

Proof. See Appendix B.

It may seem counterintuitive that the incumbent firms prefer not to collude. Notice, however, that: (i) a collusive agreement with immediate accommodation of the entrant induces an earlier entry (at T^m rather than at T^c); and (ii) each incumbent profits more under competition between the two incumbents than under a collusive agreement that includes the three firms. Of course, the entrant surely prefers the collusive scenario.

The parameter values for which the incumbents prefer competition to collusion with accommodation of the entrant are represented in Figure 1. In the painted area, the collusive agreement with entry accommodation would be sustainable, since $\mu\delta \in (\frac{1}{4}, 1)$, but the incumbents prefer to compete rather than to collude and accommodate the entrant.

Figure 1: No collusion versus collusion with entry accommodation ($K = \frac{1}{3}$). Collusion is sustainable to the right of the solid line, $\mu\delta \geq \frac{1}{4}$. Assumption 1 holds to the left of the dashed line, $\mu\delta < 1$. Assumption 2 is satisfied to the left of the dotted line, $T^m \geq 1$. In the painted region, the incumbents prefer no collusion

It should be clear that this result does not depend on the penal code. It also holds under grim trigger punishment strategies because the present values of profits under collusion and competition do not depend on the punishment strategy.

4.2 Discontinuance of collusion

Suppose now that the incumbents combine to discontinue the collusive agreement after firm 3 enters the market.²² More precisely, the collusive agreement established by the incumbents is the following:

- Before entry, produce the quantities that maximize joint profits if the agreement was honored in the previous period.
- Before entry, engage in a punishment that absorbs all the continuation value if there was a defection in the previous period.²³
- After entry, switch to stage Nash-Cournot equilibrium.

The advantage of this reaction to entry is that, by decreasing post-entry profits, it delays entry from period T^m to period T^c . The entry period is the same as if there was no collusion because the post-entry profits of the entrant are the same in both scenarios.

Before entry, the incumbents abide by the collusive agreement if and only if:

$$\sum_{s=t}^{T^c-1} \delta^{s-t} \pi_s^{m2} + \sum_{s=T^c}^{+\infty} \delta^{s-t} \pi_s^{c3} \geq \pi_t^{d2}, \quad \forall t < T^c. \quad [7]$$

Lemma 3. *Let $\mu\delta \geq \frac{1}{9}$. If the ICC [7] is satisfied in the period that immediately precedes entry, $t = T^c - 1$, it is satisfied in all previous periods, $t \in \{0, 1, \dots, T^c - 2\}$.*

Proof. Follow the same steps of the proof of Lemma 2.

Substituting $t = T^c - 1$ and the expressions for profits in the ICC [7], we obtain the following result.

Proposition 4. *The collusive agreement with discontinuance of collusion after entry is sustainable if and only if: $\mu\delta \geq \frac{1}{5}$.*

Proof. See Appendix B.

²² With grim trigger strategies, this type of collusive agreement would never be sustainable. The incumbents would surely deviate in the period that immediately precedes entry, since reversion to competition would occur in the entry period, independently of their previous decisions.

²³ The construction of this punishment is analogous to that of the collusive agreement with entry accommodation, except that T^c must be considered instead of T^m .

To investigate whether the incumbents prefer discontinuance of collusion to immediate accommodation, we compare the present value of profits in the two scenarios.

Proposition 5. *The incumbents are better off discontinuing the collusive agreement when entry occurs than accommodating the entrant if and only if: $(\mu\delta)^{T^c - T^m} < \frac{2}{3}$.*

Proof. See Appendix B.

The parameter values for which the incumbents prefer discontinuance of collusion to accommodation of the entrant are represented in Figure 2. In the painted area, they prefer discontinuance of collusion (collusion is sustainable in both scenarios), i.e. $(\mu\delta)^{T^c - T^m} < \frac{2}{3}$.

Figure 2: Discontinuance of collusion versus entry accommodation ($K = 1$). Collusion is sustainable to the right of the solid line, $\mu\delta \geq \frac{1}{6}$. Assumption 1 holds to the left of the dashed line, $\mu\delta < 1$. Assumption 2 is satisfied to the left of the dotted line, $T^m \geq 1$. In the painted region, the incumbents prefer to break the cartel in response to entry rather than to accommodate entry

The complex pattern in Figure 2 results from the discrete nature of time, which implies that a slight change in parameter values may lead to a discrete change of the entry period. We can distinguish two types of lines: the oblique lines, corresponding to jumps in T^c or T^m ; and the almost vertical lines, corresponding to the condition $(\mu\delta)^{T^c-T^m} = \frac{2}{3}$, for different values of the entry delay, $T^c - T^m$.

The relative desirability of accommodating entry or discontinuing the collusive agreement after entry depends on the type of agent (incumbent, entrant or consumer), as will be discussed in Section 4.5.

4.3 Entry deterrence

Suppose now that the incumbents can commit to the following agreement, which treats entrants as deviators (Harrington 1989; Stenbacka 1990). In period t :

- Produce quantities that maximize the industry profit, if the agreement was honored in the previous period and firm 3 did not enter the market.
- Start a punishment that reduces the continuation value to zero, if one incumbent deviated in the previous period or if firm 3 entered the market.

Since the continuation value after entry is zero, entering the market is not profitable (due to the entry cost, $K > 0$). As a result, the incumbents effectively deter entry.²⁴

Proposition 6. *The collusive agreement with entry deterrence is sustainable if and only if: $\mu\delta \geq \frac{1}{9}$.*

Proof. See Appendix B.

This is the best collusive scenario for the incumbents, as each incumbent receives half of the monopoly profit in all periods. On the other hand, this is the worst scenario for the entrant. Of course, if the incumbents are not able to commit to this punishment, the entrant will anticipate that the incumbents will not punish entry (as they receive a higher profit by sharing the market than by punishing entry), and, therefore, will enter.

²⁴ Salop (1979) suggested an alternative way for the incumbents to deter entry. In his setting, the incumbents can make expenses (e.g. in innovation, advertising) that must be matched by the entrant in order to survive. If the value of this expenditure is greater than the value of entering the market, the potential entrant prefers to remain inactive.

Predatory behavior toward new entrants is not rare. In his study of the cartel formed in the late nineteenth century by British shipping firms, Morton (1997) reported that when a new firm entered the market, the cartel either started a price war or admitted the entrant to the cartel without conflicts. Morton (1997) concluded that the reaction to entry depended on the characteristics of the entrant. Firms with low financial resources, little experience and without an established customer base were more likely to be preyed upon. Strong competitors were more likely to be accepted in the cartel.²⁵

Another case in point is the pre-insulated pipe cartel in the 1990s, in which the firm ABB took the bulk of the costs of running the cartel. In particular, when Powerpipe, a firm outside the cartel, tried to expand its activities, ABB used large resources to try to eliminate the maverick from the market. This predatory activity was multidimensional and costly, involving, for instance, a systematic campaign of luring away key employees of Powerpipe, including its then managing director (for details, see Ganslandt, Persson, and Vasconcelos (2012)).

4.4 Gradual accommodation

Based on the proposal of Friedman and Thisse (1994), we now consider the case in which the entrant is gradually accommodated in the collusive agreement. Firms maximize the industry profit in all periods (before and after entry), but there is an adjustment phase during which the entrant receives a smaller share of the industry profit than the incumbents.

Let q_{it}^g and π_{it}^g , respectively, denote the output and profit of firm $i \in \{1, 2, 3\}$ in period t . We assume that the entrant never profits less than under competition nor more than the incumbents, i.e. $\pi_{3t}^g \in [\pi_t^{c3}, \pi_t^{m3}]$. Moreover, we assume that the entrant's share of the cartel profit is non-decreasing along the accommodation phase. Formally, letting σ_s denote the entrant's share of the cartel profit s periods after entering the market, we assume that σ_s is non-decreasing in s and that

$$\sigma_s \in \left[\frac{\pi_s^{c3}}{3\pi_s^{m3}}, \frac{1}{3} \right].$$

²⁵ The "long purse" or "deep pocket" theory of predation may justify this cartel reaction to entry (Benoit 1984). If the incumbents are financially less constrained than the entrant (they have more assets and easier access to credit), they may deter entry by threatening to start a price war in response to entry. This would entail losses to all firms; however, the entrant would not be able to sustain losses for as long as the incumbents. If bankruptcy is anticipated by potential entrants, entry does not occur. Notice, however, that imperfections in the capital market play a key role in this theory.

Let T^g denote the optimal entry period.²⁶ The assumption that $\pi_{3t}^g \in [\pi_t^{c3}, \pi_t^{m3}]$, $\forall t$, implies that $T^m \leq T^g \leq T^c$.

