Topics

Yumiko Taba*

Optimal Product R&D Policies with Endogenous Quality Choices and Unilateral Spillover

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Abstract: This study derives non-cooperative and cooperative optimal product research and development (R&D) policies of a country with a high-quality firm and a country with a low-quality firm in the presence of technology spillover under Cournot and Bertrand competitions in an international duopoly. When the respective governments determine their R&D policies non-cooperatively, optimal policies for both countries involve an R&D tax (subsidy) if spillover is large (small). When the governments choose their R&D policies cooperatively, a tax is always optimal for the country with low-quality firm and a subsidy (tax) is optimal for the country with high-quality firm if spillover is large (small). In addition, we show that the non-cooperative optimal product R&D Policy is tax for a wider range of spillover effects under Cournot competition, compared to the case of Bertrand competition.

Keywords: product R&D policy, endogenous quality choices, technology spillover

JEL Classification: F13

1 Introduction

Government research and development (R&D) policies play a vital role in supporting innovation and product development by domestic firms and in determining those firms' competitiveness in international markets. Thus, the strategic use of governments R&D policies must affect domestic and rival firms.

In a market of vertically differentiated products, quality-improving product R&D receives greater attention than cost-reducing process R&D. Indeed, quality

*Corresponding author: Yumiko Taba, Faculty of Education and Integrated Arts and Sciences, Waseda University, Nishi-Waseda 1-6-1, Shinjuku-ku, Tokyo 196-8050, Japan, E-mail: taba-y@aoni.waseda.jp

competition is becoming particularly fierce in recent globalized markets. Because production costs cannot be infinitely reduced, the main strategy firms can use to position themselves in domestic and international markets is to offer upgraded and differentiated products (Fernandes and Paunov 2013; Moreira 2007). Moreover, technology spillovers have a huge impact on the firms' competitiveness, especially in such a product-differentiated market. Thus, a firm's strategic choices regarding product quality are significant for its survival.

This study aims to derive the optimal product R&D policies when governments of high-quality and low-quality firms determine these non-cooperatively and cooperatively. It also takes into account how product R&D policies affect international rivalries in the presence of unilateral technology spillover. To achieve this task, we construct a three-stage international duopoly model with endogenous quality choices within the framework of a third-country trade model. We assume zero production costs and fixed quality ordering. In particular, we focus on Cournot-quantity competition. In addition, we check the robustness of the results in the case of Bertrand-price competition. In the presence of technology spillover, a significant finding is that the non-cooperative optimal product R&D policies depend on the magnitude of the spillover. When the spillover is small, the optimal R&D policies of both governments are subsidies, whereas when the spillover is large, the optimal R&D policies for both governments are taxes irrespective of the mode of competition. However, there exists a range of spillover for which a subsidy is optimal under Bertrand competition but a tax is optimal under Cournot competition. Furthermore, the analysis shows that the range of spillover in which the optimal R&D policy is tax is wider under Cournot competition than under Bertrand competition.

Decisions regarding product R&D tax/subsidy policies with endogenous quality choices have been analyzed in several works. Previous papers can be divided into two categories based on the models they used. One employs the traditional Hotelling vertically differentiated model; this is analyzed by Park (2001), Zhou et al. (2002), Jinji (2003), Jinji and Toshimitsu (2006), and others. These previous analyses derive almost the same results as those discussed below, and the results depend on the mode of competition. Park (2001) and Zhou et al. (Zhou et al. 2002) assume the fixed quality ordering of two goods. Jinji (2003) develops the analysis under endogenous quality ordering and symmetric R&D costs. Jinji and Toshimitsu (2006) expand it with a small technology gap and asymmetric R&D costs. Toshimitsu and Jinji (2008) extend the work of Zhou et al. (2002) by assuming asymmetric marginal production costs and show that the results obtained in previous papers may be reversed.

Another category of papers uses a horizontal and vertical product differentiation model (Ishii 2014; Toshimitsu 2014; Taba and Ishii 2014).¹ The present study is based on this model.

We employ a utility function based on horizontal and vertical product differentiation because, to reflect the real world, we consider different industries from those in the previous studies that employed Hotelling utility function. These previous studies simulate R&D in high-tech industries (Jinji 2003), such as electronics (Zhou et al. 2002) and semiconductors (Park 2001). However, firms in other industries such as food processing and agriculture also engage in innovation and invest in quality-improving product R&D. As shown by Görg and Strobl (2007), in Ireland, the most important sectors for R&D are chemicals, food, metals and engineering. The increasing significance of R&D or innovation activities for the food processing sector matches the gradual surge in competition brought about by the globalized food market. It also corresponds to the continuous growth in the international trade of processed food products, which responds to a consumer demand for food variety (Ghazalian 2012).² Furthermore, in high-tech industries, a consumer may consume only one product rather than multiple products. However, in the reality of food, clothing and other industries, a consumer purchases multiple products over a wide range of quality: from cheap, low-quality products to expensive, high-quality products (Ishii 2014). Thus, to analyze the optimal R&D policies for such industries, we employ a horizontal and vertical product differentiation model instead of the Hotelling model.

These two models are mainly differentiated by demand functions derived by the utility functions assumed in the two analyses, and these induce diverse outcomes on governments' product R&D policies, especially under Cournot-quantity competition. Let us make a quick review of the models and results to clarify the differences.

Suppose that the two models assume a three-stage game under an international duopoly, in which one firm produces a high-quality product and the other firm produces a low-quality product in the absence of spillover. In the first stage, the governments of both countries determine their product R&D policies; in the second stage, the firms choose their qualities; and in the third stage, the firms compete on price or quantity in the third country's market. In the Hotelling

¹ Using the same kind of utility function, Häckner (2000) and Symeonidis (2003) compare the firm's profitability and welfare effects resulting from Bertrand and Cournot competition but do not analyze R&D policies.

² See Ghazalian (2012) for more details on the private R&D expenditures of food processing industries in OECD countries.

vertically differentiated model, there are two types of consumers: one consumes only a high-quality product and the other consumes only a low-quality product: that is, each type of consumer purchases a single product. In a horizontal and vertical product differentiation model, a consumer purchases multiple products.

In the case of Cournot-quantity competition, the decisive factor in determining governments' product R&D policies is how consumers are divided into two types (Park 2001). Under two models, an increase in a firm's own product quality enhances its own production and reduces the rival firm's quality and production. In the Hotelling model, a high-quality (low-quality) firm targets a consumer who is willing to pay for high (low) quality products. Therefore, the high-quality (low-quality) firm's government subsidizes (taxes) its domestic firm's R&D investment to promote (degrade) the quality of its domestic firm's products. In the horizontal and vertical product differentiation model, consumers are not divided into two types and they consume two differentiated products. The firms decide their strategy in order to obtain a larger market share. Thus, governments determine their product R&D policies to increase their domestic firms' production. Therefore, both governments subsidize their domestic firms' product R&D investment.

In the case of Bertrand-price competition, under both product differentiation models, a rise in the low-quality (high-quality) firm's product quality reduces (expands) the quality differentiation and intensifies (moderates) the subsequent price competition. However, the governments' optimal policies differ depending on the model. In the Hotelling vertically differentiated model, the low-quality firm's government (high-quality firm's government) subsidizes (taxes) its domestic firm. Conversely, in the horizontal and vertical product differentiation model, both firms' governments subsidize their domestic firms' product R&D investment activities. This difference arises as, in the Hotelling model, both the high-quality and low-quality firms' governments try to reduce the price competition and expand the quality differentiation between the two firms because the consumers in this case are divided into two types and each firm earns profit by concentrating on its target consumer. However, in the horizontal and vertical differentiation model, governments decide their policies to allow the domestic firms to take advantage of price competition in the market rather than to make the competition less intense. This is because, under this model, consumers purchase multiple products and the firm with the higher price can earn more profit. Thus, governments subsidize their domestic firms differently than those in the Hotelling model.

