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Advances

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Energy Taxes and Oil Price Shocks

Abstract: This paper examines if an energy price shock should be compensated by a reduction in energy taxes to mitigate its impact on consumer prices. It shows that the consumer price should not increase by as much as the producer price, implying a small reduction in the energy tax in dollars. The energy tax *rate*, on the other hand, decreases sharply. This decline is primarily due to an adjustment in the Pigouvian component: A constant marginal social damage being divided by a higher producer price. The redistributive component of the tax remains at about 10% of the social cost of energy.

Keywords: oil price shock, energy tax, Pigouvian tax, redistributive concerns
JEL Classification: H21, H23

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1 Introduction

As energy is heavily taxed in most industrialized countries, an "oil shock" (a sudden and significant increase in energy prices) often leads to various interest groups putting political pressure on their respective governments asking for tax reductions. The issue regularly comes up in US presidential election campaign as it did during the 2008 campaign. In 2012, Connecticut capped its tax on the wholesale price of gasoline and Iowa forsook a previously planned state gas tax increase.[1] Overseas, the Israeli government cuts its gasoline taxes

1 http://news.yahoo.com/malloy-signs-bill-capping-conn-wholesale-gas-tax-144423620.html and http://www.desmoinesregister.com/article/20120331/NEWS/303310028/House-speaker-puts-kibosh-increasing-state-s-fuel-tax.

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citing the price increases as the reason.[2] There has also been a similar debate in France with many interest groups (truck and fishery industries, agriculture, etc.), asking for an energy tax relief to mitigate the impact of oil price increases. Reacting to the 2012 increases in energy prices, the then socialist presidential candidate François Hollande proposed a legal cap on consumer price of gasoline.[3] Adoption of a price cap means that any future increases in world energy prices will be absorbed by a one-to-one reduction in energy taxes.[4]

Among possible justifications for energy tax reductions is the idea that consumers have difficulties to adjust to strong and sudden price shocks. This is because existing technologies and equipment limit the substitution possibilities in the short run. However, this argument would at best lead to a temporary reduction to smooth the transition. Another argument is based on the regressive character of rising energy prices. The share of energy consumption in total spending tends to decrease with income so that low-income individuals are affected more heavily, relative to their income, than high-income individuals by an oil price shock. Redistributive concerns may then call for an energy tax reduction.

This paper studies the validity of this redistributive argument using the model of optimal emission taxation developed by Cremer, Gahvari, and Ladoux (1998, 2001, 2003 and 2010). In doing this, we build on the vast body of theoretical and empirical literatures that have been developed over the past four decades or so. On the theory side, the literature has typically followed, with few exceptions, a Ramsey approach to the optimal tax problem. This means, in particular, that tax instruments have been restricted to be linear and non-linear income taxes are ignored. See, among others, Sandmo (1975), Bovenberg and de Mooij (1994), Bovenberg and van der Ploeg (1994), Fullerton (1997), Parry (1997), Cremer, Gahvari, and Ladoux (2001) and particularly Bovenberg and Goulder's (2002) survey. In consequence, what emerges in this literature as an "optimal tax" system may in fact suboptimal in that a "true" optimal tax system must implement a Pareto-efficient allocation constrained only by the information structure in the economy.

Most recently, however, this tendency has been corrected and a number of authors have studied the optimal tax design problem with externalities, and the

2 http://www.jpost.com/NationalNews/Article.aspx?id=264228.

3 http://www.bloomberg.com/news/2012-04-04/hollande-says-he-d-seek-eu-fiscal-pact-revamp-freeze-gas-prices.html.

4 There is another side to this coin. The recent January 2015 plunge in world oil prices has bolstered the proponents of energy taxes in the United States to renew their call for carbon taxes. See, e.g. http://www.cnbc.com/id/102313435 and http://dailycaller.com/2015/01/07/oil-prices-plunge-so-liberals-push-for-a-carbon-tax/.

structure of environmental taxes, in light of modern optimal tax theory à la Mirrlees (1971). This theory allows for heterogeneity among individuals and justifies the use of distortionary taxes on the basis of informational asymmetries between tax authorities and taxpayers. A hallmark of this literature is its inclusion of nonlinear tax instruments. See, among others, Kaplow (1996), Mayeres and Proost (1997), Pirttilä and Tuomala (1997) and Cremer, Gahvari, and Ladoux (1998, 2001).[5]

Turning to the empirical studies of environmental taxes, they have remained squarely in the Ramsey tradition; see, e.g. Bovenberg and Goulder (1996) or Goulder (1994). Moreover, the applied literature has been mainly concerned with welfare implications of "piecemeal" tax reforms. This approach ignores the fact that the benefits of reforming any particular tax is fundamentally linked to the way the other taxes in the system are set. The "piecemeal" approach is thus problematic from a policy perspective. It undermines the role that income taxation can play in offsetting the possible "regressive bias" of environmental taxes. Poterba (1991) estimates that, with few exceptions, the expenditure shares of such polluting goods as gasoline, fuel oil, natural gas and electricity decrease at all income deciles as income increases.

In Cremer, Gahvari, and Ladoux (2003), we made an initial attempt to break loose from this piecemeal tradition and examine quantitatively the efficiency and redistributive properties of optimal environmental taxes for the French economy within the context of the modern optimal tax theory à la Mirrlees (1971). In Cremer, Gahvari, and Ladoux (2010), we further developed this approach. There, we considered not only polluting goods but also polluting inputs. There are indeed intermediate goods that are polluting; energy being an obvious example. This is an important addition; lumping final goods and inputs together inevitably leads to incorrect policy recommendations. Diamond and Mirrlees (1971) have taught us that the tax treatment of intermediate and final goods should in general be different. Applying their production efficiency result to economies with a consumption externality, leads to the conclusion that polluting intermediate goods should be taxed only in so far as they correct externalities – a result proved by Cremer and Gahvari (2001). They also proved that, in contrast, polluting final goods is taxed for Pigouvian considerations as well as for redistributive concerns.

These two earlier empirical applications were concerned with a given state of the economy. However, their underlying models also provide a methodology

5 Mayeres and Proost (2001) have introduced consumer heterogeneity and distributional aims. However, that paper remains within the Ramsey tradition considering only linear tax instruments.

that can be adapted to study the incidence of various shocks and their implications for policy design and environmental taxation. In the current paper, as in Cremer, Gahvari, and Ladoux (2010), energy is used as a consumption good by households and as an input by the firms. We calibrate a modified version of this model on US data. Subsequently, we simulate the optimal energy taxes for different shocks in the before-tax price of energy assuming a concomitant adjustment in optimal income taxes. We show that redistributive concerns call for a subsidy on energy goods equal to about 10% of its social cost (producer price plus the associated marginal social damage of emissions). Interestingly, simulations indicate that variations in the world market price of energy have an almost negligible effect on this percentage. On the other hand, the *total* tax rate on energy (redistributive plus Pigouvian) decreases sharply as the world price of energy increases. This arises purely as an arithmetic adjustment: A constant marginal social damage is divided by a rising producer price leading to a decline in the Pigouvian tax *rate*. Nevertheless, it is also true that the consumer price does not increase dollar by dollar with the producer price. As the world price of energy increases, the dollar amount of the subsidy increases too – albeit by small amounts.

2 The Model

Consider an open economy wherein people consume two produced goods: a composite consumption good and "energy." The composite consumption good is produced domestically using "energy inputs," capital and labor. Energy, whether used as a consumption good or as a factor input, is imported from overseas. Capital services are also rented from outside.[6] Labor is the only factor of production which is supplied domestically. All imports are financed through exports of the portion of the general output that is not consumed domestically. Energy, both as a consumption good and a factor input, is polluting; the composite consumption good is not.