After entry, each incumbent receives half of the difference between the monopoly profit and the profit of the entrant:

$$\pi_{it}^g = \frac{\mu^t}{8} - \frac{\pi_{3t}^g}{2}, \quad \forall t \geq T^g.$$

Along the collusive path, the profit of firm $j \in \{1, 2, 3\}$, producing q_{jt}^g units of output, while the other two firms (jointly) produce Q_{-jt}^g , is

$$\pi_{jt}^g = \left(1 - 2Q_{-jt}^g \mu^{-t}\right) \frac{\mu^t}{4},$$

The profit of firm j if it unilaterally deviates from the collusive agreement in period t , while the other firms keep (jointly) producing Q_{-jt}^g , is

$$\pi_{jt}^{dg} = \left(1 - Q_{-jt}^g \mu^{-t}\right)^2 \frac{\mu^t}{4}.$$

As in the previous sections, we consider that firms adopt an optimal penal code to punish deviations, which reduces the continuation value after a deviation to zero.

Since the entrant's share of the cartel profit is increasing in time, it is in the entry period that the ratio between the one-shot gain from deviating (difference between the deviation profit and the collusive profit) and the continuation value under collusion is the highest.

Proposition 7. *The entrant is the most tempted to deviate in the entry period, T^g .*

Proof. See Appendix B.

In contrast, since the incumbents' share of the cartel profit is decreasing, their incentives to disrupt the collusive agreement (after entry) increase as time passes.

Proposition 8. *After entry, the incentives for the incumbents to deviate from the collusive agreement are non-decreasing over time.*

Proof. See Appendix B.

²⁶ Given a specification for the evolution over time of the entrant's share of profits, $\{\sigma_s\}_0^{+\infty}$, the expression for T^g can be obtained by following the same steps as in the case of immediate accommodation.

The assumption that the entrant's share of the cartel profit is not greater than the share of each incumbent implies that the entrant has more incentives to deviate than the incumbents.

Lemma 4. *In any period after entry, $t \geq T^g$, the incumbents have less incentives to deviate than the entrant.*

Proof. See Appendix B.

We conclude that the critical ICC for collusion to be sustainable after entry is the one for the entrant in the entry period, T^g .

Corollary 2. *Collusion is sustainable after entry if the entrant does not deviate in the entry period, T^g . This is the case if and only if: $\sum_{s=1}^{+\infty} (\mu\delta)^s \sigma_s \geq \frac{(1-\sigma_0)^2}{4}$.*

Finally, we need to analyze the incentives for the incumbents to collude before entry. They comply with the collusive agreement in period $T^g - \tau$, with $\tau \geq 1$, if and only if:

$$\sum_{s=T^g-\tau}^{T^g-1} \delta^{s-T^g+\tau} \pi_s^{m2} + \sum_{s=T^g}^{+\infty} \delta^{s-T^g+\tau} \pi_{is}^g \geq \pi_{T^g-\tau}^{d2}. \quad [8]$$

Lemma 5. *If the ICC [8] is satisfied in the period that immediately precedes entry, $T^g - 1$, it is satisfied in all previous periods, $T^g - \tau$, with $\tau \geq 1$.*

Proof. See Appendix B.

Thus, the necessary and sufficient condition for collusion to be sustainable before entry is the ICC [8] evaluated at $T^g - 1$, which can be rewritten as follows:

$$\frac{1}{8} \mu^{T^g-1} + \sum_{s=T^g}^{+\infty} \delta^{s-T^g+1} \pi_{is}^g \geq \frac{9}{64} \mu^{T^g-1} \Leftrightarrow \sum_{s=0}^{+\infty} (\mu\delta)^{s+1} (1 - \sigma_s) \geq \frac{1}{8}. \quad [9]$$

Proposition 9. *The ICC for the entrant to abide by the collusive agreement in the entry period, T^g , is the most difficult to satisfy.*

Proof. See Appendix B.

These results suggest that the increase over time of the profit share of the entrant hurts the sustainability of collusion relative to a situation in which the entrant's profit share is constant over time.

Consider an equivalent sharing of the discounted value of cartel profits using constant profit shares. More precisely, let $\sigma^* \equiv \frac{1}{1-\mu\delta} \sum_{s=0}^{+\infty} (\mu\delta)^s \sigma_s$ be the share of cartel profit that accrues to the entrant. By construction, with this constant profit-sharing rule, the discounted value of the entrant's and incumbent's profits is the same as with the sharing rule defined by the sequence $\{\sigma_s\}_0^{+\infty}$.

It follows that the entry period is not affected, since the mapping from the entry period to the entrant's discounted sum of profits is preserved. However, profit sharing in constant proportions has an advantage: collusion is easier to sustain. Instead of a situation in which only the entrant's ICC at T^g is binding, we have the entrant's ICCs equally binding at every period, but with a lower critical value of the adjusted discount factor.

In our environment, therefore, there seems to be no rationale for gradual accommodation of the entrant in the collusive agreement.

4.5 Welfare analysis

Let us now compare the different cartel reactions to entry in terms of: incumbents' profits, entrant's profits, consumer surplus and total surplus.

4.5.1 Incumbents' surplus

Entry deterrence is undoubtedly the scenario that gives the highest payoff to incumbents since they receive half of the monopoly profit in every period.

The incumbents prefer gradual to immediate accommodation ($V_i^g > V_i^a$) because: (i) gradual accommodation delays entry; and (ii) the share of incumbents in the industry profit is greater during the accommodation process than after entry.

The comparison between the scenarios of no collusion and discontinuance of collusion is also immediate ($V_i^{dc} > V_i^c$) since: (i) entry occurs in the same period; (ii) profits after entry are the same; but (iii) profits before entry are lower under competition.

Remark 1. *The preferences of the incumbents regarding the different scenarios satisfy the following partial ordering:*

$$V_i^a < V_i^g < V_i^p \quad \text{and} \quad V_i^c < V_i^{dc} < V_i^p.$$

All preference orderings compatible with this partial ordering occur for some values of the parameters. Figure 1 shows that, depending on the values of β , δ and K , we may have $V_i^a < V_i^c$ or $V_i^c < V_i^a$. Since immediate accommodation is a particular case of gradual accommodation, we may have $V_i^g < V_i^c$ or $V_i^c < V_i^g$. Figure 2 shows that, again depending on the values of β , δ and K , we may have $V_i^a < V_i^{dc}$ or $V_i^{dc} < V_i^a$.

4.5.2 Entrant's surplus

Obviously, the worst scenario for the entrant is the one in which the incumbents adopt a predatory behavior and deter entry ($V_e^p = 0$).

For the entrant, it is irrelevant whether the incumbents compete from the beginning of the game or discontinue collusion when entry occurs since entry takes place in the same period and the flow of the entrant's profits is the same ($V_e^c = V_e^{dc}$).

The reason for the entrant to prefer collusion with gradual accommodation to no collusion is easy to understand. Suppose that the entrant was forced to enter at T^c . Even in this case, collusion with gradual accommodation would be preferable because, in all periods $t > T^c$, profits would be greater than in the absence of collusion. The possibility of entering at $T^g \leq T^c$ only increases the comparative value of collusion with gradual accommodation ($V_e^g > V_e^c$).

It is straightforward that the entrant prefers to be accommodated in the collusive agreement immediately after entry rather than gradually ($V_e^a > V_e^g$).

Remark 2. *The preferences of the entrant regarding the different scenarios satisfy the following complete ordering:*

$$V_e^p < V_e^c = V_e^{dc} < V_e^g < V_e^a.$$

4.5.3 Consumers' surplus

We measure consumer welfare as the discounted sum of each period's consumer surplus:

$$CS = \sum_{t=0}^{+\infty} \delta^t \frac{Q_t(1-p_t)}{2} = \frac{1}{2} \sum_{t=0}^{+\infty} (\mu\delta)^t (1-p_t)^2.$$

Consumers are not directly affected by the timing of entry or by the cartel reaction to entry. Consumers only care about the total output and the resulting market price.

The competitive scenario is the best for consumers because output is the highest in all periods. For the same reason, consumers prefer discontinuance of collusion to entry accommodation, as there is competition after entry ($CS^a < CS^{dc}$). They are indifferent between collusion with entry deterrence or entry accommodation (immediate or gradual) because total output is at the monopoly level in all periods ($CS^p = CS^a = CS^g$).