Furthermore, when the governments jointly determine their product R&D policies, the results are different from those described above, where they decide their policies non-cooperatively. As shown by Zhou et al. (2002) in the Hotelling

model, the optimal policies for both countries are taxes under Cournot competition, whereas the low-quality (high-quality) firm's government has an incentive to tax (subsidize) its domestic firm's R&D investments under Bertrand competition. However, in the horizontal and vertical differentiation model, both governments have incentives to tax their domestic R&D activities irrespective of the mode of competition. Under both models, the main reason for switching the policies from those in a non-cooperative case is to eliminate the motive for rent extraction.

The works of Ishii (2014), Toshimitsu (2014) and Taba and Ishii (2014), among others who employ the horizontal and vertical differentiation model, have similarities with our study. They assume a similar utility function and the same setting for the model, namely, the three-stage international duopoly model provided by Spencer and Brander (1983) with endogenous quality choices, as mentioned above. Under this setting, Ishii (2014) analyzes non-cooperative R&D policy in the absence of spillover, whereas Toshimitsu (2014) takes into account both non-cooperative and cooperative policy in the presence of demand spillover. However, both authors consider only the case of Bertrand-price competition. Taba and Ishii (2014) analyze non-cooperative R&D policy in the absence of spillover under Cournot competition and show that the optimal policies of both countries are subsidies.

In what follows, we discuss the differences between the analysis of Toshimitsu (2014) and ours. First, he mainly focuses on bilateral demand spillovers rather than on unilateral technology spillover, which we assume. Second, the competition mode he presumes is Bertrand-price competition, but this paper deals especially with Cournot-quantity competition. Third, he employs a general cost function of R&D investments, while we assume a quadratic one.

When we compare Bertrand competition results analyzed in Toshimitsu (2014) with ours, the results are qualitatively the same. That is, the governments' optimal non-cooperative (cooperative) policies for high-quality and low-quality firms' R&D investment are subsidies (taxes) in the absence of spillover. In the presence of large (small) spillover, the optimal R&D policies are taxes (subsidies).

However, our work differs from Toshimitsu's (2014) in the details to reach the outcome. In particular, what factor affects the slope of the reaction function in the second quality-setting stage? When governments determine their R&D policies, the important factor is the sign of cross-effects (the slope of the reaction function) in the second quality-choice stage. If one assumes demand spillover, as in Toshimitsu's (2014) analysis, those effects are affected by the magnitude of the spillover and the degree of substitutability between products. Conversely, if one supposes the technology spillover that we consider, those effects are influenced by the technology spillover and not only the degree of substitutability

between products but also the relative marginal quality improvement of R&D investments. Therefore, this technology spillover affects the firm's activities and the governments' optimal policies more directly than demand spillover. Furthermore, we analyze the cases of both Bertrand and Cournot competition. Then, we present the difference in the area of spillover in which the governments subsidize or tax their domestic firms under Cournot and Bertrand competition, respectively. Finally, Toshimitsu (2014) assumes a general form of R&D cost function; however, even if this is quadratic, which we assume, similar results are obtained.

The rest of the paper is organized as follows: Section 2 sets up the model and the assumptions. Section 3 analyzes the non-cooperative and cooperative optimal product R&D policies under Cournot competition. Section 4 examines these policies under Bertrand competition. In Section 5 we discuss the results.

2 The Model

In this section, we introduce the model and its assumptions. The model is similar to the strategic trade model provided by Spencer and Brander (1983), except for its endogenous choice of quality and technology spillover.

Consider an international duopoly in the presence of spillover where there are two firms: one is a high-quality firm located in H-country and the other is a low-quality firm located in L-country. Each firm produces a quality-differentiated product, all of which is exported to the third country.

The consumers in the third country have a utility function that depends on the quantities as well as their qualities of the goods they consume. The utility function is defined as

$$U(x, x^*, q, q^*) = e(x + x^*) + (qx + q^*x^*) - \frac{m(x^2 + x^{*2})}{2} - nxx^* + z, (m > n) \quad [1]$$

where x and x^* are consumptions, and q and q^* are qualities of the goods from the low-quality firm and the high-quality firm, respectively. It is based on Ishii (2014).

This utility function has the following properties. According to Häckner (2000) and Symeonidis (2003), this utility function is quadratic in the consumption of two differentiated goods; and it is linear in the consumption of numéraire goods denoted by z . In addition, under this utility function, we can ignore the income effects of these goods because the consumer's expenditure on these differentiated goods is a small amount of his/her income. $e > 1$ is a positive

parameter, and $n \in [-m, m]$ shows the substitutability of the products. If n is positive, the products are substitutes; if it is negative, the products are complements. However, throughout the present model we assume that n and m are positive exogenous parameters. The variables marked with “*” below represent those associated with the high-quality firm. A consumer’s utility is zero if nothing is consumed. Utility maximization subject to a budget constraint yields the following linear inverse demand functions:

$$p = e + q - mx - nx^*, \quad [2.1]$$

$$p^* = e + q^* - mx^* - nx \quad [2.2]$$

where p and p^* are the prices of goods from each firm.

The firms engage in product R&D activities to improve the quality of their goods. They choose their levels of R&D investment, those of quality and output, to maximize their profits. We assume that there is a technology spillover from a high-quality firm to a low-quality firm. This spillover effect is defined below.

First, let us define quality functions q and q^* . In the quality functions, $\underline{q} > 0$ and $\underline{q}^* > 0$ are the initial quality levels that are determined before product R&D investments (or competition). The positive parameters θ and θ^* express the marginal quality improvement in terms of investments and we assume that $\theta^* > \theta > 0$. The product R&D investments are $I \geq 0$ and $I^* \geq 0$. The quality function of the low-quality firm is defined as

$$q = \underline{q} + \theta(I + \varepsilon I^*), \quad [3.1]$$

where $\varepsilon \in [0, 1]$ is spillover effect from the high-quality firm to the low-quality firm.³

The quality function of the high-quality firm is defined as

$$q^* = \underline{q}^* + \theta^* I^*. \quad [3.2]$$

Moreover, each firm incurs an R&D investment cost. The R&D cost functions are defined as

$$C^I(I) = \frac{\gamma}{2} I^2, \quad [4.1]$$

$$C^{*I}(I^*) = \frac{\gamma^*}{2} I^{*2}, \quad [4.2]$$

³ Even if we assume general quality functions and spillover effects such as $q = q(I, I^*)$ and $q^* = q^*(I^*)$, and $\varepsilon = (\partial q / \partial I^*) / (\partial q / \partial I)$, we can obtain similar results, but the analysis becomes more complicated.

where $\gamma^I > 0$ and $\gamma^{*I} > 0$.⁴ We assume that the quality of the low-quality firm is inferior to that of the high-quality firm and that there is no possibility of changing the quality ordering across firms. That is, the following condition always holds:

$$\gamma^I > \gamma^{*I}. \quad [4.3]$$

Equation [4.3] shows that there is a large difference between the R&D investment costs of firms so that the firm in L-country has no incentive to choose a quality level that is higher than that of the firm in H-country.