Labor is heterogeneous with different groups of individuals having different productivity levels and different tastes. There are four types of individuals. Denote a person's type by j, his productivity factor by n^j and the proportion of people of type j in the economy by π^j (where the population size is normalized at 1). Preferences of a j-type person depend on his consumption of non-polluting goods, x^j, consumption

6 The assumptions that energy and capital services are rented from outside are made to ensure that the world price of energy and returns to capital will not be affected by domestic decisions.

of polluting goods, y^j, labor supply, L^j, and the total level of emissions in the atmosphere, E. This construct is based on Cremer, Gahvari, and Ladoux's (2010) model. To make this paper self-contained, we first review its main features.[7]

2.1 Preferences

Consumers' preferences are represented by nested constant elasticity of substitution (CES) utility functions, first in goods and labor supply and then in the two categories of consumer goods. All consumer types have identical elasticities of substitution between leisure and non-leisure goods, ρ, and between polluting and non-polluting goods, ω. Differences in tastes are captured by differences in other parameter values of the posited utility function (a^j and b^j in eqs [2] and [3]). Assume further that emissions enter the utility function linearly. The preferences for a person of type j can then be represented by

$$\upsilon^j = U(x, y, L^j; \theta^j) - \phi E, \qquad j = 1, 2, 3, 4, \tag{1}$$

where θ^j reflects the "taste parameter," ϕ represents the marginal social damage of emissions assumed to be constant, and

$$U(x, y, L^j, \theta^j) = \left(b^j Q^{j\frac{\rho-1}{\rho}} + (1 - b^j)(1 - L^j)^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}}, \tag{2}$$

$$Q^j = \left(a^j x^{\frac{\omega-1}{\omega}} + (1 - a^j) y^{\frac{\omega-1}{\omega}} \right)^{\frac{\omega}{\omega-1}}. \tag{3}$$

Consumers choose their consumption bundles by maximizing eqs [1]–[3] subject to their budget constraints. These will be nonlinear functions when the income tax schedule is nonlinear. However, for the purpose of uniformity in exposition, we characterize the consumers' choices, as the solution to an optimization problem in which each person faces a (type-specific) *linearized and possibly truncated* budget constraint. To do this, introduce a "virtual income," G^j, into each type's budget constraint. Denote the j-type's net-of-tax wage by w_n^j. We can then write j's budget constraint as

$$px^j + qy^j = G^j + M^j + w_n^j L^j, \tag{4}$$

where p and q are the consumer prices of x and y, G^j is the income adjustment term (virtual income) needed for linearizing the budget constraint (or the lump-sum rebate if the tax function is linear) and M^j is the individual's exogenous income. The first-order conditions for a j-type's optimization problem are

7 For more details, see Cremer, Gahvari, and Ladoux (2010).

$$\frac{1-a^j}{a^j}\left(\frac{x^j}{y^j}\right)^{\frac{1}{\omega}}=\frac{q}{p}, \qquad [5]$$

$$\frac{(1-b^j)(x^j/(1-L^j))^{\frac{1}{\rho}}}{a^j b^j\left[a^j+(1-a^j)(x^j/y^j)^{\frac{1-\omega}{\omega}}\right]^{\frac{\omega-\rho}{\rho(1-\omega)}}}=\frac{w_n^j}{p}. \qquad [6]$$

Equations [4]–[6] determine x^j, y^j and L^j as functions of p, q, w_n^j and G^j+M^j.

2.2 Production Technology

The production process, for the composite consumption good, uses three inputs: capital K, labor L and energy D. The technology of production is represented by a nested CES,

$$O=O(L,K,D)=B\left[(1-\beta)L^{\frac{\sigma-1}{\sigma}}+\beta\Gamma^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \qquad [7]$$

$$\Gamma=A\left[\alpha K^{\frac{\delta-1}{\delta}}+(1-\alpha)D^{\frac{\delta-1}{\delta}}\right]^{\frac{\delta}{\delta-1}}, \qquad [8]$$

where A and B are constants, σ and δ represent the elasticities of substitution between L and Γ and between K and D (given Γ) respectively. Substituting eq. [8] in eq. [7] yields,

$$O=B\left[(1-\beta)L^{\frac{\sigma-1}{\sigma}}+\beta A^{\frac{\sigma-1}{\sigma}}\left[\alpha K^{\frac{\delta-1}{\delta}}+(1-\delta)D^{\frac{\delta-1}{\delta}}\right]^{\frac{\delta(\sigma-1)}{\sigma(\delta-1)}}\right]^{\frac{\sigma}{\sigma-1}}. \qquad [9]$$

Aggregate output, O, is the numeraire.

Capital services and energy inputs are imported at constant world market prices of r and p_D where the units of D is chosen such that *initially* $p_D=1$. Assume that there are no *producer taxes* on labor and capital.[8] Let w denote the

8 It is not optimal to tax capital in this setting. With the world rental price of capital fixed, a tax on capital income will have to be totally borne domestically. This can only distort the production decisions of domestic producers and is thus suboptimal. Formally, a tax on capital income does not affect the economy's resource constraint [15]. Consequently, the first-order condition [26] in Appendix A, as well as its simplified version [40] there, which characterizes the optimal usage of capital, remains the same with a tax on capital as without. That is, $O_K(L,K,D)=r$ characterizes the optimal condition for hiring capital by the economy. As shown by eq. [11], this same relationship determines the equilibrium condition for hiring capital by the firms *in the absence* of capital income taxes.

price of one unit of *effective labor* and τ_D denote the per-unit tax on energy input. The first-order conditions for the firms' input-hiring decisions are assuming competitive markets,

$$O_L(L, K, D) = w, \quad [10]$$

$$O_K(L, K, D) = r, \quad [11]$$

$$O_D(L, K, D) = p_D + \tau_D. \quad [12]$$

Equations [9]–[12] determine the equilibrium values of O, L, K and D as functions of w, r and $p_D + \tau_D$ (where r and p_D are determined according to world prices).

As different types of people have different productivities, labor is a heterogeneous factor of production. When a j-type person with productivity n^j works for L^j hours, his effective labor is $n^j L^j$ resulting in aggregate effective labor supply $\sum_{j=1}^{4} \pi^j n^j L^j$. Equating this with aggregate demand gives,

$$L = \sum_{j=1}^{4} \pi^j n^j L^j.$$

We choose the units of emissions so that a unit of energy, consumed or used as an input, results in one unit of emissions. Total emissions are then given by,

$$E = \sum_{j=1}^{4} \pi^j y^j + D,$$

where π^j is the proportion of people of type j in the economy. Observe also that with the population size being normalized to one, $\sum_{j=1}^{4} \pi^j y^j$ represents total households' energy consumption.

2.3 Optimal Tax Policy

The optimal tax policy maximizes an isoelastic social welfare function

$$W = \frac{1}{1-\eta} \sum_{j=1}^{4} \pi^j (\mho^j)^{1-\eta} \qquad \eta \neq 1 \qquad \text{and} \qquad 0 \leq \eta < \infty, \quad [13]$$

where η is the "inequality aversion index." The value of η dictates the desired degree of redistribution in the economy: The higher is η the more the society cares about equality, here we retain a relatively low value, $\eta = 0.1$.[9]

9 As is well-known, $\eta = 0$ implies a utilitarian social welfare function and $\eta \to \infty$ a Rawlsian. The value we use is chosen according to the observed degree of redistribution of existing tax systems; see Bourguignon and Spadaro (2000).

The feasibility of tax instruments depends on information available to the tax administration. Generally, this information allows for linear commodity taxes and non-linear income taxation. This is why we restrict our analysis to this case even if other possibilities could be considered.[10] Under linear commodity taxation, all consumers face the same commodity prices.