Remark 3. *The preferences of consumers regarding the different scenarios satisfy the following complete ordering:*

$$CS^p = CS^g = CS^a < CS^{dc} < CS^c.$$

4.5.4 Social welfare

We define social welfare as the sum of consumers' surplus with the discounted sum of the industry profits. As production costs are null, social welfare is increasing in total output. In addition, *ceteris paribus*, the later the entry occurs (i.e. the lower the discounted value of the entry cost is), the higher the social welfare is.

It is clear that social welfare is higher if there is no collusion than if there is collusion with immediate accommodation because the output is higher in every period and the entry occurs later. Likewise, discontinuance of collusion is socially better than entry accommodation, but worse than competition ($W^a < W^{dc} < W^c$).

The output is the same under immediate accommodation, gradual accommodation and entry deterrence. However, social welfare is higher when accommodation is gradual than when it is immediate because the entry cost is incurred later; and it is even higher when entry is deterred because the entry cost is not even incurred ($W^a < W^g < W^p$).

The comparison between discontinuance of collusion and entry deterrence is not straightforward. On the one hand, total output is greater when collusion is discontinued. On the other hand, the entry cost is not incurred under entry deterrence. We find that the output effect more than compensates the entry cost effect.

Remark 4. *The different scenarios satisfy the following complete ordering in terms of social welfare:*

$$W^a < W^g < W^p < W^{dc} < W^c.$$

Proof. See Appendix B.

5 Extensions

In this section, we consider one variation and one extension of the baseline model. More precisely, we analyze the case in which firms set prices instead of quantities and allow for an arbitrary number of firms (incumbents and entrants).

5.1 Price-setting firms

Until now, we have considered that the decision variable of firms is the quantity to produce and sell in the market. Now, we will consider that firms set prices.

In this case, since firms produce homogeneous goods, Bertrand competition implies that all firms receive zero profits. Thus, one way of implementing a null continuation value after a deviation is through the permanent reversion to Bertrand competition after a deviation. Grim trigger strategies constitute, therefore, an optimal punishment.

With firms setting prices instead of quantities, collusion becomes harder to sustain. The reason is simple. The collusive profits and the continuation value after a deviation do not depend on whether firms choose prices or quantities. However, the one-shot deviation profit is higher when firms set prices since the deviator is able to receive the monopoly profit by slightly undercutting the collusive price.

For the sake of completeness, we derive the critical discount factor for collusion to be sustainable (before and after entry) when firms set prices.

Proposition 10. *If firms set prices, collusion is sustainable after entry if and only if: $\mu\delta \geq \frac{2}{3}$.*

Proof. See Appendix B.

Comparing the critical discount factor for collusion to be sustainable after entry when firms set quantities (Proposition 1) and when firms set prices (Proposition 10), it immediately follows that collusion is more difficult to sustain in the latter case.

Proposition 11. *If firms set prices, collusion is sustainable before entry if and only if: $\mu\delta \geq \frac{3}{5}$.*

Proof. See Appendix B.

We conclude that collusion is more difficult to sustain when firms set prices than when they set quantities, both before and after entry. Regardless of the firms' decision variable, the ICC for collusion to be sustainable after entry is more demanding than the ICC for collusion to be sustainable before entry.

5.2 Multiple entrants

The assumption that there is a single entrant may not be the most suitable to study a market that expands forever. We now study the sustainability of collusion when there are multiple entrants.²⁷ More precisely, we consider an industry with $N_0 \geq 2$ incumbent firms and $N - N_0$ entrants, focusing our attention on the scenario in which entrants are immediately accommodated in the collusive agreement.

As in the single-entrant case, the active firms produce quantities that maximize the industry profit as long as no firm defects; if some firm breaks the collusive agreement, a harsh punishment is started, which drives the continuation value of all active firms to zero. Such an optimal penal code can be constructed as in the case of a single entrant (see Section 3.2 and Appendix A).

For simplicity, we assume that all entrants incur the same entry cost, $K > 0$. This generates some indeterminacy in the order of entry, which is not relevant for the sustainability of collusion.²⁸ Whenever it is optimal for one firm to enter, but not for a second firm to enter simultaneously, all inactive firms will want to be that single entrant. We assume that there exists a simple coordinating device that randomly and equiprobably selects one of the inactive firms to become active.

We start by obtaining the (optimal) entry period of the last entrant. Since there are no more potential entrants in the market, the last entrant will start its activity when the discounted value of its profits, V_N , is maximal. Using expression [3], we obtain the optimal entry period of the last entrant under collusion:

²⁷ Vasconcelos (2008) briefly discussed the impacts of considering two entrants when firms use grim trigger strategies. Henry and Ponce (2011) also studied optimal entry of two firms. In their model, the entry cost results from the acquisition of an essential knowledge (without which a firm is not competitive). Entrants may buy it from the firm already established in the market, or undertake a costly imitation process. The authors found that, in equilibrium, the first entrant purchases the knowledge from the incumbent at the imitation cost, while the second entrant gets the knowledge for free (as a result of competition between the incumbent and the first entrant to sell the knowledge). Being aware of this, both entrants aim at being the second entrant. Hence, by allowing the first entrant to resale knowledge, the incumbent can effectively delay entry of both firms and enjoy a monopoly position for a given period of time.

²⁸ Assuming sufficiently asymmetric entry costs could solve this indeterminacy (firms with lower entry costs would enter first). However, this would entail additional notation and complications.

$$T_N^m = \left\lceil \frac{\ln[4NK(1-\delta)]}{\ln(\mu)} \right\rceil.$$

Let us now assume the role of the penultimate entrant and denote the discounted flow of its profits by $V_{N-1}(T)$ if it enters the market in period T (anticipating that firm N will enter at T_N^m). Firm $N-1$ should enter in the first period, $T < T_N^m$, that verifies the following condition: $V_{N-1}(T) > V_N(T_N^m)$; or (if there is no such period) at $T_{N-1}^m = T_N^m$. The reason is as follows: if the market value of the penultimate entrant was not higher than the market value of the last entrant, the penultimate entrant would prefer to enter in the next period. If it did not enter immediately after $V_{N-1}(T)$ became higher than $V_N(T_N^m)$, the last entrant would enter in its turn.

The entry period of firm $N-2$ is only slightly more difficult to obtain. After the entry of firm $N-2$, the remaining firms have an expected value given by the simple average between $V_{N-1}(T_{N-1}^m)$ and $V_N(T_N^m)$, which we denote by EV_{N-1} . The entry of firm $N-2$ will occur, therefore, in the first period, $T < T_{N-1}^m$, that verifies the following condition: $V_{N-2}(T) > EV_{N-1}$; or (if there is no such period) at $T_{N-2}^m = T_{N-1}^m$. The same logic applies to earlier entrants.

To study the sustainability of collusion with $N - N_0$ entrants, it is necessary to consider $N - N_0 - 1$ incentive compatibility conditions. We start by obtaining the critical discount factor for collusion to be sustainable after entry, i.e. in periods $t \geq T_N^m$:

$$\sum_{s=t}^{+\infty} \delta^{s-t} \pi_s^{mN} \geq \pi_s^{mN} \Leftrightarrow \frac{(\mu\delta)^t}{4N(1-\mu\delta)} \geq \frac{(N+1)^2}{16N^2} (\mu\delta)^t \Leftrightarrow \mu\delta \geq \left(\frac{N-1}{N+1}\right)^2. \quad [10]$$

As expected, the greater the number of entrants, the more difficult it is to sustain collusion after entry.

Consider now a period $t \in \{T_n^m, \dots, T_{n+1}^m - 1\}$, with $n \in \{N_0, \dots, N-1\}$ and $T_{N_0}^m = 0$. The n active firms abide by the collusive agreement in this period if and only if the following ICC is satisfied:

$$\sum_{s=t}^{T_{n+1}^m-1} \delta^{s-t} \pi_s^{m,n} + \sum_{s=T_{n+1}^m}^{T_{n+2}^m-1} \delta^{s-t} \pi_s^{m,n+1} + \dots + \sum_{s=T_{N-1}^m}^{T_N^m-1} \delta^{s-t} \pi_s^{m,N-1} + \sum_{s=T_N^m}^{+\infty} \delta^{s-t} \pi_s^{m,N} \geq \pi_t^{d,n}. \quad [11]$$

Lemma 6. If $\mu\delta \geq \left(\frac{n-1}{n+1}\right)^2$ and collusion is sustainable in period $T_{n+1}^m - 1$, it is sustainable $\forall t \in \{T_n^m, \dots, T_{n+1}^m - 1\}$.

Proof. See Appendix B.