The profit functions of the low-quality and high-quality firms are given by

$$\begin{aligned} \pi &= p \cdot x - \frac{\gamma^I}{2} I^2 (1 - s) \\ &= (e + q - mx - nx^*)x - \frac{\gamma^I}{2} I^2 (1 - s), \end{aligned} \quad [5.1]$$

$$\begin{aligned} \pi^* &= p^* \cdot x^* - \frac{\gamma^{*I}}{2} I^{*2} (1 - s^*) \\ &= (e + q^* - mx^* - nx)x^* - \frac{\gamma^{*I}}{2} I^{*2} (1 - s^*), \end{aligned} \quad [5.2]$$

where s and s^* are government subsidies (taxes) if these are positive (negative). We assume that only the governments determine R&D policies for their domestic firms. Thus, each government maximizes a social surplus equal to the firm's profit net of aggregate subsidy payments to the domestic firm, which is defined by

$$W(s, s^*) = \pi - \frac{\gamma^I}{2} I^2 s, \quad [6.1]$$

$$W^*(s, s^*) = \pi^* - \frac{\gamma^{*I}}{2} I^{*2} s^*. \quad [6.2]$$

The analysis is performed within the framework of a three-stage game, and the entire game is classified as a non-cooperative game. In the first stage, each government chooses its optimal R&D policies. In the second stage, each firm simultaneously chooses the levels of R&D investments (qualities) given the governments' R&D policies implemented in the first stage and considering spillover effects. Finally, in the third stage, the firms compete in quantities in a third market. The solution is the sub-game perfect Nash equilibrium.

⁴ As with the quality functions, even supposing that we use more general R&D cost functions, we can qualitatively reach the same results by setting the model in the present.

3 Cournot Competition in the Third Stage

The first step is to derive the perfect Nash equilibrium in the third stage, viz. Cournot-Nash equilibrium outputs. Given the qualities of the goods $q(I, I^*)$, $q^*(I^*)$ or the amount of investments (I, I^*) chosen in the second stage and the R&D policies (s, s^*) implemented by the governments in the first stage, the firms simultaneously choose the output levels to maximize their profits. The first-order conditions of each firm are

$$\frac{\partial \pi}{\partial x} = e + q(I, I^*) - nx^* - 2mx = 0 \quad [7.1]$$

$$\frac{\partial \pi^*}{\partial x^*} = e + q^*(I^*) - nx - 2mx^* = 0 \quad [7.2]$$

The second-order conditions and the stability conditions are satisfied as follows: $\pi_{xx} = \pi_{x^*x^*}^* = -2m < 0$, $\pi_{xx^*} = \pi_{x^*x}^* = -n < 0$ and $\pi_{xx}\pi_{x^*x^*}^* - \pi_{xx^*}\pi_{x^*x}^* = 4m^2 - n^2 > 0$, where $m > n$. Then, the outputs (x and x^*) are strategic substitutes. $\partial^2 \pi / \partial j \partial i$ is abbreviated as π_{ij} , and $\partial^2 \pi^* / \partial j \partial i$ is abbreviated as π_{ij}^* . The Cournot-Nash equilibrium outputs in the third stage are solved as

$$x = \frac{(2mq - nq^*) + e(2m - n)}{\Delta} \quad [8.1]$$

$$x^* = \frac{(2mq^* - nq) + e(2m - n)}{\Delta} \quad [8.2]$$

where $\Delta = 4m^2 - n^2 > 0$.

3.1 Comparative Statics in the Third Stage

We are now interested in how the qualities or R&D investments of both firms affect the equilibrium outputs in the presence of spillover effect. Partially differentiating the outputs of the low-quality firm eq. [8.1] with respect to (I, I^*) yields

$$\frac{\partial x}{\partial I} = \frac{2m\theta}{\Delta} > 0, \quad [9.1]$$

$$\frac{\partial x}{\partial I^*} = \frac{(2m\varepsilon\theta - n\theta^*)}{\Delta} \begin{cases} < 0 & \text{if } 0 \leq \varepsilon < \frac{n}{2m} \frac{\theta^*}{\theta} \\ = 0 & \text{if } \varepsilon = \frac{n}{2m} \frac{\theta^*}{\theta} \\ > 0 & \text{if } \frac{n}{2m} \frac{\theta^*}{\theta} < \varepsilon \leq 1. \end{cases} \quad [9.2]$$

Similarly, by partially differentiating eq. [8.2], the effects on the output of the high-quality firm are given by

$$\frac{\partial x^*}{\partial I^*} = \frac{(2m\theta^* - n\varepsilon\theta)}{\Delta} > 0, \quad [9.3]$$

$$\frac{\partial x^*}{\partial I} = -\frac{n\theta}{\Delta} < 0, \quad [9.4]$$

where spillover depends on the substitutability between two goods denoted by $n/2m$, and the relative marginal quality improvements in terms of the R&D investment shown as θ^*/θ . We define a small spillover as a spillover in the area $0 \leq \varepsilon < \frac{n}{2m} \frac{\theta^*}{\theta}$ and a large spillover as one in the area $\frac{n}{2m} \frac{\theta^*}{\theta} < \varepsilon \leq 1$; these are shown in Figure 1. In eq. [9.3], as depicted in Figure 1, the sign of $\partial x^*/\partial I^*$ is negative if and only if $\varepsilon > 2m\theta^*/n\theta$, and it is positive if and only if $0 \leq \varepsilon < 2m\theta^*/n\theta$. However, we restrict the range of $\varepsilon \in [0, 1]$ by assumption. Thus, we obtain Lemma 1.

Figure 1: Spillover and cross-effects with Cournot competition.

Lemma 1 *In the presence of spillover from a high-quality firm to a low-quality firm, (1) an increase in the firm's own investment raises its own output, (2) an increase in a low-quality firm's investment always decreases the rival firm's output, and (3) an increase in a high-quality firm's investment raises (decreases) the rival firm's output if $\frac{n}{2m} \frac{\theta^*}{\theta} < \varepsilon \leq 1$ ($0 \leq \varepsilon < \frac{n}{2m} \frac{\theta^*}{\theta}$).*

These results are intuitive. An increase in the low-quality firm's investment raises its own output through its own quality improvement. Conversely, an increase in the high-quality firm's investment can have two effects, depending on the degree of spillover. First, if spillover is large ($\varepsilon > n\theta^*/2m\theta$), it improves the low-quality firm's quality and then increases its output. Second, when spillover

is small ($0 \leq \varepsilon < n\theta^*/2m\theta$), if the high-quality firm increases its investment, spillover does not raise the low-quality firm's quality significantly; rather, it increases the high-quality firm's quality and output, which in turn decreases the low-quality firm's output. If $\varepsilon = n\theta^*/2m\theta$, an increase in the high-quality firm's investment does not affect the low-quality firm's output.

3.2 Quality Choices in the Second Stage

The firms in both countries determine their qualities in order to maximize their own profits given the R&D policies provided by the low-quality and high-quality firms' governments in the first stage. Differentiating each firm's profit, as given in eqs [5.1] and [5.2], with respect to its own investment (I or I^*), obtains the best-response functions. The first-order condition of the low-quality firm is

$$\frac{\partial \pi}{\partial I} = \frac{4m^2\theta x}{\Delta} - \gamma^I \cdot I(1-s) = 0 \quad [10.1]$$

and the second-order condition is $\partial^2 \pi / \partial I^2 = (8m^3\theta^2/\Delta^2) - \gamma^I(1-s) < 0$ if and only if $s < 1 - (8m^3\theta^2/\Delta^2\gamma^I)$. The first-order condition of the high-quality firm is

$$\frac{\partial \pi^*}{\partial I^*} = \frac{2mx^*}{\Delta} (2m\theta^* - n\varepsilon\theta) - \gamma^{*I} \cdot I^*(1-s^*) = 0 \quad [10.2]$$

and the second-order condition is $\partial^2 \pi^* / \partial I^{*2} = 2m(2m\theta^* - n\varepsilon\theta)^2/\Delta^2 - \gamma^{*I}(1-s^*) < 0$ if and only if $s^* < 1 - (2m(2m\theta^* - n\varepsilon\theta)^2/\Delta^2\gamma^{*I})$. We assume that the second-order conditions in the second stage are satisfied. If the second-order and stability conditions are satisfied, then from eqs [10.1] and [10.2], the Nash-equilibrium R&D investments for both firms are obtained. These are represented as the function of both countries' R&D policies: $I = I(s, s^*)$ and $I^* = I^*(s, s^*)$.