With prices to be determined endogenously and optimally, the social welfare function [13], and thus the j-type's utility function υ^j, must be rewritten as a function of prices. Denote the after-tax income (outlay) of a j-type household by c^j. Maximizing the utility function [1] with respect to the budget constraint

$$px^j + qy^j = c^j,$$

one obtains the demand functions for x^j and y^j as $x^j = \mathbf{x}(p, q, c^j; \theta^j)$ and $y^j = \mathbf{y}(p, q, c^j; \theta^j)$. Substituting these equations into the j-type person's utility function [1], we have

$$\mathbf{V}\left(p, q, c^j, \frac{I^j}{wn^j}; \theta^j\right) = \mathbf{U}\left(\mathbf{x}(p, q, c^j; \theta^j), \mathbf{y}(p, q, c^j; \theta^j), \frac{I^j}{wn^j}, \theta^j\right),$$

where

$$I^j \equiv wn^j L^j.$$

There are four feasible tax instruments in this model: two commodity taxes, an input tax and an income tax. Given that the demand functions for goods, and the labor supply function, are all homogeneous of degree zero in consumer prices, one can always normalize one of the consumer prices to one. Put differently, there is no loss of generality in setting one of the tax rates to zero. Given that the properties of optimal pollution taxes are at the center of our study, and with energy being the good that creates pollution, we set the tax rate on non-energy goods at zero.

The optimal tax structure is derived as the solution to

$$\max_{q, c^j, I^j, K, D, w} \frac{1}{1-\eta} \sum_{j=1}^{4} \pi^j \left[\mathbf{V}\left(p, q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \phi \sum_{j=1}^{4} \pi^j \mathbf{y}(p, q, c^j; \theta^j) - \phi D\right]^{1-\eta} \quad [14]$$

under the resource constraint,

10 In Cremer et al. (1998) different possibilities are examined, including the case where all taxes are non-linear.

$$\mathbf{O}(L,K,D) - \sum_{j=1}^{4} \pi^j \mathbf{x}(p,q,c^j;\theta^j) - rK - p_D \left[\sum_{j=1}^{4} \pi^j \mathbf{y}(p,q,c^j;\theta^j) + D \right] - \bar{R} \geq 0, \quad [15]$$

the incentive compatibility constraints,

$$\mathbf{V}\left(p,q,c^j,\frac{I^j}{wn^j};\theta^j\right) \geq \mathbf{V}\left(p,q,c^k,\frac{I^k}{wn^j};\theta^j\right), \quad [16]$$

the endogeneity of wage condition,

$$w - \mathbf{O}_L(L,K,D) = 0, \quad [17]$$

with

$$L = \sum_{j=1}^{4} \pi^j n^j L^j = \sum_{j=1}^{4} \pi^j \frac{I^j}{w}.$$

The analytical results of Cremer, Gahvari, and Ladoux (2010) can easily be extended to show that the optimal tax on energy inputs (τ_D) is Pigouvian and equal to the marginal social damage of emissions. The optimal tax on the *consumption* of energy, on the other hand, is generally different from its Pigouvian level; see Appendix A. Cremer and Gahvari (2001) established these properties in a somewhat similar setting. They stand in striking contrast with many of the results obtained in the literature on environmental taxation in the presence of pre-existing distortions; see, e.g. Bovenberg and de Mooij (1994), Bovenberg and van der Ploeg (1994), Fullerton (1997), Parry (1997) and the survey by Bovenberg and Goulder (2002).[11] These authors, and the vast majority of other authors who write on environmental taxation, typically follow a Ramsey approach to the optimal tax problem, which means that they restrict tax instruments to be linear and ignore non-linear income taxes. What emerges as an "optimal tax" system under these formulations is in fact suboptimal in that a "true" optimal tax system must implement a Pareto-efficient allocation constrained only by the information structure in the economy. Tax interactions in these suboptimal tax systems then generally lead to distortions in all tax instruments.

Mirrleesian optimal tax systems too may imply tax distortions but only if they lead to a better screening (relaxing an otherwise binding self-selection constraint). Thus, as long as emissions per se cannot be used for screening,

11 These authors do not allow for a direct emission tax. Gahvari (2010, 2012) uses a similar approach while allowing for a direct emission tax.

taxation of polluting intermediate goods will have no direct effect on the incentive constraints.[12] They should not then be taxed because that will entail no benefits; only costs that arise due to the induced inefficiencies in production. Now the structure of preferences in our model is such that it renders emissions useless as a screening device.[13] It is the absence of nonlinear income taxes that makes taxation of polluting inputs non-Pigouvian for this preference structure.

3 Data and Calibration

To solve our model numerically, one must know the values of the parameters of the utility functions (ρ, ω, a^j, b^j, ϕ) and the values of the parameters of the production function (σ, δ, α, β, A, B). The data sources are the PSID (Panel Study of Income Dynamics)[14], US Bureau of Labor Statistics (BLS)[15] and the US Bureau of Economic Analysis. The first two gives data on households' consumption, income and labor. The latter reports macroeconomic data from the EUKLEMS database on capital, labor and energy. The calibration process follows the one we have used in our previous paper (see Cremer et al. 1998).[16] The data allow us to identify four types of households, "managers and professionals" (type 1), "technical sales and clerical workers" (type 2), "service workers, operators, fabricators and laborers" (type 3) and "construction workers and mechanics" (type 4). Table 1 provides a summary of the data and parameter values. Finally, observe that our optimal tax calculations are based on the assumption that the government's external revenue requirement (share in GDP of expenditures on non-transfer payments) remains unchanged. The details of the calibration method are summarized in Appendix B (B1 and B2).

12 One may still want to tax intermediate goods, polluting or not, if taxing them affects the relative gross-of-tax wages of different workers. This can also open doors to better screening; see Cremer, Gahvari, and Ladoux (2010) and Naito (1999). This avenue is closed in our model as well as in all the optimal tax papers written in the Ramsey tradition.

13 It implies that two persons with the same labor income face identical marginal damage of emissions even if they may have different preferences and different labor supplies.

14 See http://psidonline.isr.umich.edu.

15 See http://www.bls.gov/.

16 However, the actual calibration is completely different because we use US data in the current paper, while the earlier papers presented applications based on French data.

Table 1: Calibrated parameters (monetary figures are in US dollars).

	Managers and professionals (type 1)	Technical sales and clerical workers (type 2)	Service workers, operators, fabricators and laborers (type 3)	Construction workers and mechanics (type 4)
π	35.18%	28.90%	28.86%	7.06%
I	68,712	40,147	31,887	44,111
px	51,134	34,742	29,155	37,498
qy	3,051	2,612	2,520	3,100
n	1.33620	0.90094	0.71472	0.88815
L	0.50731	0.43961	0.44015	0.48998
t	28.0%	15.0%	15.0%	15.0%
G	9,797	2,195	2,280	2,363
M	−5085	1,034	2,290	741
a	0.99997	0.99993	0.99989	0.99991
b	0.53201	0.39970	0.39438	0.46747
Type-independent figures				
	$p = 1.00000$	$q = 1.00000$	$\sigma = 0.8$	$\delta = 0.42141$
	$\rho = 0.66490$	$\omega = 0.26892$	$\alpha = 0.98662$	$\beta = 0.54242$
	$A = 1.28395$	$B = 0.74215$		

4 Results

Optimal energy taxes/subsidies are determined by solving the calibrated version of our model. The forces at work in their determination are twofold. One is Pigouvian in nature. To correct for the marginal social damage of emissions, one wants to impose a correcting tax on energy. In case of energy inputs, this is the only force at work. Another force comes into play in case of energy consumption goods. This arises because of the distributional considerations. Now because the share of energy expenditures tends to decrease with one's income, one may want to subsidize energy consumption goods to offset this regressive bias. It is true that an optimally designed income tax mitigates this regressive bias, but in a world of asymmetric information (where first best lump-sum taxes are unavailable), it cannot eliminate it completely (as long as the Atkinson and Stiglitz Theorem does not apply so that Pareto-efficient tax structures include commodity taxes). There still remains a role for energy subsidies; see Cremer, Gahvari, and Ladoux (1998, 2001, 2003, 2010).