Since the periods that immediately precede each entry are the critical moments for collusion to be sustainable before entry, we can replace $t = T_{n+1}^m - 1$ in the ICC [11] that applies to periods in which there are $n < N$ active firms. We obtain a single ICC for each value of $n \in \{N_0, \dots, N - 1\}$. Comparing these ICCs, we conclude that they become stricter as the number of active firms increases. Finally, we compare the ICC before the last entry with the ICC after entry to arrive at the following result.

Proposition 12. *If collusion is sustainable after entry, it is also sustainable before entry. Collusion is globally sustainable if and only if: $\mu\delta \geq \left(\frac{N-1}{N+1}\right)^2$.*

Proof. See Appendix B.

We conclude that, with optimal penal codes, collusion is more difficult to sustain after entry than before entry regardless of the number of entrants. We also conclude that the higher the total number of firms, the harder it is to sustain collusion.

Observe that the critical discount factor depends on the total number of firms (N) but not on the split between incumbents and entrants (N_0). The reason for this is the following. As stated in Proposition 12, the binding ICC for collusion to be sustainable is the one corresponding to periods after entry. After the last entry, all firms are symmetric (entrants and incumbents). Thus, the incentives for the incumbents to comply with the collusive agreement are exactly the same as those of the entrants.

6 Conclusions

In this paper, we have studied the sustainability of collusion when the market growth triggers entry. In a similar model, Vasconcelos (2008) assumed that, after a deviation from the collusive agreement, firms permanently reverted to the Cournot equilibrium (grim trigger strategies). However, firms can increase the sustainability of the collusive agreement by adopting Abreu-type punishment strategies. Motivated by this idea, we have modified the model of Vasconcelos (2008) by considering penal codes that drive the continuation value after a deviation to zero.²⁹

²⁹ In the laboratory experiments carried out by Fudenberg, Rand, and Dreber (2012) and Wright (2013), individuals tended to avoid harsh punishment strategies as grim trigger strategies or

Following Vasconcelos (2008), we started by considering that the incumbents immediately accommodate the entrant in a more inclusive agreement. In contrast to Vasconcelos (2008), we concluded that collusion is more difficult to sustain after entry than before entry. This finding conforms to the idea that the higher the number of firms in the market, the less sustainable is collusion. The origin of the discrepancy between the results of Vasconcelos (2008) and ours is the ability of the incumbents to profitably delay entry by deviating from the collusive agreement. With grim trigger strategies, a deviation may profitably delay entry (since individual profits are higher when two firms compete than when three firms collude). In contrast, with optimal penal codes, breaking the agreement leads to a punishment that absorbs all future profits. As a result, the entry delay is irrelevant.³⁰ The finding that, with optimal penal codes, collusion is more difficult to sustain after entry than before entry is robust to the consideration of price-setting instead of quantity-setting and to the number of incumbents and entrants in the market.

Surprisingly, incumbents may prefer to compete (since the beginning of the game) rather than establish a collusive agreement that accommodates entry. This result is, again, explained by the fact that entry occurs later under competition than under collusion, together with the fact that competition before entry is more profitable than collusion after entry.

We have studied alternative reactions to entry and compared them in terms of the surplus of firms and consumers. The incumbents' surplus is the highest when they are able to deter entry by regarding entry as a deviation from the collusive agreement. Not surprisingly, this is worst scenario for the entrant. The entrant is best off when the incumbents immediately accommodate entry. Depending on the parameters of the model, the incumbents may prefer discontinuance of collusion or immediate accommodation, while the entrant always prefers immediate accommodation. Finally, incumbents prefer gradual to immediate accommodation, while the entrant prefers the opposite. Consumers are best off when firms compete in all periods, since the incumbents and the entrant sell homogeneous products and competition is the scenario in which total output is the highest. Consumers are indifferent between collusion with entry deterrence or (immediate or gradual)

optimal penal codes. Instead, individuals tended to: (i) be more lenient (e.g. waiting some periods before punishing deviations); (ii) adopt gradual punishments, like tit-for-tat and (iii) adjust the magnitude of the punishment to the magnitude of the deviation. In future research, we would like to study the impact of considering alternative punishment strategies encompassing these features.

30 In a related contribution, Miklós-Thal (2011) considered optimal penal codes instead of grim trigger strategies and reexamined the conclusions of Bae (1987) and Harrington (1991a) regarding the effect of cost asymmetries on the sustainability of collusion.

entry accommodation, since total output is always at the monopoly level. Among the collusive scenarios that were considered, discontinuance of collusion after entry is the one that harms consumers the less.

Our results embody some important competition policy implications. First, by proposing a framework wherein the number of market participants depends on the evolution of demand, our analysis warns that market growth is potentially detrimental to collusion because entry becomes easier in growing markets, and future entry may hinder firms' ability to engage in a collusive agreement. Second, we offer a more comprehensive analysis than previous works, which often focus attention on a specific strategy regarding incumbents' reaction to entry (e.g. entry accommodation). More specifically, by comparing alternative cartel reactions to entry, we show, among other things, that incumbents can coordinate on carefully designed punishment processes that threaten the potential entrant with the prospect of a null post-entry discounted value of the flow of profits, thereby deterring entry for any level of the entry cost (as long as this cost is positive). As a consequence, a significant degree of collusion can be sustained in equilibrium by incumbents in a growing market even if the entry costs are low. Lastly, and perhaps most importantly, the obtained results are important for the evaluation of the coordinated effects of mergers in markets with growing demand.³¹ In particular, it is shown that, in markets where demand growth may trigger future entry, whether the most severe coordinated effects arise after or before entry takes place crucially depends on whether deviations trigger optimal (i.e. security level) punishments or not.

Appendix A

Optimal penal code with a positive security level

We assume that the security level is relatively low, so that the entrant does not profit by entering the market, deviating and exiting immediately afterward.

Temporary reversion to zero profit equilibrium

The penal code is as follows. If a deviations occurs in period t , and there are $n \in \{2, 3\}$ active firms in the market:

³¹ When addressing the coordinated effects of a proposed merger, competition authorities investigate whether the structural change implied by a merger creates more favorable conditions for collusion to arise between the remaining firms in the industry.

- In periods $\tau \leq T - 2$ (with T calculated below), firms produce quantities that completely satiate the market (implying that the price is zero), even if one of the firms deviates and produces nothing:

$$q_{\tau}^{pn} = \frac{1}{n-1} \mu^{\tau}$$

- At $T - 1$, firms produce quantities such that profits in this period are equal to $[v - V(T)]\delta^{T-1-(t+1)}$.
- If no firm deviates along the punishment path, the collusive agreement is reinstated in period T . Otherwise, the punishment is restarted.

We still need to determine the period T at which the collusive agreement must be reinstated for the continuation value to coincide with the security level, $v > 0$.

If collusion is reinstated in a period T after entry (i.e. $T \geq T_m$), the continuation value after the deviation (discounted to $t + 1$) is given by:

$$V(T) = \sum_{s=T}^{+\infty} \frac{1}{12} \mu^s \delta^{s-(t+1)} = \frac{(\mu\delta)^T}{12(1-\mu\delta)} \delta^{-(t+1)}.$$

If collusion is reinstated in a period T after entry (i.e. $T < T_m$), there is collusion with two firms between T and $T^m - 1$ and collusion with three firms afterward. In this case, the continuation value after the deviation is equal to:

$$\begin{aligned} V(T) &= \sum_{s=T}^{T^m-1} \frac{1}{8} \mu^s \delta^{s-(t+1)} + \sum_{s=T^m}^{+\infty} \frac{1}{12} \mu^s \delta^{s-(t+1)} = \\ &= \frac{(\mu\delta)^T - (\mu\delta)^{T^m}}{8(1-\mu\delta)} \delta^{-(t+1)} + \frac{(\mu\delta)^{T^m}}{12(1-\mu\delta)} \delta^{-(t+1)}. \end{aligned}$$

For the continuation value to be exactly equal to the security level, v : the period T must be the earliest period for which $V(T) \leq v$; and the output level at $T - 1$ must be so that profits in this period are equal to $[v - V(T)]\delta^{T-1-(t+1)}$.

Stick-and-carrect punishment with dissipative costs

Suppose now that firms have access to some kind of dissipative cost. Consider further that the punishment lasts for one single period (in the case of being respected by all firms).³²

³² It would be straightforward to adapt the optimal penal code to the case in which the punishment lasts for more periods.

Suppose that one of the cartel members deviates in period t . Denote by A_{t+1} the value of the dissipative cost incurred in the punishment period, $t + 1$. This value must be such that the continuation value after a deviation is equal to the security level, v . In addition, the output of each firm in the punishment period must be high enough for the market price to be null, even if there is a deviation. Therefore, as production costs are null, the profit of each firm in the punishment period is $\pi_{t+1}^p = -A_{t+1}$.