The cross-effect of the low-quality firm is given by

$$\frac{\partial^2 \pi}{\partial I^* \partial I} = \frac{4m^2\theta}{\Delta^2} (2m\varepsilon\theta - n\theta^*) \begin{cases} < 0 & \text{if } 0 \leq \varepsilon < \frac{n\theta^*}{2m\theta} \\ = 0 & \text{if } \varepsilon = \frac{n\theta^*}{2m\theta} \\ > 0 & \text{if } \frac{n\theta^*}{2m\theta} < \varepsilon \leq 1 \end{cases} \quad [11.1]$$

and that of the high-quality firm is

$$\frac{\partial^2 \pi^*}{\partial I \partial I^*} = -\frac{2mn\theta}{\Delta^2} (2m\theta^* - n\varepsilon\theta) < 0 \Leftrightarrow \varepsilon \leq 1. \quad [11.2]$$

where $\pi_{I^*,I}^* \leq 0$ if $\varepsilon \leq 2m\theta^*/n\theta$; however, we omit the area $\varepsilon > 1$ by construction.

In Figure 1, we do not consider the range where the magnitude of spillover is larger than 1 because of the assumption that spillover is smaller than 1. By using these conditions, we can obtain effects on the firms' R&D investments when the optimal R&D policies change.

In the presence of spillover effects, the cross-effect of the high-quality firm is always negative and its reaction function is downward sloping. Thus, the two firms' investments are strategic substitutes. The cross-effect of the low-quality firm, however, depends on the magnitude of spillover. That is, if spillover is small ($\varepsilon < n\theta^*/2m\theta$), the cross-effect is negative and the reaction function slopes downward. Thus, the firms' investments are strategic substitutes. In this case, an increase of the high-quality firm's investment decreases the low-quality firm's investment and its quality, output and profit. Conversely, if spillover is large ($n\theta^*/2m\theta \leq \varepsilon < 1$), the reaction function slopes upward and thus, the firms' investments are strategic complements. In this case, an increase in the high-quality firm's investment increases the low-quality firm's investment, quality, output and profit. If $\varepsilon = \frac{n}{2m} \frac{\theta^*}{\theta}$, the high-quality firm's investment does not affect the low-quality firm's marginal profit in terms of investment.

3.3 Comparative Statics in the Second Stage

This section examines how governments' R&D policies influence the activities of firms. The results of the subsequent analysis for comparative statics are used in determining governments' R&D policies, which are analyzed in the first stage. Totally differentiating eqs [10.1] and [10.2] yields

$$\begin{bmatrix} \pi_{II} & \pi_{II^*} \\ \pi_{I^*I}^* & \pi_{I^*I^*}^* \end{bmatrix} \begin{bmatrix} dI \\ dI^* \end{bmatrix} = \begin{bmatrix} -ds \\ -ds^* \end{bmatrix} \quad [12]$$

where $\pi_{II}\pi_{I^*I^*}^* - \pi_{II^*}\pi_{I^*I}^* = \delta > 0$. From eq. [12], the effects of R&D subsidies on the firms' R&D investments are given by

$$\frac{\partial I}{\partial s} = -\frac{\pi_{I^*I}^* \gamma^I I}{\delta} > 0, \quad \frac{\partial I^*}{\partial s^*} = -\frac{\pi_{II} \gamma^{*I} I^*}{\delta} > 0. \quad [13.1]$$

While each government's R&D subsidy raises the investments of its domestic firm, the effects of an increase in a rival's R&D policy depend on the degree of spillover and the strategic nature in the second stage. That is,

$$\frac{\partial I}{\partial s^*} = \frac{\pi_{II^*} \gamma^* I^*}{\delta} \begin{cases} < 0 & \text{if } 0 \leq \varepsilon < \frac{n}{2m} \frac{\theta^*}{\theta} \\ = 0 & \text{if } \varepsilon = \frac{n}{2m} \frac{\theta^*}{\theta} \\ > 0 & \text{if } \frac{n}{2m} \frac{\theta^*}{\theta} < \varepsilon \leq 1 \end{cases}, \quad \frac{\partial I^*}{\partial s} = \frac{\pi_{I^*I}^* \gamma^* I}{\delta} < 0. \quad [13.2]$$

An increase in L-country's R&D subsidy always decreases the high-quality firm's R&D investments because the strategic nature in this stage is negative irrespective of the magnitude of spillover. Conversely, an increase in H-country's R&D subsidy decreases the low-quality firm's R&D investment if spillover is small, whereas it raises its investment if spillover is large. Therefore, we can obtain Lemma 2.

Lemma 2 *In the presence of spillover from a high-quality firm to a low-quality firm, (1) an increase in the respective government's R&D subsidy enhances its own domestic firms' investment, (2) an increase in L-country government's R&D subsidy always reduces the rival firm's R&D investments, and (3) an increase in H-country government's R&D subsidy enhances (reduces) the rival firm's R&D investments if and only if $\frac{n}{2m} \frac{\theta^*}{\theta} \leq \varepsilon < 1$ ($0 \leq \varepsilon < \frac{n}{2m} \frac{\theta^*}{\theta}$).*

These effects are very intuitive. They show the direct effects of respective governments' subsidies on the investments of their domestic firms and the rival firms. Basically, these effects depend on the cross-effects in the second stage. Clearly, a government's R&D subsidy enhances its domestic firm's investment and L-country's subsidy lowers the high-quality firm's investment irrespective of the magnitude of spillover. Conversely, the effects of H-country's subsidy on the low-quality firm's R&D investment depend on the magnitude of spillover. If spillover is small ($0 \leq \varepsilon < \frac{n}{2m} \frac{\theta^*}{\theta}$), an increase in H-country's subsidy lowers the low-quality firm's investment through the increase of the high-quality firm's investment. If spillover is large, it increases the low-quality firm's investment. If $\varepsilon = n\theta^*/2m\theta$, an increase in H-country's subsidy does not affect the low-quality firm's investment.

Although the governments' R&D policies directly affect the firms' R&D investments, the potential impact on quality is of more interest. For analyzing these effects, the partially differentiating quality functions of the low-quality firm $q(I(s, s^*), I^*(s, s^*))$ and the high-quality firm $q^*(I^*(s, s^*))$ in terms of s and s^* give

$$\frac{\partial q}{\partial s} = \theta \left(\frac{\partial I}{\partial s} + \varepsilon \frac{\partial I^*}{\partial s} \right) \geq 0, \quad \frac{\partial q^*}{\partial s^*} = \theta^* \frac{\partial I^*}{\partial s^*} > 0 \quad [14.1]$$

where we use Lemma 2. The governments' R&D subsidies ameliorate their domestic firms' quality. The effects of the rival government's R&D subsidies are as follows:

$$\frac{\partial q}{\partial s^*} = \theta \left(\frac{\partial I}{\partial s^*} + \varepsilon \frac{\partial I^*}{\partial s^*} \right) \begin{cases} < 0 & \text{if } 0 \leq \varepsilon < \frac{4m^2 n \theta \theta^*}{\Delta^2 \gamma^I (1-s)} \\ = 0 & \text{if } \varepsilon = \frac{4m^2 n \theta \theta^*}{\Delta^2 \gamma^I (1-s)} \\ > 0 & \text{if } \frac{4m^2 n \theta \theta^*}{\Delta^2 \gamma^I (1-s)} < \varepsilon \leq 1 \end{cases}, \quad \frac{\partial q^*}{\partial s} = \theta^* \frac{\partial I^*}{\partial s} < 0$$