Let τ^{pig} denote the Pigouvian tax (equal to the marginal social damage of emission) as defined by Cremer, Gahvari, and Ladoux (1998),[17]

17 This is an implicit expression with τ^{pig} being expressed in units of the numeraire output.

$$\tau^{\text{pig}} \equiv \sum_{j=1}^{4} \pi^j \left[\mathbf{V}\left(p, q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \phi \sum_{j=1}^{4} \pi^j \mathbf{y}(p, q, c^j; \theta^j) - \phi D \right]^{-\eta} \frac{\phi}{\mu}. \qquad [18]$$

In the case of inputs, the optimal energy tax is equal to τ^{pig}; see Proposition A2 and eq. [39] in Appendix A. On the other hand, in the case of energy consumption goods, both forces are at work in determining the energy tax, $q - p_D$. This is also shown in Appendix A where we prove that the optimal per unit tax on energy goods is given by

$$q - p_D = \tau^{\text{pig}} + \frac{\sum_{j=1}^{4} \sum_{k \neq j} \lambda^{kj} \left\{ \mathbf{V}_c\left(q, c^j, \frac{I^j}{wn^k}; \theta^k\right) \left[\mathbf{y}(q, c^j; \theta^j) - \mathbf{y}(q, c^j; \theta^k) \right] \right\}}{\mu \sum_{j=1}^{4} \pi^j \widetilde{\mathbf{y}}_q(q, c^j; \theta^j)}. \qquad [19]$$

To calculate these two taxes, we normalize the initial before-tax price of energy at 1 and set $\phi = 0.24$ which is the value used by Cremer, Gahvari, and Ladoux (2003) corresponding to a Pigouvian tax rate of approximately 50% of the initial producer price of one. The optimal prices of energy input and energy goods are reported in columns 2 and 3, and their optimal taxes in columns 4 and 5, of Table 2 (as the before-tax price of imported energy, relative to the domestic producer price of output in the economy, increases from one to two in column 1).

The optimal tax calculations reveal that while the optimal tax on energy input remains basically invariant to the world price of energy, the optimal tax on

Table 2: Optimal prices of, and taxes on, energy input and energy goods when

World price of energy (p_D)	Optimal price of energy input ($q^{\text{pig}} = p_D + \tau^{\text{pig}}$)	Optimal price of energy consumption goods (q)	Optimal tax on energy input ($\tau_D = \tau^{\text{pig}}$).	Optimal tax on energy consumption goods ($q - p_D$)
1.0	1.4823	1.3359	0.4823	0.3359
1.1	1.5769	1.4210	0.4769	0.3210
1.2	1.6718	1.5064	0.4718	0.3064
1.3	1.7668	1.5920	0.4668	0.2920
1.4	1.8622	1.6777	0.4622	0.2777
1.5	1.9577	1.7637	0.4577	0.2637
1.6	2.0534	1.8499	0.4534	0.2499
1.7	2.1493	1.9362	0.4493	0.2362
1.8	2.2454	2.0226	0.4454	0.2226
1.9	2.3416	2.1092	0.4416	0.2092
2.0	2.4380	2.1960	0.4380	0.1960

energy goods decreases as the world price of energy increases. That the tax on energy input does not change reflects the fact that the tax corrects for the externality caused by emissions – a cost that is basically independent of the producer price of energy. On the other hand, the decrease in the taxes on energy goods indicate that the tax should indeed be used to mitigate the impact of the increasing world energy prices on consumers.

The above observations concerning the behavior of optimal energy taxes are in terms of specific taxes. To the extent that these taxes may be levied on an ad valorem basis, the tax *rates* will show a different pattern. Specifically, with the producer prices increasing, one would expect the tax rates to be decreasing. This is indeed the case as shown in columns 2 and 3 of Table 3. Both tax *rates* show sharp declines.

Table 3: Optimal tax rate on energy inputs and energy consumption goods with its Pigouvian and redistributive components when $\phi = 0.24$.

World price of energy p_D	Optimal tax rate on energy inputs $\frac{\tau^{\text{pig}}}{p_D}$	Optimal tax rate on energy goods $\frac{q-p_D}{p_D}$	Pigouvian tax component $\frac{q^{\text{pig}}-p_D}{p_D}$	Redistributive tax component $\frac{q-q^{\text{pig}}}{p_D}$	Redistributive component as a % of social price $\frac{q-q^{\text{pig}}}{q^{\text{pig}}}$
1.0	48.23%	33.59%	48.23%	−14.64%	−9.88%
1.1	43.35%	29.19%	43.35%	−14.17%	−9.88%
1.2	39.32%	25.53%	39.32%	−13.78%	−9.89%
1.3	35.91%	22.46%	35.91%	−13.45%	−9.90%
1.4	33.01%	19.84%	33.01%	−13.18%	−9.90%
1.5	30.51%	17.58%	30.51%	−12.93%	−9.91%
1.6	28.34%	15.62%	28.34%	−12.72%	−9.91%
1.7	26.43%	13.89%	26.43%	−12.54%	−9.92%
1.8	24.74%	12.37%	24.74%	−12.38%	−9.92%
1.9	23.24%	11.01%	23.24%	−12.23%	−9.92%
2.0	21.90%	9.80%	21.90%	−12.10%	−9.93%

The behavior of energy input tax rates warrant no further discussion. However, in the case of energy goods, it will be instructive to look into its different components separately. In this way, one will be able to discern how each of these components responds to the rise in the world price of energy. To this end, we first define the concept of a "Pigouvian price" to reflect the "social opportunity cost" of energy. Using Cremer, Gahvari, and Ladoux s (1998) definition of the Pigouvian tax, τ^{pig}, this is defined by

$$q^{\text{pig}} = p_D + \tau^{\text{pig}}. \qquad [20]$$

One can then divide the optimal tax on energy consumption goods into two parts:

$$q - p_D = \left(q^{\text{pig}} - p_D\right) + \left(q - q^{\text{pig}}\right),$$

with $q^{\text{pig}} - p_D = \tau^{\text{pig}}$ denoting the Pigouvian component and $q - q^{\text{pig}}$ the redistributive component. Similarly, in terms of tax *rates*, we have

$$\frac{q - p_D}{p_D} = \frac{q^{\text{pig}} - p_D}{p_D} + \frac{q - q^{\text{pig}}}{p_D}.$$

Columns 4 and 5 in Table 3 show the calculations for these components. Interestingly, the redistributive (incentive) is negative throughout indicating that energy goods should be subsidized relative to other goods. This is in line with the previous literature on this subject and the empirical evidence that suggests energy taxes are regressive.

More specifically, consider Table 2 again. Initially, when the world price of energy is one dollar and the external emission damage is $0.4823, the Pigouvian price $1.4823. Under this circumstance, redistributive concerns call for a subsidy of $0.1464 resulting in a consumer price of energy equal to $1.3359. As the price of energy doubles, the Pigouvian tax changes only slightly (from $0.4823 to $0.4380). The redistributive subsidy, on the other hand, changes substantially (from $0.1464 to $0.2420 or an increase of 65%). Translating these changes into relative terms in Table 3, one observes that it is the Pigouvian tax rate which changes dramatically (from 48.23% of the energy price to 21.90% in column 4). On the other hand, the redistributive subsidy does not change much (decreasing from 14.64% to 12.10% in column 5).