Keep in mind that the decision of firm 3 regarding the entry period only depends on the flow of profits that it expects to obtain after entry. Recall that if the firm expects to get one third of the monopoly profit in all periods, entry occurs in period T^m . The firm does not care about the existence of a deviation in any period $t \leq T^m - 2$ because the collusive agreement would be restored at T^m . The entry decision is only affected if one incumbent deviates at $T^m - 1$. In that case, the punishment takes place at T^m and firm 3 delays its entry to period $T^m + 1$.

Suppose that the deviation occurs at $t \leq T^m - 3$. The punishment will take place at $t + 1$; there will be collusion with two firms from $t + 2$ until $T^m - 1$; entry will occur at T^m ; and there will be collusion with three firms from T^m onward. In this case, for the continuation value (after the deviation at t) to be equal to the security level, v , the dissipative cost must satisfy the following condition:

$$V = -A_{t+1} + \sum_{s=t+2}^{T^m-1} \delta^{s-(t+1)} \pi_s^{m2} + \sum_{s=T^m}^{+\infty} \delta^{s-(t+1)} \pi_s^{m2} \Leftrightarrow$$

$$A_{t+1} = \delta^{-(t+1)} \left[\sum_{s=t+2}^{T^m-1} \frac{(\mu\delta)^s}{8} + \sum_{s=T^m}^{+\infty} \frac{(\mu\delta)^s}{12} \right] - v = \frac{\mu\delta \left[3 - (\mu\delta)^{T^m-t-2} \right]}{24(1 - \mu\delta)} \mu^{t+1} - v.$$

If the deviation occurs at $t = T^m - 2$, entry will still take place at T^m , and there will be collusion with three firms from $t + 2$ onward. Therefore,

$$A_{t+1} = \sum_{s=t+2}^{+\infty} \delta^{s-(t+1)} \pi_s^{m3} - v = \frac{\mu\delta}{12(1 - \mu\delta)} \mu^{t+1} - v.$$

If the deviation occurs at $t = T^m - 1$, the punishment will be carried out in period T^m . In that case, firm 3 delays entry to period $T^m + 1$ (in which firms will be colluding). After the punishment period, there will be collusion with three firms. Again, the amount of dissipative advertising must be exactly that which absorbs the value of collusion with three from $t + 2$ onward in excess of the security level.

If a firm deviates after entry, at $t \geq T^m$, there will also be collusion with three firms from $t + 2$ onward. Therefore, the same expression for A_{t+1} applies.

In short, the amount spent on dissipative costs in the punishment period, $t + 1$, is equal to

$$A_{t+1} = \begin{cases} \frac{\mu\delta[3 - (\mu\delta)^{T^m-t-2}]}{24(1-\mu\delta)}\mu^{t+1} - v, & \text{if } t \leq T^m-3 \\ \frac{\mu\delta}{12(1-\mu\delta)}\mu^{t+1} - v, & \text{if } t \geq T^m-2. \end{cases}$$

Appendix B

Proofs

Proof of Lemma 1

Firm 3 prefers to enter at T rather than at $T + 1$ if and only if:

$$\begin{aligned} V_3(T) \geq V_3(T+1) &\Leftrightarrow \sum_{t=T}^{+\infty} \Pi\mu^t\delta^t - \delta^T K \geq \sum_{t=T}^{+\infty} \Pi\mu^t\delta^t - \delta^{T+1}K \\ &\Leftrightarrow \frac{\Pi}{1-\pi\delta} \left[(\pi\delta)^T - (\pi\delta)^{T+1} \right] \geq K(\delta^T - \delta^{T+1}) \\ &\Leftrightarrow \Pi\mu^T \geq K(1-\delta) \Leftrightarrow T \geq \frac{\ln\left[\frac{K(1-\delta)}{\Pi}\right]}{\ln(\mu)}. \end{aligned}$$

As time is discrete, the optimal entry period is the smallest integer that is greater than or equal to this threshold. $\square$

Stick-and-carrot punishment with production costs

Suppose that firms have constant unit costs equal to $c > 0$. Suppose further that, in period t , one of the n_t (active) firms deviates from the collusive agreement. Let us describe a punishment scheme that drives the continuation value of the deviator to zero.

In period $t + 1$, the firms that complied with the collusive agreement in period t must produce quantities that satiate the market (even if the deviator produces nothing), i.e.

$$p^{pn_{t+1}} = 0 \Leftrightarrow 1(n_{t+1} - 1)q_{t+1}^{pn_{t+1}}\mu^t = 0 \Leftrightarrow q_{t+1}^{pn_{t+1}} = \frac{1}{n_{t+1} - 1}\mu^{t+1}.$$

Assuming that the collusive agreement is restored in period $t + 2$, the value for the deviator of being active in the market (from moment $t + 1$ onward) is null if and only if:

$$\begin{aligned} \pi_{t+1}^{pdn_{t+1}}(c) + \sum_{s=t+2} \delta^{s-(t+1)} \pi_s^{mn_s}(c) &= 0 \Leftrightarrow (0 - c)q_{t+1}^{pdn_{t+1}} = - \sum_{s=t+2} \delta^{s-(t+1)} \pi_s^{mn_s}(c) \\ \Leftrightarrow q_{t+1}^{pdn} &= \frac{1}{c} \sum_{s=t+2}^{+\infty} \delta^{s-(t+1)} \pi_s^{mn_s}(c), \end{aligned}$$

where $\pi_s^{mn_s}(c)$ is the collusive profit if firms have unit costs equal to c , and n_s denotes the number of active firms in period s .

Proof of Proposition 1

In period t , the profit function of firm i is given by:

$$\pi_t(q_t) = (1 - q_t\mu^{-t} - Q_{-t}\mu^{-t})q_t, \quad [12]$$

where Q_{-t} is the sum of the quantities produced by the remaining firms in period t . Profit-maximization yields the following best-response function:

$$q_1(Q_{-t}) = \frac{1}{2}\mu^t - \frac{Q_{-t}}{2}.$$

Substituting $Q_{-t} = \frac{1}{3}\mu^t$ in eq. [12], we obtain the deviation output, $q_t^{d3} = \frac{1}{3}\mu^t$, which yields $\pi_t^{d3} = \frac{1}{9}\mu^t$. Substituting the expressions for profits in ICC [5], we obtain

$$\frac{1}{12}\delta^{-t} \sum_{s=t}^{+\infty} (\mu\delta)^s \geq \frac{1}{9}\mu^t \Leftrightarrow \mu\delta \geq \frac{1}{4}.$$

□

Proof of Lemma 2

Substituting $Q_{-t} = \frac{1}{4}\mu^t$ in eq. [12], we obtain the deviation quantity, $q_t^{d2} = \frac{3}{8}\mu^t$, and the corresponding profit, $\pi_t^{d2} = \frac{9}{64}\mu^t$. Substituting the expressions for profits in ICC [6], we obtain

$$\frac{1}{8} \sum_{s=t}^{T^m-1} (\mu\delta)^s + \frac{1}{12} \sum_{s=T^m}^{+\infty} (\mu\delta)^s \geq \frac{9}{64} (\mu\delta)^t.$$

The ICC in period $t = T^m - \tau$, with $1 \leq \tau \leq T^m$, is given by:

$$\frac{1}{8} \sum_{s=T^m-\tau}^{T^m-1} (\mu\delta)^s + \frac{1}{12} \sum_{s=T^m}^{+\infty} (\mu\delta)^s \geq \frac{9}{64} (\mu\delta)^{T^m-\tau}. \quad [13]$$

We want to show that if eq. [13] is satisfied for $\tau = k$, then it is also satisfied for $\tau = k + 1$. Our hypothesis is, therefore, that:

$$\frac{1}{8} \sum_{s=T^m-k}^{T^m-1} (\mu\delta)^s + \frac{1}{12} \sum_{s=T^m}^{+\infty} (\mu\delta)^s - \frac{9}{64} (\mu\delta)^{T^m-k} \geq 0. \quad [14]$$

For $\tau = k + 1$, the ICC [13] can be written as:

$$\frac{1}{8} \sum_{s=T^m-k-1}^{T^m-1} (\mu\delta)^s + \frac{1}{12} \sum_{s=T^m}^{+\infty} (\mu\delta)^s - \frac{9}{64} (\mu\delta)^{T^m-k-1} \geq 0. \quad [15]$$