[14.2]

Lemma 3 *In the presence of spillover from a high-quality firm to a low-quality firm, (1) an increase in the respective government's R&D subsidy improves its own domestic firm's quality, (2) an increase in L-country's R&D subsidy always degrades the high-quality firm's product quality and (3) an increase in H-country's R&D subsidy degrades (improves) the rival firm's quality if $0 \leq \varepsilon < \frac{4m^2 n \theta \theta^*}{\Delta^2 \gamma^I (1-s)} \left(\frac{4m^2 n \theta \theta^*}{\Delta^2 \gamma^I (1-s)} < \varepsilon \leq 1 \right)$.*

This is because each country's subsidy enhances the domestic firm's quality; L-country's subsidy always decreases the high-quality firm's quality because it increases its domestic firm's R&D investment, and the best response of the high-quality firm is to decrease its R&D investment because of the downward slope of the reaction function. However, H-country's subsidy has two effects, depending on the magnitude of spillover. It decreases the low-quality firm's quality if spillover is small $\left(0 \leq \varepsilon < \frac{4m^2 n \theta \theta^*}{\Delta^2 \gamma^I (1-s)} \right)$ because, in this case, the slope of the reaction function is sloping downward. It enhances the low-quality firm's quality if spillover is large $\left(\frac{4m^2 n \theta \theta^*}{\Delta^2 \gamma^I (1-s)} < \varepsilon \leq 1 \right)$ because, in this case, the slope of the reaction function of the high-quality firm is upward.

Intuitively, in the presence of spillover effects, an increase in H-country's subsidy has two effects on the low-quality firm's quality. One is an indirect effect, which the subsidy affects the low-quality firm's investment through spillover. The other is a direct effect caused by the subsidy on the high-quality firm's R&D investment. This increases the high-quality firm's investment, but spillover lowers the magnitude of this effect. If spillover is large, an increase in H-country's subsidy improves the low-quality firm's investment and quality, and the increase in the low-quality firm's investment decreases the high-quality firm's investment. However, a marginal increase of investment in the

high-quality firm in terms of government's R&D subsidy ($\partial I^*/\partial s^*$) does not take a negative sign because we assume that spillover is less than and equal to one. Conversely, if spillover is small, an increase in H-country's subsidy decreases the low-quality firm's investment by enhancing the high-quality firm's investment. If $\varepsilon = \frac{4m^2n\theta\theta^*}{\Delta^2\gamma^I(1-s)}$, an increase in H-country's subsidy does not affect the low-quality firm's investment.

3.4 The Choice of Optimal R&D Policies in the First Stage

Let us now derive the policies of both governments non-cooperatively and cooperatively. We define a non-cooperative policy as a government maximizes the social surplus, while a cooperative policy is determined by maximizing the total social surplus of both countries. Even in cases where governments jointly maximize the social surplus of the countries in question, each firm maximizes its own profits, independently.

3.5 Non-cooperative Product R&D Policy

Governments determine their optimal product R&D policies in order to maximize their social surplus, which is defined in eqs [6.1] and [6.2]. By differentiating eq. [6.1] with respect to s , the first-order condition of the low-quality firm's country in the first stage is

$$\frac{\partial W}{\partial s} = \left[\frac{\partial \pi}{\partial x^*} \left(\frac{\partial x^*}{\partial q^*} \frac{\partial q^*}{\partial I^*} + \frac{\partial x^*}{\partial q} \frac{\partial q}{\partial I^*} \right) + \frac{\partial \pi}{\partial q} \frac{\partial q}{\partial I^*} \right] \frac{\partial I^*}{\partial s} - \gamma^I I s \frac{\partial I}{\partial s} = 0$$

The optimal R&D policy of that country is given by

$$s^c = \frac{2mx(2m\varepsilon\theta - n\theta^*) \cdot \partial I^*/\partial s}{\Delta\gamma^I I \cdot \partial I/\partial s} \begin{cases} < 0 & \text{if } \frac{n}{2m} \frac{\theta^*}{\theta} < \varepsilon \leq 1 \\ = 0 & \text{if } \varepsilon = \frac{n}{2m} \frac{\theta^*}{\theta} \\ > 0 & \text{if } 0 \leq \varepsilon < \frac{n}{2m} \frac{\theta^*}{\theta} \end{cases} \quad [15.1]$$

L-country has an incentive to subsidize (tax) its domestic firm's R&D activities if spillover is small (large). This depends on the degree of spillover and how the governments' R&D policies affect the firm's R&D activities, namely, the sign of $\partial I^*/\partial s$ derived from Lemma 2. However, if the spillover is less than and equal to 1, $\partial I^*/\partial s$ is negative. Therefore, the subsidy depends on only the magnitude of

spillover. Hence, if the spillover is small (large), the product R&D policy is a subsidy (tax).

Differentiating eq. [6.2] with respect to s^* yields the first-order condition of the high-quality firm's country as

$$\frac{\partial W^*}{\partial s^*} = \frac{\partial \pi^*}{\partial x} \frac{\partial x}{\partial q} \frac{\partial q}{\partial I} \frac{\partial I}{\partial s^*} - \gamma^* I^* s^* \frac{\partial I^*}{\partial s^*} = 0.$$

The optimal policy of the high-quality firm's country is represented by

$$s^{*c} = \frac{-2mnx^*\theta \cdot \partial I / \partial s^*}{\Delta \gamma^* I^* \cdot \partial I^* / \partial s^*} \begin{cases} < 0 & \text{if } \frac{n}{2m} \frac{\theta^*}{\theta} < \varepsilon \leq 1 \\ = 0 & \text{if } \varepsilon = \frac{n}{2m} \frac{\theta^*}{\theta} \\ > 0 & \text{if } 0 \leq \varepsilon < \frac{n}{2m} \frac{\theta^*}{\theta}. \end{cases} \quad [15.2]$$

As in the case of the low-quality firm's country, the optimal policy of the high-quality firm's country depends on the degree of spillover, which relates to the sign of $\partial I / \partial s^*$ derived in Lemma 2. If the spillover effect is large, ($\partial I / \partial s^* > 0$), the optimal policy of the high-quality firm's country is a tax. In contrast, if the spillover effect is small ($\partial I / \partial s^* < 0$), the government's optimal R&D policy is a subsidy.