To summarize these findings, the rise in the world price of energy does not affect the Pigouvian tax but increases the required redistributive subsidy measured in dollars. In percentage terms, on the other hand, the rise in the world price of energy lowers the Pigouvian tax drastically but does not affect the subsidy rate by much. Of these latter two changes, that the Pigouvian tax rate decreases is basically an arithmetic artifact: a constant marginal social damage is divided by a higher world price.[18]

In contrast, the behavior of the redistributive subsidy reflects a fundamental point: the subsidy rates, *as a percentage of the "social price,"* are constant. This finding is borne out by the calculations reported in the last column in Table 4. They show the subsidy rate on energy goods as a percentage of the Pigouvian

18 If the social damage function is convex, marginal social damage decreases as world price of energy increases (because consumption of energy decreases). This will reinforce the reduction in the Pigouvian tax rate.

Table 4: Subsidy rates on energy consumption goods as a percentage of their social price when $\phi = 0$.

p_D	q	$p_D - q$	$\frac{p_D - q}{p_D}$
1.0	0.8993	0.1007	10.07%
1.1	0.9892	0.1108	10.07%
1.2	1.0791	0.1209	10.07%
1.3	1.1690	0.1310	10.07%
1.4	1.2589	0.1411	10.08%
1.5	1.3488	0.1512	10.08%
1.6	1.4387	0.1613	10.08%
1.7	1.5287	0.1713	10.08%
1.8	1.6186	0.1814	10.08%
1.9	1.7085	0.1915	10.08%
2.0	1.7984	0.2016	10.08%

price or the social price – reflecting both the producer price of energy as well as the social damage of emissions – as the producer price of energy doubles (increases from one to two). Similarly, Figure 1 depicts the Pigouvian price, the optimal price and the implicit subsidy $\left(q^{\text{pig}} - q\right)$ in dollars as the world price of energy doubles.

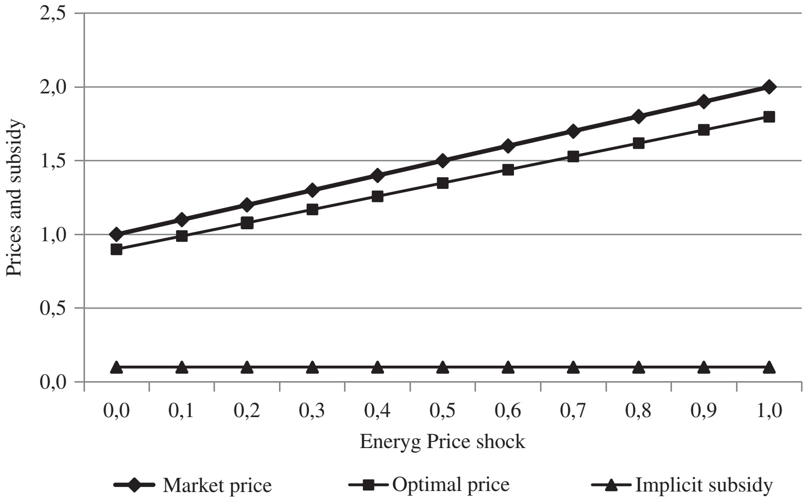

Figure 1: Prices and subsidies when $\phi = 0.24$.

Finally, it is interesting to note that the constancy of the tax/subsidy rate as a percentage of the social price of energy holds regardless of the emission costs and even if there are no such costs. Table 4 and Figure 2 depict the results in the absence of externality ($\phi = 0$). Under this circumstance, the world price of energy, p_D, reflects the social cost of a unit of energy and the consumer price, q, differs from the energy price by the redistributive subsidy $(q - p_D)$. The subsidy rate is then equal to $(q - p_D)/p_D$.

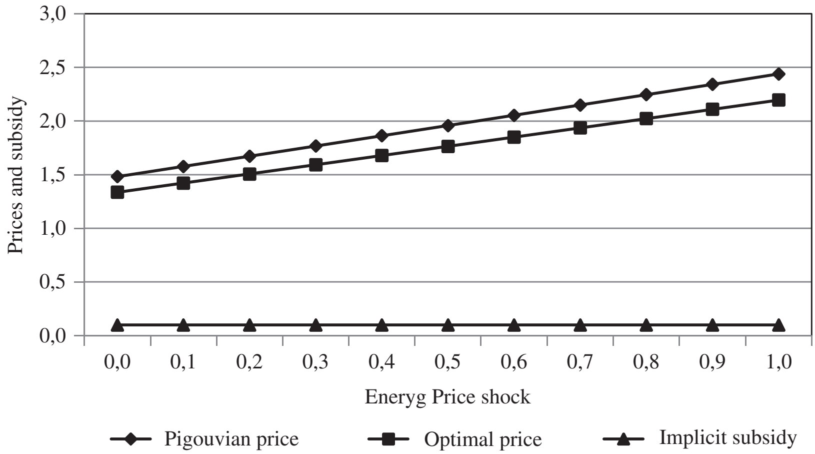

Figure 2: Prices and subsidies when $\phi = 0$.

5 Conclusion

This paper examines if an energy price shock should be compensated by a reduction in energy taxes to mitigate its impact on consumer prices. Such an adjustment is often debated and advocated for redistributive reasons. Our investigation is based on a modified version of the model for optimal environmental taxation developed by Cremer, Gahvari, and Ladoux (1998, 2001, 2003, 2010) and Cremer and Gahvari (2001). This model allows us to characterize second-best optimal taxes in the presence of externalities generated by the use of energy in consumption and in production. The current paper has calibrated a version of this model on the US data and has run simulations on the calibrated model to examine how one may want, in light of redistributive concerns, to alter energy tax rates as world oil prices increase. Quite importantly, these calculations account for the government's ability to also use the income tax schedule for effecting its redistributive goals.

The paper has shown that optimal energy taxes on consumption goods, as opposed to inputs, are affected by redistributive consideration and that the optimal energy tax is *less* than the Pigouvian tax (marginal social damage). The difference is a subsidy representing roughly 10% of the Pigouvian price of energy (its true social cost). Assuming that energy prices are subject to an exogenous shock, we have calculated the optimal tax mix, including income, commodity and energy taxes, for different levels of this shock. Simulations show that variation in the energy price has an almost negligible effect on the subsidy *rate* as a percentage of the Pigouvian price. On the other hand, the Pigouvian tax *rate* decreases substantially as the price of energy increases. This latter effect is simply a purely arithmetic adjustment due to the fact that the marginal social damage does not change. Nevertheless, it is also true that the dollar subsidy to the consumer price of energy increases by a small amount so that, in dollars, the consumer price should not increase by as much as world energy prices.

Appendix A

The Optimal General Income Plus Linear Commodity Taxes

The Lagrangian for the second-best problem is (where p is set equal to 1),

$$\begin{aligned}\pounds = {} & \frac{1}{1-\eta}\sum_{j=1}^{4}\pi^j\left[\mathbf{V}\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \phi\sum_{j=1}^{4}\pi^j\mathbf{y}\left(q, c^j; \theta^j\right) - \phi D\right]^{1-\eta} \\ & + \mu\left\{\mathbf{O}(K, L, D) - \sum_{j=1}^{4}\pi^j\mathbf{x}\left(q, c^j; \theta^j\right) - rK - p_D\left[\sum_{j=1}^{4}\pi^j\mathbf{y}\left(q, c^j; \theta^j\right) + D\right] - \overline{R}\right\} \\ & + \sum_j\sum_{k\neq j}\lambda^{jk}\left[\mathbf{V}\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \mathbf{V}\left(q, c^k, \frac{I^k}{wn^j}; \theta^j\right)\right] + \gamma[w - \mathbf{O}_L(K, D, L)].\end{aligned} \qquad [21]$$