Subtracting the left-hand side of eq.[14] from that of eq.[15], we obtain

$$\frac{1}{8} (\mu\delta)^{T^m-k-1} - \frac{9}{64} \left[(\mu\delta)^{T^m-k-1} - (\mu\delta)^{T^m-k} \right] = \frac{9\mu\delta - 1}{64} (\mu\delta)^{T^m-k-1}.$$

This expression is positive, meaning that eq. [14] implies eq. [15], if and only if $\mu\delta \geq \frac{1}{9}$. $\square$

Proof of Proposition 2

Collusion is sustainable at $T^m - 1$ (and, therefore, at all $t < T^m$) if and only if:

$$\frac{1}{8} (\mu\delta)^{T^m-1} + \frac{1}{12} \sum_{s=T^m}^{+\infty} (\mu\delta)^s \geq \frac{9}{64} (\mu\delta)^{T^m-1} \Leftrightarrow \frac{(\mu\delta)^{T^m}}{(1-\mu\delta)} \geq \frac{3}{16} (\mu\delta)^{T^m-1} \Leftrightarrow \mu\delta \geq \frac{3}{19}.$$

$\square$

Proof of Proposition 3

The incumbents prefer no collusion to collusion with immediate accommodation if and only if:

$$\sum_{s=0}^{T^m-1} \delta^s (\pi_s^{c2} - \pi_s^{m2}) + \sum_{s=T^m}^{T^c-1} \delta^s (\pi_s^{c2} - \pi_s^{m3}) + \sum_{s=T^c}^{+\infty} \delta^s (\pi_s^{c3} - \pi_s^{m3}) \geq 0.$$

Manipulating this condition, we obtain

$$\begin{aligned} & -\frac{1}{72} \sum_{s=0}^{T^m-1} (\mu\delta)^s + \frac{1}{36} \sum_{s=T^m}^{T^c-1} (\mu\delta)^s - \frac{1}{48} \sum_{s=T^c}^{+\infty} (\mu\delta)^s \geq 0 \Leftrightarrow \\ & -2 \left[1 - (\mu\delta)^{T^m} \right] + 4 \left[(\mu\delta)^{T^m} - (\mu\delta)^{T^c} \right] - 3(\mu\delta)^{T^c} \geq 0 \Leftrightarrow \\ & -2 + 6(\mu\delta)^{T^m} - 7(\mu\delta)^{T^c} \geq 0. \end{aligned}$$

□

Proof of Proposition 4

Replacing the expressions for profits, we can write the ICC [7] as follows:

$$\frac{1}{8} \sum_{s=t}^{T^c-1} (\mu\delta)^s + \frac{1}{16} \sum_{s=T^c}^{+\infty} (\mu\delta)^s \geq \frac{9}{64} (\mu\delta)^t.$$

Lemma 3 allows us to consider $t = T^c - 1$ to obtain the critical discount factor:

$$\frac{1}{8} (\mu\delta)^{T^c-1} + \frac{1}{16} \sum_{s=T^c}^{+\infty} (\mu\delta)^s \geq \frac{9}{64} (\mu\delta)^{T^c-1} \Leftrightarrow \frac{(\mu\delta)^{T^c}}{16(1-\mu\delta)} \geq \frac{(\mu\delta)^{T^c-1}}{64} \Leftrightarrow \mu\delta \geq \frac{1}{5}.$$

□

Proof of Proposition 5

The present value of profits of an incumbent firm in the collusive agreement with immediate accommodation of the entrant is

$$V^m = \sum_{t=0}^{T^m-1} \delta^t \pi_t^{m2} + \sum_{t=T^m}^{+\infty} \delta^t \pi_t^{m3}.$$

while, in the case of reversion to competition, it is

$$V^{rc} = \sum_{t=0}^{T^c-1} \delta^t \pi_t^{m2} + \sum_{t=T^c}^{+\infty} \delta^t \pi_t^{c3}.$$

Therefore, incumbents prefer reversion to competition relatively to accommodation if and only if:

$$V^m > V^{rc} \Leftrightarrow \sum_{t=T^m}^{T^c-1} \delta^t (\pi_t^{m2} - \pi_t^{m3}) + \sum_{t=T^c}^{+\infty} \delta^t (\pi_t^{c3} - \pi_t^{m3}) > 0 \Leftrightarrow$$

$$\frac{1}{24} \sum_{t=T^m}^{T^c-1} (\mu\delta)^t - \frac{1}{48} \sum_{t=T^c}^{+\infty} (\mu\delta)^t > 0 \Leftrightarrow (\mu\delta)^{T^c-T^m} < \frac{2}{3}.$$

□

Proof of Proposition 6

Since the continuation value after a deviation is null, the incumbents are willing to collude in period $t \geq 0$ if and only if:

$$\sum_{s=t}^{+\infty} \delta^{s-t} \pi_s^{m2} \geq \pi_t^{d2} \Leftrightarrow \frac{1}{8} \sum_{s=t}^{+\infty} (\mu\delta)^s \geq \frac{9}{64} (\mu\delta)^t \Leftrightarrow \frac{(\mu\delta)^t}{1-\mu\delta} \geq \frac{9}{8} (\mu\delta)^t \Leftrightarrow \mu\delta \geq \frac{1}{9}.$$

□

Proof of Proposition 7

The collusive and deviating profit of the entrant in period t can be written as follows:

$$\pi_{3t}^g = \sigma_{t-T^g} \frac{\mu^t}{4} \quad \text{and} \quad \pi_{3t}^{dg} = (1 + \sigma_{t-T^g})^2 \frac{\mu^t}{16}.$$

The entrant complies with the collusive agreement in the entry period, T^g , if and only if:

$$\begin{aligned} \pi_{3T^g}^g + \sum_{s=T^g+1}^{+\infty} \delta^{s-T^g} \pi_{3s}^g &\geq \pi_{3T^g}^{dg} \Leftrightarrow \sum_{s=T^g+1}^{+\infty} \delta^{s-T^g} \sigma_{s-T^g} \frac{\mu^s}{4} \geq \frac{(1-\sigma_0)^2}{16} \mu^{T^g} \\ &\Leftrightarrow \sum_{s=T^g+1}^{+\infty} (\mu\delta)^{s-T^g} \sigma_{s-T^g} \geq \frac{(1-\sigma_0)^2}{4} \\ &\Leftrightarrow \sum_{s=1}^{+\infty} (\mu\delta)^s \sigma_s \geq \frac{(1-\sigma_0)^2}{4}. \end{aligned} \quad [16]$$

Consider now a period $T^g + \tau$, with $\tau \geq 0$, the ICC is the following:

$$\begin{aligned} \pi_{3T^g+\tau}^g + \sum_{s=T^g+\tau+1}^{+\infty} \delta^{s-T^g-\tau} \pi_{3s}^g &\geq \pi_{3T^g+\tau}^{dg} \Leftrightarrow \sum_{s=T^g+\tau+1}^{+\infty} \delta^{s-T^g-\tau} \sigma_{s-T^g} \mu^s \geq \frac{(1-\sigma_\tau)^2}{4} \mu^{T^g+\tau} \\ &\Leftrightarrow \sum_{s=1}^{+\infty} (\mu\delta)^s \sigma_{\tau+s} \geq \frac{(1-\sigma_\tau)^2}{4}. \end{aligned} \quad [17]$$

Assuming that eq. [16] is satisfied, since σ_s is non-decreasing in s , we obtain

$$\sum_{s=1}^{+\infty} (\mu\delta)^s \sigma_{\tau+s} \geq \sum_{s=1}^{+\infty} (\mu\delta)^s \sigma_s \geq \frac{(1+\sigma_0)^2}{4} \geq \frac{(1+\sigma_\tau)^2}{4}.$$

This means that, if the ICC is satisfied in the entry period, T^g , it is satisfied in all following periods. $\square$

Proof of Proposition 8

In period $t \geq T^g$, the collusive and the deviation profits of an incumbent are, respectively, given by:

$$\pi_{it}^g = (1 - \sigma_{t-T^g}) \frac{\mu^t}{8} \quad \text{and} \quad \pi_{it}^{dg} = (3 - \sigma_{t-T^g})^2 \frac{\mu^t}{64}. \quad [18]$$

Incumbents abide by the collusive agreement in period $T^g + \tau$, for $\tau \geq 0$, if and only if:

$$\begin{aligned} \sum_{s=T^g+\tau}^{+\infty} \delta^{s-T^g-\tau} \pi_{is}^g &\geq \pi_{iT^g+\tau}^{dg} \Leftrightarrow \sum_{s=T^g+\tau}^{+\infty} (\mu\delta)^{s-T^g-\tau} (1 - \sigma_{s-T^g}) \geq \frac{(3 - \sigma_\tau)^2}{8} \\ &\Leftrightarrow \sum_{s=T^g+\tau+1}^{+\infty} (\mu\delta)^{s-T^g-\tau} (1 - \sigma_{s-T^g}) \geq \frac{(1 + \sigma_\tau)^2}{64} \\ &\Leftrightarrow \sum_{s=1}^{+\infty} (\mu\delta)^s (1 - \sigma_{\tau+s}) \geq \frac{(1 + \sigma_\tau)^2}{64}. \end{aligned} \quad [19]$$