Proposition 1 *When the governments of L-country and H-country determine their product R&D policies non-cooperatively, in the presence of spillover from a high-quality firm to a low-quality firm, (I) optimal R&D policies of both countries are subsidies (taxes) if $0 \leq \varepsilon < \frac{n}{2m} \frac{\theta^*}{\theta}$ ($\frac{n}{2m} \frac{\theta^*}{\theta} < \varepsilon \leq 1$), (II) the optimal R&D tax/subsidy rate for both countries is just equal to zero if $\varepsilon = \frac{n}{2m} \frac{\theta^*}{\theta}$.*

In the presence of spillover effect, an increase in its own R&D investment affects the firm's strategic nature at the quality choice stage. Therefore, the associated governments' policies change as follows. For the low-quality firm, an increase in its own investment always lowers the rival firm's investment. Therefore, L-country's government backs its domestic firm by subsidizing. However, when spillover is large, its domestic firm can increase its investment through spillover, and then, the government imposes an R&D tax to cut down on its spending and prevent over-investment of the domestic firm. For the high-quality firm, an increase in its own investment raises (lowers) the rival firm's investment if spillover is large (small). Therefore, H-country's government has an incentive to tax (subsidize) its domestic firm's investment if spillover is large (small). When $\varepsilon = \frac{n}{2m} \frac{\theta^*}{\theta}$, spillover does not affect either firms' investments or their profits. Therefore, the governments' optimal R&D subsidy or tax rate is just equal to zero

because the spillover effect offsets the strategic effects. This is a major difference with or without spillover.⁵

3.6 Cooperative Product R&D Policy

As in many previous studies, this section investigates the cooperative policies provided by both governments. These policies are characterized by maximizing the joint social surplus of both countries. Thus, the cooperative social surplus that both governments take into account is defined as follows:

$$W^{Ic}(s, s^*) = W(s, s^*) + W^*(s, s^*). \quad [16]$$

The first-order condition of the low-quality firm's government is

$$\begin{aligned} \frac{\partial W^{Ic}}{\partial s} &= \frac{\partial W}{\partial s} + \frac{\partial W^*}{\partial s} \\ &= \frac{\partial I^*}{\partial s} \left\{ \frac{\partial \pi}{\partial x^*} \left(\frac{\partial x^*}{\partial q} \frac{\partial q}{\partial I^*} + \frac{\partial x^*}{\partial q^*} \frac{\partial q^*}{\partial I^*} \right) + \frac{\partial \pi}{\partial q} \frac{\partial q}{\partial I^*} \right\} + \frac{\partial I}{\partial s} \frac{\partial \pi^*}{\partial x} \frac{\partial x}{\partial q} \frac{\partial q}{\partial I} \\ &\quad - \gamma^I I s \frac{\partial I}{\partial s} - \gamma^* I^* s^* \frac{\partial I^*}{\partial s} = 0. \end{aligned} \quad [17.1]$$

The first-order condition of the high-quality firm's government is

$$\begin{aligned} \frac{\partial W^{Ic}}{\partial s^*} &= \frac{\partial W}{\partial s^*} + \frac{\partial W^*}{\partial s^*} \\ &= \frac{\partial I^*}{\partial s^*} \left\{ \frac{\partial \pi}{\partial x^*} \left(\frac{\partial x^*}{\partial q} \frac{\partial q}{\partial I^*} + \frac{\partial x^*}{\partial q^*} \frac{\partial q^*}{\partial I^*} \right) + \frac{\partial \pi}{\partial q} \frac{\partial q}{\partial I^*} \right\} + \frac{\partial I}{\partial s^*} \frac{\partial \pi^*}{\partial x} \frac{\partial x}{\partial q} \frac{\partial q}{\partial I} \\ &\quad - \gamma^I I s \frac{\partial I}{\partial s^*} - \gamma^* I^* s^* \frac{\partial I^*}{\partial s^*} = 0. \end{aligned} \quad [17.2]$$

The optimal cooperative product R&D policies are determined using eqs [17.1] and [17.2]. The optimal R&D policy of the low-quality firm's government is

$$s^{Ic} = \frac{1}{\gamma^I I} \frac{\partial \pi^*}{\partial x} \frac{\partial x}{\partial q} \frac{\partial q}{\partial I} < 0. \quad [18.1]$$

⁵ The optimal product R&D policies in the absence of spillover under the Cournot competition are analyzed in Taba and Ishii (2014) in the similar model setting. They use general quality functions and assume constant production costs. Then they show that the optimal product R&D policies of both countries are subsidies.

The optimal policy of the high-quality firm's government is

$$s^{Jc} = \frac{1}{\gamma^* I^*} \left[\frac{\partial \pi}{\partial x^*} \left(\frac{\partial x^*}{\partial q} \frac{\partial q}{\partial I^*} + \frac{\partial x^*}{\partial q^*} \frac{\partial q^*}{\partial I^*} \right) + \frac{\partial \pi}{\partial q} \frac{\partial q}{\partial I^*} \right]$$

$$= \frac{2mkx}{\Delta \gamma^* I^*} (2m\varepsilon\theta - n\theta^*) \begin{cases} > 0 & \text{if } \frac{n}{2m} \frac{\theta^*}{\theta} < \varepsilon \leq 1 \\ = 0 & \text{if } \varepsilon = \frac{n}{2m} \frac{\theta^*}{\theta} \\ < 0 & \text{if } 0 \leq \varepsilon < \frac{n}{2m} \frac{\theta^*}{\theta}. \end{cases} \quad [18.2]$$

Thus, we have

Proposition 2 *When governments of L-country and H-country jointly determine their R&D policies, (I) in the absence of spillover, both governments tax their domestic firms' product R&D investments, (II) in the presence of spillover from a high-quality firm to a low-quality firm, (II.1) L-country's government always taxes its domestic firm, (II.2.a) H-country's government taxes (subsidizes) its domestic firm's R&D investment if $0 \leq \varepsilon < \frac{n}{2m} \frac{\theta^*}{\theta}$ ($\frac{n}{2m} \frac{\theta^*}{\theta} < \varepsilon \leq 1$), and (II.2.b) the optimal R&D tax/subsidy rate for H-country's government is just equal to zero if $\varepsilon = \frac{n}{2m} \frac{\theta^*}{\theta}$.*

These results are in sharp contrast with those seen when governments determine their optimal R&D policies non-cooperatively, as shown in Section 3.5. The primary reason is that policy cooperation between the countries eliminates the strategic effect so that they do not have incentives to deprive each other of market share in the international market. Therefore, in the presence of spillover, because the low-quality firm can improve its quality without incurring additional costs, the government imposes a tax rather than a subsidy in order to maximize the joint social surplus.

Conversely, the high-quality firm loses profit as a result of quality improvement by the low-quality firm when spillover is large. Thus, the high-quality firm's government backs its domestic firm up by subsidizing its product R&D investment. If spillover is small, the externality is so small that overinvestment occurs, which increases the market share of high-quality firm's products in the international market and discourages the low-quality firm's profit. Therefore, the high-quality firm's government imposes a product R&D tax on the domestic firm. In the absence of spillover, the governments impose taxes on their domestic firms. The reasons are the same as in the case of a small spillover. As with the case of non-cooperative R&D policies, H-country's optimal R&D tax/subsidy rate is just equal to zero when $\varepsilon = \frac{n}{2m} \frac{\theta^*}{\theta}$.

4 Bertrand Competition

This section considers Bertrand-price competition using the same setting of the model as in our discussion of Cournot competition. Utility maximization subject to a budget constraint yields the following linear demand functions for each firm:

$$x = \frac{e(m-n) + (mq - nq^*) + (np^* - mp)}{m^2 - n^2} \quad [19.1]$$

$$x^* = \frac{e(m-n) + (mq^* - nq) + (np - mp^*)}{m^2 - n^2} \quad [19.2]$$

The profit function of each firm is defined, respectively, as follows:

$$\begin{aligned} \pi &= p \cdot x - \frac{\gamma^I}{2} I^2 (1-s) \\ &= p \left(\frac{e(m-n) + (mq - nq^*) + (np^* - mp)}{m^2 - n^2} \right) - \frac{\gamma^I}{2} I^2 (1-s) \end{aligned} \quad [20.1]$$

$$\begin{aligned} \pi^* &= p^* \cdot x^* - \frac{\gamma^{*I}}{2} I^{*2} (1-s^*) \\ &= p^* \left(\frac{e(m-n) + (mq^* - nq) + (np - mp^*)}{m^2 - n^2} \right) - \frac{\gamma^{*I}}{2} I^{*2} (1-s^*). \end{aligned} \quad [20.2]$$