where μ, λ^{jk} and γ are the multipliers associated, respectively, with the resource constraints, the incentive constraint and the endogenous wage condition. The first-order conditions are, for $j = 1, 2, 3, 4$,

$$\begin{aligned}\frac{\partial \pounds}{\partial q} = & \sum_{j=1}^{4} \pi^j \left[\mathbf{V}\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \phi \sum_{j=1}^{4} \pi^j \mathbf{y}(q, c^j; \theta^j) - \phi D \right]^{-\eta} \\ & \times \left[\mathbf{V}_q\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \phi \sum_{j=1}^{4} \pi^j \mathbf{y}_q(q, c^j; \theta^j) \right] \\ & - \mu \sum_{j=1}^{4} \pi^j \left[\mathbf{x}_q(q, c^j; \theta^j) + p_D \mathbf{y}_q(q, c^j; \theta^j) \right] \\ & + \sum_{j=1}^{4} \sum_{k \neq j} \lambda^{jk} \left[\mathbf{V}_q\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \mathbf{V}_q\left(q, c^k, \frac{I^k}{wn^j}; \theta^j\right) \right] = 0,\end{aligned} \quad [22]$$

$$\begin{aligned}\frac{\partial \pounds}{\partial c^j} = & \pi^j \left[\mathbf{V}\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \phi \sum_{j=1}^{4} \pi^j \mathbf{y}(q, c^j; \theta^j) - \phi D \right]^{-\eta} \mathbf{V}_c\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) \\ & - \phi \pi^j \mathbf{y}_c(q, c^j; \theta^j) \sum_{j=1}^{4} \pi^j \left[\mathbf{V}\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \phi \sum_{j=1}^{4} \pi^j \mathbf{y}(q, c^j; \theta^j) - \phi D \right]^{-\eta} \\ & - \mu \pi^j \left[\mathbf{x}_c(q, c^j; \theta^j) + p_D \mathbf{y}_c(q, c^j; \theta^j) \right] + \sum_{k \neq j} \lambda^{jk} \mathbf{V}_c\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) \\ & - \sum_{k \neq j} \lambda^{kj} \mathbf{V}_c\left(q, c^j, \frac{I^j}{wn^k}; \theta^k\right) = 0,\end{aligned} \quad [23]$$

$$\begin{aligned}\frac{\partial \pounds}{\partial I^j} = & \pi^j \left[\mathbf{V}\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \phi \sum_{j=1}^{4} \pi^j \mathbf{y}(q, c^j; \theta^j) - \phi D \right]^{-\eta} \frac{1}{wn^j} \mathbf{V}_L\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) \\ & + \mu \mathbf{O}_L(L, K, D) \frac{\pi^j}{w} + \sum_{k \neq j} \lambda^{jk} \frac{1}{wn^j} \mathbf{V}_L\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) \\ & - \sum_{k \neq j} \lambda^{kj} \frac{1}{wn^k} \mathbf{V}_L\left(q, c^j, \frac{I^j}{wn^k}; \theta^k\right) - \gamma \frac{\pi^j}{w} \mathbf{O}_{LL}(L, K, D) = 0,\end{aligned} \quad [24]$$

$$\begin{aligned}\frac{\partial \pounds}{\partial D} = & -\sum_{j=1}^{4} \pi^j \left[\mathbf{V}\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \phi \sum_{j=1}^{4} \pi^j \mathbf{y}(q, c^j; \theta^j) - \phi D \right]^{-\eta} \phi \\ & + \mu[\mathbf{O}_D(L, K, D) - p_D] - \gamma \mathbf{O}_{LD}(L, K, D) = 0,\end{aligned} \quad [25]$$

$$\frac{\partial \pounds}{\partial K} = \mu[\mathbf{O}_K(L, K, D) - r] - \gamma \mathbf{O}_{LK}(L, K, D) = 0, \quad [26]$$

$$
\begin{aligned}
\frac{\partial \pounds}{\partial w} = & \sum_{j=1}^{4} \pi^j \left[\mathbf{V}\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \phi \sum_{j=1}^{4} \pi^j \mathbf{y}(q, c^j; \theta^j) - \phi D \right]^{-\eta} \\
& \left(\frac{-I^j}{n^j w^2}\right) \mathbf{V}_L\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) + \mu\, \mathbf{O}_L(L, K, D) \frac{-1}{w^2} \sum_{j=1}^{4} \pi^j I^j \\
& + \sum_{j=1}^{4} \sum_{k \neq j} \lambda^{jk} \left(\frac{-I^j}{n^j w^2}\right) \mathbf{V}_L\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) \\
& + \sum_{j=1}^{4} \sum_{k \neq j} \lambda^{jk} \left(\frac{I^k}{n^j w^2}\right) \mathbf{V}_L\left(q, c^k, \frac{I^k}{wn^j}; \theta^j\right) \\
& + \gamma \left[1 + \frac{1}{w^2} \sum_{j=1}^{4} \pi^j I^j\, \mathbf{O}_{LL}(L, K, D) \right] = 0.
\end{aligned}
\qquad [27]
$$

We now show that whereas the optimal tax on the polluting good is non-Pigouvian (Proposition A1), the optimal tax on polluting input is Pigouvian (Proposition A2). Consider first the polluting good tax. We have:

Proposition A1 *The optimal tax on the polluting good is non-Pigouvian.*

Proof. Multiply eq. [23] by $\mathbf{y}(q, c^j; \theta^j)$, sum over j and add the resulting equation to eq. [22]. Simplifying, using Roy's identity, results in

$$
\begin{aligned}
& -\phi \sum_{j=1}^{4} \pi^j \left[\mathbf{V}\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \phi \sum_{j=1}^{4} \pi^j \mathbf{y}(q, c^j; \theta^j) - \phi D \right]^{-\eta} \\
& \times \left[\sum_{j=1}^{4} \pi^j \left[\mathbf{y}_q(q, c^j; \theta^j) + \mathbf{y}(q, c^j; \theta^j) \mathbf{y}_c(q, c^j; \theta^j) \right] \right] \\
& - \mu \sum_{j=1}^{4} \pi^j \left[\mathbf{x}_q(q, c^j; \theta^j) + \mathbf{y}(q, c^j; \theta^j) \mathbf{x}_c(q, c^j; \theta^j) \right] \\
& - \mu p_D \sum_{j=1}^{4} \pi^j \left[\mathbf{y}_q(q, c^j; \theta^j) + \mathbf{y}(q, c^j; \theta^j) \mathbf{y}_c(q, c^j; \theta^j) \right] \\
& - \sum_{j=1}^{4} \sum_{k \neq j} \left[\lambda^{kj} \mathbf{y}(q, c^j; \theta^j) \mathbf{V}_c\left(q, c^j, \frac{I^j}{wn^k}; \theta^k\right) + \lambda^{jk} \mathbf{V}_q\left(q, c^k, \frac{I^k}{wn^j}; \theta^j\right) \right] = 0.
\end{aligned}
\qquad [28]
$$

To simplify eq. [28], partially differentiate the j-type individual's budget constraint, $\mathbf{x}(q, c^j; \theta^j) + q\mathbf{y}(q, c^j; \theta^j) = c^j$, once with respect to c^j and once with respect to q. This yields

$$
\mathbf{x}_c(q, c^j; \theta^j) + q\mathbf{y}_c(q, c^j; \theta^j) = 1, \qquad [29]
$$

$$\mathbf{x}_q(q, c^j; \theta^j) + q\mathbf{y}_q(q, c^j; \theta^j) = -\mathbf{y}(q, c^j; \theta^j). \qquad [30]$$