Consider now the ICC in the next period, $T^g + \tau + 1$:

$$\sum_{s=T^g+\tau+1}^{+\infty} \delta^{s-T^g-\tau} \pi_{is}^g \geq \pi_{iT^g+\tau+1}^{dg} \Leftrightarrow \sum_{s=1}^{+\infty} (\mu\delta)^s (1 - \sigma_{\tau+1+s}) \geq \frac{(1 + \sigma_{\tau+1})^2}{64}.$$

Assume that the last inequality is satisfied. As σ_t is non-decreasing in t , we obtain

$$\sum_{s=1}^{+\infty} (\pi\delta)^s (1 - \sigma_{s+\tau}) \geq \sum_{s=1}^{+\infty} (\mu\delta)^s (1 - \sigma_{s+\tau+1}) \geq \frac{(1 + \sigma_{\tau+1})^2}{64} \geq \frac{(1 + \sigma_{\tau})^2}{64}.$$

By induction, we conclude that, as time passes the ICC becomes more restrictive. $\square$

Proof of Lemma 4

Consider a period $T^g + \tau$, with $\tau \geq 0$. The incumbents abide by the collusive agreement if and only if inequality [19] is satisfied:

$$\sum_{s=T^g+\tau+1}^{+\infty} (\mu\delta)^{s-T^g-\tau} (1 - \sigma_{s-T^g}) \geq \frac{(1 + \sigma_{\tau})^2}{64}.$$

A similar condition, given in eq. [17], must hold for the entrant not to deviate in this period:

$$\sum_{s=T^g+\tau+1}^{+\infty} (\mu\delta)^{s-T^g-\tau} \sigma_{s-T^g} \geq \frac{(1 - \sigma_{\tau})^2}{4}.$$

Recall that the entrant's share of the cartel profit (along the collusive path) never exceeds the share of each incumbent. Hence, we know that $1 - \sigma_{s-T^g} > \sigma_{s-T^g}$, $\forall s \geq T^g$. If the ICC for the entrant holds and $\sigma_{\tau} \leq \frac{1}{3}$, we can make the following ordering:

$$\sum_{s=T^g+\tau+1}^{+\infty} (\mu\delta)^{s-T^g-\tau} (1 - \sigma_{s-T^g}) > \sum_{s=T^g+\tau+1}^{+\infty} (\mu\delta)^{s-T^g-\tau} \sigma_{s-T^g} \geq \frac{(1 - \sigma_{\tau})^2}{4} \geq \frac{(1 + \sigma_{\tau})^2}{64},$$

which implies the compliance of the incumbents with the collusive agreement. $\square$

Proof of Lemma 5

Replacing the expressions for profits, obtained in eqs [1] and [18], the ICC for collusion to be sustainable in period $T^g - 1$ can be written as follows:

$$\begin{aligned} & \frac{1}{8} (\mu\delta)^{T^g-1} + \sum_{s=T^g}^{+\infty} \delta^{s-T^g+1} \pi_{is}^g \geq \frac{9}{64} (\mu\delta)^{T^g-1} \\ \Leftrightarrow & \sum_{s=T^g}^{+\infty} (\mu\delta)^{s-T^g+1} (1 - \sigma_{s-T^g}) - \frac{1}{8} \geq 0 \Leftrightarrow \sum_{s=1}^{+\infty} (\mu\delta)^s (1 - \sigma_{s-1}) - \frac{1}{8} \geq 0. \end{aligned} \quad [20]$$

Similarly, collusion is sustainable in period $T^g - \tau$ if and only if:

$$\begin{aligned}
 & \frac{1}{8}\mu^{T^g-\tau} + \frac{1}{8} \sum_{s=T^g-\tau+1}^{T^g-1} \delta^{s-T^g+\tau} \mu^s + \frac{1}{8} \sum_{s=T^g}^{+\infty} \delta^{s-T^g+\tau} \mu^s (1 - \sigma_{s-T^g}) \geq \frac{9}{64} \mu^{T^g-\tau} \\
 \Leftrightarrow & \sum_{s=T^g-\tau+1}^{T^g-1} (\mu\delta)^{s-T^g+\tau} + \sum_{s=T^g}^{+\infty} (\mu\delta)^{s-T^g+\tau} (1 - \sigma_{s-T^g}) - \frac{1}{8} \geq 0 \\
 \Leftrightarrow & \frac{\mu\delta - (\mu\delta)^\tau}{1 - \mu\delta} + \sum_{s=1}^{+\infty} (\mu\delta)^{s+\tau-1} (1 - \sigma_{s-1}) - \frac{1}{8} \geq 0.
 \end{aligned} \tag{21}$$

Subtracting the LHS of eq. [20] from the LHS of eq. [21], we obtain

$$\begin{aligned}
 & \mu\delta \frac{1 - (\mu\delta)^{\tau-1}}{1 - \mu\delta} - \left[1 - (\mu\delta)^{\tau-1} \right] \sum_{s=1}^{+\infty} (\mu\delta)^s (1 - \sigma_{s-1}) \\
 & = \left[1 - (\mu\delta)^{\tau-1} \right] \left[\frac{\mu\delta}{1 - \mu\delta} - \sum_{s=1}^{+\infty} (\mu\delta)^s (1 - \sigma_{s-1}) \right] \\
 & > \left[1 - (\mu\delta)^{\tau-1} \right] \left[\frac{\mu\delta}{1 - \mu\delta} - \sum_{s=1}^{+\infty} (\mu\delta)^s \right] = 0.
 \end{aligned}$$

This implies that the LHS of eq. [21] is greater than the LHS of eq. [20], which ends the proof. $\square$

Proof of Proposition 9

According to Corollary 2, collusion is sustainable after entry if the ICC for the entrant to abide by the collusive agreement is satisfied in period T^g , i.e. if:

$$\sum_{s=1}^{+\infty} (\mu\delta)^s \sigma_s \geq \frac{(1 - \sigma_0)^2}{4}.$$

Further, Lemma 5 states that collusion is sustainable before entry if the ICC [9] for the incumbents to abide by the collusive agreement is satisfied in period $T^g - 1$, i.e. if:

$$\sum_{s=1}^{+\infty} (\mu\delta)^s (1 - \sigma_{s-1}) \geq \frac{1}{8}.$$

The particular case of immediate accommodation ($\sigma_s = \frac{1}{3}$, $\forall s$) is, simultaneously, the most difficult case for sustainability before entry and the most favorable case

for sustainability after entry. Even in this case, sustainability after entry is more restrictive than sustainability before entry.

Replacing $\sigma_s = \frac{1}{3}$, $\forall s$, above, we obtain $\mu\delta \geq \frac{1}{4}$ for sustainability after entry and $\mu\delta \geq \frac{3}{19}$ for sustainability before entry. Since $\frac{1}{4} > \frac{3}{19}$, the proof is finished. $\square$

Proof of Remark 4

We only need to show that $W^p < W^{dc}$.