4.1 The Third Stage: Price Competition

In the third stage, the firms from H- and L-countries compete in the third-country market under price competition. The firms determine their prices. The first-order condition of each firm is

$$\frac{\partial \pi}{\partial p} = x - p \cdot \frac{m}{m^2 - n^2} = 0, \quad [21.1]$$

$$\frac{\partial \pi^*}{\partial p^*} = x^* - p^* \cdot \frac{m}{m^2 - n^2} = 0. \quad [21.2]$$

The second-order conditions are $\pi_{pp} = \pi_{p^*p^*}^* = -m/(m^2 - n^2) < 0$. The stability conditions are also satisfied: $\pi_{pp}\pi_{p^*p^*}^* - \pi_{pp^*}\pi_{p^*p}^* > 0$, where $\pi_{pp^*} = \pi_{p^*p}^* = n/(m^2 - n^2) > 0$. From the first-order conditions in eqs [20.1] and [20.2] and the stability condition, the Nash equilibrium prices are determined as follows:

$$p = \frac{e(m-n)(2m+n) + q(2m^2 - n^2) - mnq^*}{4m^2 - n^2}, \quad [22.1]$$

$$p^* = \frac{e(m-n)(2m+n) + q^*(2m^2 - n^2) - mnq}{4m^2 - n^2}. \quad [22.2]$$

4.2 The Second Stage: Quality Choice

In the second stage, the firms choose their product qualities so as to maximize their profits. The first-order condition of the low-quality firm is

$$\frac{\partial \pi}{\partial I} = \frac{\partial \pi}{\partial p^*} \frac{\partial p^*}{\partial q} \frac{\partial q}{\partial I} + \frac{\partial \pi}{\partial q} \frac{\partial q}{\partial I} + \frac{\partial \pi}{\partial I} = 0. \quad [23.1]$$

That of the high-quality firm is

$$\frac{\partial \pi^*}{\partial I^*} = \frac{\partial \pi^*}{\partial p} \left(\frac{\partial p}{\partial q} \frac{\partial q}{\partial I^*} + \frac{\partial p}{\partial q^*} q'^*(\cdot) \right) + \frac{\partial \pi^*}{\partial q^*} q'^*(\cdot) + \frac{\partial \pi^*}{\partial q} \frac{\partial q}{\partial I^*} + \frac{\partial \pi^*}{\partial I^*} = 0. \quad [23.2]$$

We assume that the second-order conditions hold: $\pi_{II} < 0$ if $s < 1 - 2m(2m^2 - n^2)^2 \theta^2 / (m^2 - n^2)(4m^2 - n^2)^2 \gamma^I$, and $\pi_{I^*I^*}^* < 0$ if $s^* < 1 - 2m(mn\theta\varepsilon - (2m^2 - n^2)\theta^*)^2 / (m^2 - n^2)(4m^2 - n^2)^2 \gamma^{*I}$. The cross-effects of both firms are given as

$$\frac{\partial^2 \pi}{\partial I^* \partial I} \begin{cases} < 0 & \text{if } 0 \leq \varepsilon < \frac{mn}{(2m^2 - n^2)} \frac{\theta^*}{\theta} \\ = 0 & \text{if } \varepsilon = \frac{mn}{(2m^2 - n^2)} \frac{\theta^*}{\theta} \\ > 0 & \text{if } \frac{mn}{(2m^2 - n^2)} \frac{\theta^*}{\theta} < \varepsilon \leq 1 \end{cases}, \quad \frac{\partial^2 \pi^*}{\partial I \partial I^*} < 0 \quad \text{if } 0 \leq \varepsilon \leq 1. \quad [24]$$

From the second-order conditions and eq. [24], the stability condition in the second stage is satisfied: $\pi_{II}\pi_{I^*I^*}^* - \pi_{II^*I}^*\pi_{I^*I}^* > 0$. From these conditions, the subgame perfect Nash equilibrium investments $I = I(s, s^*)$ and $I^* = I^*(s, s^*)$ of firms are obtained.

From eq. [24], the relationship between spillover and cross-effects are depicted in Figure 2. In Figure 2, we do not consider the area in which spillover is larger than 1 by assumption. As in the case of Cournot competition, the slope of the reaction function of the high-quality firm is always negative, while that of the low-quality firm depends on the magnitude of spillover. If spillover is small, the slope of reaction function is negative, and if spillover is large, it is positive. If $\varepsilon = \frac{mn}{(2m^2 - n^2)} \frac{\theta^*}{\theta}$, the rival firm's R&D investment does not affect its own profit.

Figure 2: Spillover and cross-effects with Bertrand competition.

4.3 The First Stage: Governments' Optimal R&D Policies

Governments determine their R&D policies so as to maximize their domestic social surplus. The first-order condition of L-country's government is

$$\frac{\partial W}{\partial s} = \left[\frac{\partial \pi}{\partial p^*} \left(\frac{\partial p^*}{\partial q} \frac{\partial q}{\partial I^*} + \frac{\partial p^*}{\partial q^*} q'^*(\cdot) \right) + \frac{\partial \pi}{\partial q} \frac{\partial q}{\partial I^*} + \frac{\partial \pi}{\partial q^*} q'^*(\cdot) \right] \frac{\partial I^*}{\partial s} - \gamma^I I s = 0. \quad [25]$$

The optimal product R&D policy of L-country's government is

$$s^b = \frac{2m^2 p(2m\theta\varepsilon - n\theta^*) \frac{\partial I^*}{\partial s}}{(m^2 - n^2)(4m^2 - n^2)\gamma^I I} \begin{cases} > 0 & \text{if } 0 \leq \varepsilon < \frac{mn}{(2m^2 - n^2)} \frac{\theta^*}{\theta} \\ = 0 & \text{if } \varepsilon = \frac{mn}{(2m^2 - n^2)} \frac{\theta^*}{\theta} \\ < 0 & \text{if } \frac{mn}{(2m^2 - n^2)} \frac{\theta^*}{\theta} < \varepsilon \leq 1 \end{cases} \quad [26]$$

where $\partial I^* / \partial s < 0$. The first-order condition of H-country's government is

$$\frac{\partial W^*}{\partial s^*} = \frac{\partial \pi^*}{\partial q} \frac{\partial q}{\partial I} \frac{\partial I}{\partial s^*} - \gamma^* I^* s^* = 0. \quad [27]$$

The optimal product R&D policy of H-country's government is

$$s^{*b} = \frac{-m\theta p^* \frac{\partial I}{\partial s^*}}{(m^2 - n^2)\gamma^* I^*} \begin{cases} > 0 & \text{if } 0 \leq \varepsilon < \frac{mn}{(2m^2 - n^2)} \frac{\theta^*}{\theta} \\ = 0 & \text{if } \varepsilon = \frac{mn}{(2m^2 - n^2)} \frac{\theta^*}{\theta} \\ < 0 & \text{if } \frac{mn}{(2m^2 - n^2)} \frac{\theta^*}{\theta} < \varepsilon \leq 1 \end{cases} \quad [28]$$

where $\frac{\partial I}{\partial s^*} > 0$ if $\frac{mn\theta^*}{(2m^2-n^2)\theta} < \varepsilon \leq 1$ and $\frac{\partial I}{\partial s^*} < 0$ if $0 \leq \varepsilon < \frac{mn\theta^*}{(2m^2-n^2)\theta}$.