Multiply eq. [29] by $\mathbf{y}(q, c^j; \theta^j)$ and add the resulting equation to eq. [30]. We get

$$\begin{aligned}&\mathbf{x}_q(q, c^j; \theta^j) + \mathbf{y}(q, c^j; \theta^j)\mathbf{x}_c(q, c^j; \theta^j)\\ &\quad = -q\left[\mathbf{y}_q(q, c^j; \theta^j) + \mathbf{y}(q, c^j; \theta^j)\mathbf{y}_c(q, c^j; \theta^j)\right].\end{aligned} \qquad [31]$$

Substituting eq. [31] into eq. [28], the latter equation is rewritten as

$$\begin{aligned}&\sum_{j=1}^{4}\pi^j\left[\mathbf{y}_q(q, c^j; \theta^j) + \mathbf{y}(q, c^j; \theta^j)\mathbf{y}_c(q, c^j; \theta^j)\right]\\ &\quad\times\left\{\mu(q - p_D) - \phi\sum_{j=1}^{4}\pi^j\left[\mathbf{V}\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \phi\sum_{j=1}^{4}\pi^j\mathbf{y}(q, c^j; \theta^j) - \phi D\right]^{-\eta}\right\}\\ &\quad-\sum_{j=1}^{4}\sum_{k\neq j}\left[\lambda^{kj}\mathbf{y}(q, c^j; \theta^j)\mathbf{V}_c\left(q, c^j, \frac{I^j}{wn^k}; \theta^k\right) + \lambda^{jk}\mathbf{V}_q\left(q, c^k, \frac{I^k}{wn^j}; \theta^j\right)\right] = 0.\end{aligned} \qquad [32]$$

Next, rewrite the last term on the left-hand side of eq. [32] as

$$\begin{aligned}&\sum_{j=1}^{4}\sum_{k\neq j}\lambda^{jk}\mathbf{V}_q\left(q, c^k, \frac{I^k}{wn^j}; \theta^j\right) = \sum_{j=1}^{4}\sum_{k\neq j}\lambda^{kj}\mathbf{V}_q\left(q, c^j, \frac{I^j}{wn^k}; \theta^k\right)\\ &\quad = \sum_{j=1}^{4}\sum_{k\neq j}\lambda^{kj}\mathbf{V}_c\left(q, c^j, \frac{I^j}{wn^k}; \theta^k\right)\mathbf{y}(q, c^j; \theta^k),\end{aligned} \qquad [33]$$

where going from the second to the last expression, we have made use of Roy's identity. Now substituting eq. [33] into eq. [32] results in

$$\begin{aligned}&\sum_{j=1}^{4}\pi^j\left[\mathbf{y}_q(q, c^j; \theta^j) + \mathbf{y}(q, c^j; \theta^j)\mathbf{y}_c(q, c^j; \theta^j)\right]\\ &\quad\times\left\{\mu(q - p_D) - \phi\sum_{j=1}^{4}\pi^j\left[\mathbf{V}\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \phi\sum_{j=1}^{4}\pi^j\mathbf{y}(q, c^j; \theta^j) - \phi D\right]^{-\eta}\right\}\\ &\quad-\sum_{j=1}^{4}\sum_{k\neq j}\lambda^{kj}\left\{\mathbf{V}_c\left(q, c^j, \frac{I^j}{wn^k}; \theta^k\right)\left[\mathbf{y}(q, c^j; \theta^j) - \mathbf{y}(q, c^j; \theta^k)\right]\right\} = 0.\end{aligned}$$

Denote the compensated demand function for y by $\tilde{\mathbf{y}}(q, c^j; \theta^j)$. Substituting $\tilde{\mathbf{y}}_q(q, c^j; \theta^j)$ for $\mathbf{y}_q(q, c^j; \theta^j) + \mathbf{y}(q, c^j; \theta^j)\mathbf{y}_c(q, c^j; \theta^j)$ in the above, dividing the resulting equation by $\mu\sum_{j=1}^{4}\pi^j\tilde{\mathbf{y}}_q(q, c^j; \theta^j)$, and rearranging yields

$$q - p_D = \frac{\phi}{\mu}\sum_{j=1}^{4}\pi^j\left[\mathbf{V}\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \phi\sum_{j=1}^{4}\pi^j\mathbf{y}(q, c^j; \theta^j) - \phi D\right]^{-\eta}$$
$$+\frac{\sum_{j=1}^{4}\sum_{k\neq j}\lambda^{kj}\left\{\mathbf{V}_c\left(q, c^j, \frac{I^j}{wn^k}; \theta^k\right)\left[\mathbf{y}(q, c^j; \theta^j) - \mathbf{y}(q, c^j; \theta^k)\right]\right\}}{\mu\sum_{j=1}^{4}\pi^j\tilde{\mathbf{y}}_q(q, c^j; \theta^j)}. \qquad [34]$$

This proves that $q - p_D$ is non-Pigouvian unless the polluting good demand depends only on one's income but not on his taste so that the second expression on the right-hand side of eq. [34] will be zero. ■

Second, we prove that the input tax is Pigouvian regardless of individuals' tastes. The proof is facilitated through the following lemma.

Lemma A1 *In the optimal income tax problem [21]) and characterized by the first-order conditions [22]–[27], the Lagrange multiplier associated with the constraint* $w = O_L(K, D, L)$, γ, *is equal to zero.*

Proof. Multiply eq. [24] through by I^j/w, sum over j, and simplify to get

$$\frac{1}{w^2}\sum_{j=1}^{4}\frac{\pi^j I^j}{n^j}\left[\mathbf{V}\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \phi\sum_{j=1}^{4}\pi^j\mathbf{y}(q, c^j; \theta^j) - \phi D\right]^{-\eta}\mathbf{V}\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right)$$
$$+\mu L + \frac{1}{w^2}\sum_j\sum_{k\neq j}\left[\left(\frac{I^j}{n^j}\right)\lambda^{jk}\mathbf{V}\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \left(\frac{I^j}{n^k}\right)\lambda^{kj}\mathbf{V}\left(q, c^j, \frac{I^j}{wn^k}; \theta^k\right)\right]$$
$$-\frac{1}{w^2}\gamma\,\mathbf{O}_{LL}(L, K, D)(wL) = 0. \qquad [35]$$

Substituting eq. [35] into eq. [27] and simplifying, we get

$$\sum_j\sum_{k\neq j}\left(\frac{I^j}{n^k}\right)\lambda^{kj}\mathbf{V}_L\left(q, c^j, \frac{I^j}{wn^k}; \theta^k\right) = \gamma w^2 + \sum_j\sum_{k\neq j}\left(\frac{I^k}{n^j}\right)\lambda^{jk}\mathbf{V}_L\left(q, c^k, \frac{I^k}{wn^j}; \theta^j\right). \qquad [36]$$

Then rewrite the left-hand side of eq. [36] as

$$\sum_j\sum_{k\neq j}\left(\frac{I^j}{n^k}\right)\lambda^{kj}\mathbf{V}_L\left(q, c^j, \frac{I^j}{wn^k}; \theta^k\right) = \sum_j\sum_{k\neq j}\left(\frac{I^k}{n^j}\right)\lambda^{jk}\mathbf{V}_L\left(q, c^k, \frac{I^k}{wn^j}; \theta^j\right). \qquad [37]$$

Substituting eq. [37] into eq. [36] implies

$$\gamma = 0.$$ ■

Observe that Lemma A1 is in fact an application of the production efficiency result as it tells us that $w = \mathbf{O}_L(K, D, L)$ imposes no constraint on our second-best problem. Using this lemma, we can easily show:

Proposition A2 *The optimal tax on energy input is Pigouvian.*

Proof. Using the result that $\gamma = 0$ in the first-order conditions [24]–[27], simplifies them to

$$\frac{\partial \pounds}{\partial I^j} = \pi^j \left[\mathbf{V}\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \phi \sum_{j=1}^{4} \pi^j \mathbf{y}(q, c^j; \theta^j) - \phi D \right]^{-\eta} \frac{1}{wn^j} \mathbf{V}_L\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right)$$
$$+ \mu\pi^j + \sum_{k \neq j} \lambda^{jk} \frac{1}{wn^j} \mathbf{V}_L\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \sum_{k \neq j} \lambda^{kj} \frac{1}{wn^k} \mathbf{V}_L\left(q, c^j, \frac{I^j}{wn^k}; \theta^k\right) = 0, \tag{38}$$

$$\frac{\partial \pounds}{\partial D} = -\sum_{j=1}^{4} \pi^j \left[\mathbf{V}(q, c^j, I^j wn^j; \theta^j) - \phi \sum_{j=1}^{4} \pi^j \mathbf{y}(q, c^j; \theta^j) - \phi D \right]^{-\eta} \phi$$
$$+ \mu[\mathbf{O}_D(L, K, D) - p_D] = 0, \tag{39}$$

$$\frac{\partial \pounds}{\partial K} = \mu[\mathbf{O}_K(L, K, D) - r] = 0, \tag{40}$$

$$\frac{\partial \pounds}{\partial w} = \sum_{j=1}^{4} \pi^j \left[\mathbf{V}\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \phi \sum_{j=1}^{4} \pi^j \mathbf{y}(q, c^j; \theta^j) - \phi D \right]^{-\eta}$$
$$\left(\frac{-I^j}{n^j w^2}\right) \mathbf{V}_L\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) + \mu \mathbf{O}_L(L, K, D) \frac{-1}{w^2} \sum_{j=1}^{4} \pi^j I^j$$
$$+ \sum_{j=1}^{4} \sum_{k \neq j} \lambda^{jk} \left(\frac{-I^j}{n^j w^2}\right) \mathbf{V}_L\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right)$$
$$+ \sum_{j=1}^{4} \sum_{k \neq j} \lambda^{jk} \left(\frac{I^k}{n^j w^2}\right) \mathbf{V}_L\left(q, c^k, \frac{I^k}{wn^j}; \theta^j\right). \tag{41}$$

That the input tax is Pigouvian follows immediately from eq. [39]. ■

Appendix B

B1: Calibration of the Parameters of the Nested CES Production Function

The data used from EUKLEMS include:
- gross value added,
- capital compensation,
- labor compensation,
- intermediate energy compensation,
- gross value added at current basic prices,
- capital services, volume indices, 1995 = 100,
- total hours worked by persons engaged (millions),
- number of persons engaged (thousands),
- intermediate energy inputs, volume indices, 1995 = 100,
- gross value added, volume indices, 1995 = 100,
- capital-labor elasticity (from the literature).

From these data, K, D, L, r, w, $p_E = p_D(1+\tau_D)$ can easily be derived. These data also allow to build price indexes for O and Γ, respectively, p_O and p_Γ. To do this define the following notation:

$$S_\Gamma \equiv \frac{p_\Gamma \Gamma}{C} = \frac{p_K K + p_D D}{C}$$

$$\sigma_{KD} = \text{elasticity of substitution between } K \text{ and } D,$$

$$\sigma_{KL} = \text{elasticity of substitution between } K \text{ and } L,$$

$$\sigma_{LD} = \text{elasticity of substitution between } L \text{ and } D.$$

Using the properties of our nested CES function it follows that,

$$\begin{aligned} \sigma_{KL} &= \sigma_{LD} = \sigma \\ \sigma_{KD} &= \frac{1}{S_\Gamma}\delta + \left(1 - \frac{1}{S_\Gamma}\right)\sigma \end{aligned} \qquad [42]$$

Econometric estimations found in the literature show that σ is close to 0.8[19]. There is less consensus about the value of σ_{KD} but it is generally shown to be

19 See, for instance, Diewert and Wales (1987).

weakly complement in the short run and substitutable in the long run. We take the value $\sigma_{KD} = -0.2$ and derive the value of the parameter δ from eq. [42].

The parameters α, β, A and B are then the solution to the following system of equations[20]

$$O = B\left[(1-\beta)L^{\frac{\sigma-1}{\sigma}} + \beta\Gamma^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}},$$

$$\Gamma = A\left[\alpha K^{\frac{\delta-1}{\delta}} + (1-\alpha)D^{\frac{\delta-1}{\delta}}\right]^{\frac{\delta}{\delta-1}},$$

$$O_L(L,K,D) = w,$$

$$O_K(L,K,D) = r,$$

$$O_D(L,K,D) = p_D(1+\tau_D),$$

where their analytical expressions as derived from the CES forms combining O and Γ are substituted for $O_L(L,K,D) = w$, $O_K(L,K,D) = r$, $O_D(L,K,D) = p_D(1+\tau_D)$.

B2: Calibration of the Parameters of the Nested CES Utility Functions

We combine data from the BLS and the PSID. We need those two sources because the BLS does not give any information on labor supply. The data from the BLS are given for different categories of households while the data from the PSID are individual data about 7,407 households. Consequently we must allocate the 7,407 households of the PSID to the different categories of households found in the BLS nomenclature. This is performed by considering the PSID information about the main occupation of the family head. We consider four categories of households:

- Managers and professionals
- Technical sales and clerical workers
- Service workers, operators, fabricators and laborers
- Construction workers and mechanics

The data used from the PSID are as follows:

- Family interview ID number
- Labor income of the head
- Labor income of the wife

20 Let us note that the same values for α, β, A and B can be obtained by using the duality properties of CES cost and production functions.

- Main occupation of the head
- Main occupation of the wife
- Work weeks by the head
- Work hours by the head
- Wage rate by the head
- Work weeks by the wife
- Work hours by the wife
- Wage rate by the wife

The data used from the BLS are as follows:

- Number of consumer units
- Income before and after taxes
- Wages and salaries
- Personal taxes (federal income taxes, state and local income taxes, other taxes)
- Average number of persons, of child under 18, of persons 65 and over, of earners, of vehicle in the consumer unit (sex of reference, person, age, etc.)
- Average annual expenditures

The last item is given in a very detailed nomenclature of goods and services.

We use those data to build the small data set we need to calibrate the parameters of the utility functions of the different categories of households listed above. This data set includes the following variables:

- Labor supply (L)
- Before and after tax wage (respectively w and w_n)
- Non-energy consumption and energy consumption (respectively, x and y)

The j's consumer budget constraint is linearized and thus written,

$$px^j + qy^j = G^j + M^j + w_n^j L^j,$$

where p and q are the consumer prices of x and y, G^j is the income adjustment term needed for linearizing the budget constraint (virtual income that can be estimated by using the data above together with the official US tax Schedule) and M^j is the individual's exogenous income. The first-order conditions for a j-type's optimization problem are given by

$$\frac{1-a^j}{a^j}\left(\frac{x^j}{y^j}\right)^{\frac{1}{\omega}} = \frac{q}{p},$$

$$\frac{(1-b^j)(x^j/(1-L^j))^{\frac{1}{\rho}}}{a^j b^j \left[a^j + (1-a^j)(x^j/y^j)^{\frac{1-\omega}{\omega}}\right]^{\frac{\omega-\rho}{\rho(1-\omega)}}} = \frac{w_n^j}{p}.$$

As the energy and non-energy taxes are almost the same in the United States, the relative price of energy is not distorted by the tax system, we therefore set $p = q = 1$. We have three equations and five unknown parameters, a^j, b^j, ω, ρ and M^j to estimate. As shown in the above equations, the parameters of the elasticities of substitution are the same for all categories of households. The value of these parameters has been chosen by the help of some very simple econometric estimations and values found in the existing literature. The sensitivity of the results to the choice of ω is very small; the sensitivity to the choice ρ is larger but the results are not qualitatively affected by the choice of ρ.

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