In the scenario of discontinuance of collusion, the sum of the incumbents' profits with the consumer surplus is

$$\sum_{t=0}^{T^c-1} \frac{1}{4} (\mu\delta)^t + \sum_{t=T^c}^{+\infty} \frac{1}{8} (\mu\delta)^t + \frac{1}{2} \sum_{t=0}^{T^c-1} \frac{1}{4} (\mu\delta)^t + \frac{1}{2} \sum_{t=T^c}^{+\infty} \frac{9}{16} (\mu\delta)^t = \frac{12 + (\mu\delta)^{T^c}}{32(1 - \mu\delta)}.$$

Under entry deterrence, it is lower:

$$\sum_{t=0}^{+\infty} \frac{1}{4} (\mu\delta)^t + \frac{1}{2} \sum_{t=0}^{+\infty} \frac{1}{4} (\mu\delta)^t = \frac{12}{32(1 - \mu\delta)}.$$

This concludes the proof, since the entrant's surplus is surely positive (otherwise, it would not enter the market). $\square$

Proof of Proposition 10

If firms are price-setters, collusion is sustainable in period $t \geq T^m$ if the following ICC is satisfied:

$$\sum_{s=t}^{+\infty} \delta^{s-t} \pi_s^{m3} \geq 3\pi_s^{m3} \Leftrightarrow \sum_{s=t}^{+\infty} \delta^{s-t} \frac{1}{12} \mu^s \geq \frac{1}{4} \mu^t \Leftrightarrow \frac{(\mu\delta)^t}{1 - \mu\delta} \geq 3(\mu\delta)^t \Leftrightarrow \mu\delta \geq \frac{2}{3}.$$

$\square$

Proof of Proposition 11

If firms are price-setters, the collusive agreement is sustainable in period $t \leq T^m - 1$ if and only if:

$$\sum_{s=t}^{T^m-1} \delta^{s-t} \pi_s^{m2} + \sum_{s=T^m}^{+\infty} \delta^{s-t} \pi_s^{m3} \geq 3\pi_s^{m3} \Leftrightarrow \frac{1}{8} \sum_{s=t}^{T^m-1} (\mu\delta)^s + \frac{1}{12} \sum_{s=T^m}^{+\infty} (\mu\delta)^s \geq \frac{1}{4} (\mu\delta)^t$$

$$\Leftrightarrow \frac{3 - 3(\mu\delta)^{T^m-t}}{1 - \mu\delta} + \frac{2(\mu\delta)^{T^m-t}}{1 - \mu\delta} \geq 6 \Leftrightarrow \frac{3 - (\mu\delta)^{T^m-t}}{1 - \mu\delta} \geq 6.$$

It follows immediately that: the higher is $T^m - t$, the higher is the LHS of the last inequality. In other words, $T^m - 1$ is the period in which the ICC is the most difficult to satisfy. Replacing $t = T^m - 1$, we obtain

$$\frac{3 - \mu\delta}{1 - \mu\delta} \geq 6 \Leftrightarrow \mu\delta \geq \frac{3}{5}.$$

□

Proof of Lemma 6

Substituting the expressions [1] and [2] for profits in ICC [11], we obtain

$$\frac{1}{4n} \sum_{s=t}^{T_{n+1}^m-1} (\mu\delta)^s + \frac{1}{4(n+1)} \sum_{s=T_{n+1}^m}^{T_{n+2}^m-1} (\mu\delta)^s + \dots + \frac{1}{4N} \sum_{s=T_N^m}^{+\infty} (\mu\delta)^s - \frac{(n+1)^2}{16n^2} (\mu\delta)^t \geq 0. \quad [22]$$

Evaluating it at $t = T_{n+1}^m - 1$, we obtain

$$\frac{1}{4n} (\mu\delta)^{T_{n+1}^m-1} + \frac{1}{4(n+1)} \sum_{s=T_{n+1}^m}^{T_{n+2}^m-1} (\mu\delta)^s + \dots + \frac{1}{4N} \sum_{s=T_N^m}^{+\infty} (\mu\delta)^s - \frac{(n+1)^2}{16n^2} (\mu\delta)^{T_{n+1}^m-1} \geq 0. \quad [23]$$

We only need to show that the LHS of inequality [22] is greater than the LHS of [23]. Subtracting the LHS of inequality [22] from the LHS of [23], we obtain

$$\begin{aligned} & \frac{(n+1)^2}{16n^2} \left[(\mu\delta)^t - (\mu\delta)^{T_{n+1}^m-1} \right] - \frac{1}{4n} \sum_{s=t}^{T_{n+1}^m-2} (\mu\delta)^s \\ &= \left[\frac{(n+1)^2}{16n^2} - \frac{1}{4n(1 - \mu\delta)} \right] \left[(\mu\delta)^t - (\mu\delta)^{T_{n+1}^m-1} \right]. \end{aligned}$$

The sign of this expression is negative if and only if:

$$\frac{(n+1)^2}{4n} < \frac{1}{1 - \mu\delta} \Leftrightarrow \mu\delta \geq \left(\frac{n-1}{n+1} \right)^2,$$

which is our hypothesis.

□

Proof of Proposition 12

Using Lemma 6, we know that collusion is sustainable for $t \in \{T_n^m, \dots, T_{n+1}^m - 1\}$, if inequality [23] is satisfied. Expanding the sums, we obtain

$$\begin{aligned}
 & \frac{1}{4n} (\mu\delta)^{T_{n+1}^m-1} + \frac{(\mu\delta)^{T_{n+1}^m} - (\mu\delta)^{T_{n+2}^m}}{4(n+1)(1-\mu\delta)} + \frac{(\mu\delta)^{T_{n+2}^m} - (\mu\delta)^{T_{n+3}^m}}{4(n+2)(1-\mu\delta)} \\
 & + \dots + \frac{(\mu\delta)^{T_{N-1}^m} - (\mu\delta)^{T_N^m}}{4(N-1)(1-\mu\delta)} + \frac{(\mu\delta)^{T_N^m}}{4N(1-\mu\delta)} \geq \frac{(n+1)^2}{16n^2} (\mu\delta)^{T_{n+1}^m-1} \\
 \Leftrightarrow & \frac{1}{n+1} (\mu\delta)^{T_{n+1}^m} + \left(\frac{1}{n+2} - \frac{1}{n+1} \right) (\mu\delta)^{T_{n+2}^m} + \left(\frac{1}{n+3} - \frac{1}{n+2} \right) (\mu\delta)^{T_{n+3}^m} \\
 & + \dots + \left(\frac{1}{N} - \frac{1}{N-1} \right) (\mu\delta)^{T_N^m} \geq \frac{(n-1)^2}{4n^2} (1-\mu\delta) (\mu\delta)^{T_{n+1}^m-1} \\
 \Leftrightarrow & -\frac{(n-1)^2}{4n^2} (\mu\delta)^{T_{n+1}^m-1} + \left[\frac{(n-1)^2}{4n^2} + \frac{1}{n+1} \right] (\mu\delta)^{T_{n+1}^m} + \left(\frac{1}{n+2} - \frac{1}{n+1} \right) (\mu\delta)^{T_{n+2}^m} \\
 & + \left(\frac{1}{n+3} - \frac{1}{n+2} \right) (\mu\delta)^{T_{n+3}^m} + \dots + \left(\frac{1}{N} - \frac{1}{N-1} \right) (\mu\delta)^{T_N^m} \geq 0 \\
 \Leftrightarrow & -\frac{(n-1)^2}{4n^2} + \left[\frac{(n-1)^2}{4n^2} + \frac{1}{n+1} \right] (\mu\delta) + \left(\frac{1}{n+2} - \frac{1}{n+1} \right) (\mu\delta)^{T_{n+2}^m - T_{n+1}^m + 1} \\
 & + \left(\frac{1}{n+3} - \frac{1}{n+2} \right) (\mu\delta)^{T_{n+3}^m - T_{n+1}^m + 1} + \dots + \left(\frac{1}{N} - \frac{1}{N-1} \right) (\mu\delta)^{T_N^m - T_{n+1}^m + 1} \geq 0
 \end{aligned}$$

The worst case for this inequality to hold is when $T_{n+1}^m = T_{n+2}^m = \dots = T_N^m$ (since all constants that multiply $(\mu\delta)^x$ are negative). In this extreme case, the last inequality becomes

$$-\frac{(n-1)^2}{4n^2} + (\mu\delta) \left[\frac{(n-1)^2}{4n^2} + \frac{1}{N} \right] \geq 0 \Leftrightarrow \mu\delta \geq \frac{\left(\frac{n-1}{2n} \right)^2}{\left(\frac{n-1}{2n} \right)^2 + \frac{1}{N}}.$$

This expression for the adjusted discount factor is increasing in $\frac{n-1}{2n}$, thus, it is increasing in n . A sufficient condition for the sustainability of collusion before entry is, therefore,

$$\mu\delta \geq \frac{\left(\frac{N-2}{2N-2}\right)^2}{\left(\frac{N-2}{2N-2}\right)^2 + \frac{1}{N}} = \frac{(N-2)^2 N}{N^3 - 4N + 4}.$$

We now need to compare this expression with the critical discount factor that we obtained for the sustainability of collusion after entry, given by eq. [10]:

$$\begin{aligned} \frac{(N-1)^2}{(N+1)^2} \geq \frac{(N-2)^2 N}{N^3 - 4N + 4} &\Leftrightarrow (N-1)^2(N^3 - 4N + 4) - (N+1)^2(N-2)^2 N \geq 0 \\ &\Leftrightarrow 4(2N^2 - 4N + 1) \geq 0. \end{aligned}$$

The last inequality is satisfied for $N \geq 2$, which is the admissible range for N . Therefore, we conclude that the ICC for collusion to be sustainable after all firms have entered is the one that is binding. $\square$

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