Proposition 3 *In the presence of spillover from a high-quality firm to a low-quality firm, (I) the optimal R&D policies of both countries are subsidies (taxes) if $0 \leq \varepsilon < \frac{mn}{(2m^2-n^2)} \frac{\theta^*}{\theta}$ ($\frac{mn}{(2m^2-n^2)} \frac{\theta^*}{\theta} < \varepsilon \leq 1$), and (II) the optimal R&D tax/subsidy rate for both countries is just equal to zero if $\varepsilon = \frac{mn}{(2m^2-n^2)} \frac{\theta^*}{\theta}$.*

These results are the same as with Cournot competition, as described above. In the presence of spillover, the results depend on how each firm's investment changes when the rival government's subsidy increases ($\frac{\partial I^*}{\partial s}$ and $\frac{\partial I}{\partial s^*}$) and on the magnitude of the spillover. For the low-quality firm's government, an increase in its own subsidy always decreases the high-quality firm's R&D investment. Thus, the sign of the policy is determined by the magnitude of the spillover. On the other hand, for the high-quality firm's government, an increase in its own investment increases (decreases) the rival firm's investment if spillover is large (small). The sign of the policy is determined depending on this effect. The results are similar to those of Toshimitsu (2014). However, his demand spillover analysis does not include the case in which the policy rate is equal to zero.

4.4 Cooperative Product R&D Policy: Bertrand Competition

This section considers cooperative policy by both governments under Bertrand competition and compares the results with those under Cournot competition, as analyzed in Section 3.6. The aggregate social surplus is the same as in eq. [16]. The first-order condition of L-country's government is

$$\begin{aligned} \frac{\partial W^L}{\partial s} &= \frac{\partial W}{\partial s} + \frac{\partial W^*}{\partial s} \\ &= \frac{\partial I^*}{\partial s} \left\{ \frac{\partial \pi}{\partial p^*} \left(\frac{\partial p^*}{\partial q} \frac{\partial q}{\partial I^*} + \frac{\partial p^*}{\partial q^*} \frac{\partial q^*}{\partial I^*} \right) + \frac{\partial \pi}{\partial q} \frac{\partial q}{\partial I^*} + \frac{\partial \pi}{\partial q^*} \frac{\partial q^*}{\partial I^*} \right\} \\ &\quad + \frac{\partial I}{\partial s} \left(\frac{\partial \pi^*}{\partial p} \frac{\partial p}{\partial q} \frac{\partial q}{\partial I} + \frac{\partial \pi^*}{\partial q} \frac{\partial q}{\partial I} \right) - \gamma^L I s \frac{\partial I}{\partial s} - \gamma^{*L} I^* s^* \frac{\partial I^*}{\partial s} = 0. \end{aligned} \quad [29.1]$$

The first-order condition of H-country's government is

$$\begin{aligned}
\frac{\partial W^{J_b}}{\partial s^*} &= \frac{\partial W}{\partial s^*} + \frac{\partial W^*}{\partial s^*} \\
&= \frac{\partial I^*}{\partial s^*} \left\{ \frac{\partial \pi}{\partial p^*} \left(\frac{\partial p^*}{\partial q} \frac{\partial q}{\partial I^*} + \frac{\partial p^*}{\partial q^*} \frac{\partial q^*}{\partial I^*} \right) + \frac{\partial \pi}{\partial q} \frac{\partial q}{\partial I^*} + \frac{\partial \pi}{\partial q^*} \frac{\partial q^*}{\partial I^*} \right\} \\
&\quad + \frac{\partial I}{\partial s^*} \left(\frac{\partial \pi^*}{\partial p} \frac{\partial p}{\partial q} \frac{\partial q}{\partial I} + \frac{\partial \pi^*}{\partial q} \frac{\partial q}{\partial I} \right) - \gamma^I I s \frac{\partial I}{\partial s^*} - \gamma^{*I} I^* s^* \frac{\partial I^*}{\partial s^*} = 0.
\end{aligned} \tag{29.2}$$

From eqs [29.1] and [29.2], the optimal cooperative product R&D policies are determined. That of L-country's government is

$$s^{J_b} = \frac{1}{\gamma^I I} \left(\frac{\partial \pi^*}{\partial p} \frac{\partial p}{\partial q} + \frac{\partial \pi^*}{\partial q} \right) \frac{\partial q}{\partial I} < 0. \tag{30.1}$$

That of H-country's government is

$$\begin{aligned}
s^{J_b} &= \frac{1}{\gamma^{*I} I^*} \left[\left(\frac{\partial \pi}{\partial p^*} \frac{\partial p^*}{\partial q} + \frac{\partial \pi}{\partial q} \right) \frac{\partial q}{\partial I^*} + \left(\frac{\partial \pi}{\partial p^*} \frac{\partial p^*}{\partial q^*} + \frac{\partial \pi}{\partial q^*} \right) \frac{\partial q^*}{\partial I^*} \right] \\
&= \frac{2m[(2m^2 - n^2)\varepsilon\theta - mn\theta^*]p}{\gamma^{*I} I^* (m^2 - n^2)(4m^2 - n^2)} \left\{ \begin{array}{ll} < 0 & \text{if } 0 \leq \varepsilon < \frac{mn}{(2m^2 - n^2)} \frac{\theta^*}{\theta} \\ = 0 & \text{if } \varepsilon = \frac{mn}{(2m^2 - n^2)} \frac{\theta^*}{\theta} \\ > 0 & \text{if } \frac{mn}{(2m^2 - n^2)} \frac{\theta^*}{\theta} < \varepsilon \leq 1. \end{array} \right. \tag{30.2}
\end{aligned}$$

Proposition 4 When the governments of H-country and L-country jointly determine their product R&D policies, (I) in the absence of spillover, both governments tax their domestic firms' R&D investment, (II.1) in the presence of spillover, L-country's government always taxes its domestic firm's R&D investment, (II.2.a) H-country's government taxes (subsidizes) its domestic firm's R&D investment if $0 \leq \varepsilon < \frac{mn}{(2m^2 - n^2)} \frac{\theta^*}{\theta}$ ($\frac{mn}{(2m^2 - n^2)} \frac{\theta^*}{\theta} < \varepsilon \leq 1$), and (II.2.b) the optimal R&D tax/subsidy rate for both countries is just equal to zero if $\varepsilon = \frac{mn}{(2m^2 - n^2)} \frac{\theta^*}{\theta}$.

The results in the case of Bertrand competition are qualitatively the same as those in the case of Cournot competition. In the absence of spillover, both governments impose taxes on their domestic firms' R&D activities because, due to policy cooperation, the firms eliminate strategic behavior. In the presence of spillover, both firms avoid outcompeting their rival, but as spillover becomes

larger, the social surplus and profits of H-country decrease. Thus, H-country's government subsidizes its domestic firm's R&D investment.

From the above analysis, we can state that the same qualitative results are obtained irrespective of the mode of competition. This is in sharp contrast to previous studies that used a traditional Hotelling product differentiation model, which derives an asymmetric outcome between two competition modes.

4.5 Comparison between Cournot and Bertrand

We are interested in how the area in which the optimal policy is a tax (subsidy) changes if the mode of competition changes when the governments determine their R&D policies non-cooperatively.

Figure 3 depicts the relationship between the sign of the non-cooperative optimal policies and the magnitude of spillover. The superscripts c and b represent the mode of competition: (c is Cournot and b is Bertrand). If $0 < \varepsilon < n\theta^*/2m\theta$, tax is optimal under Bertrand and Cournot competition. It shows that the area in which the optimal policy is a subsidy (tax) is wider than in the case of Bertrand (Cournot). The reason is simply because the competition in the third stage is fiercer under Bertrand competition than under Cournot competition. For that matter, if the governments jointly determine their R&D policies, the signs of optimal policy for H-country are opposite those in the non-cooperative case under each mode, while the optimal policy for L-country is always tax over the full range of spillover, irrespective of the mode of competition.

Figure 3: Comparing policies under Cournot and Bertrand competition.

5 Conclusion

This study examined non-cooperative and cooperative optimal product R&D policies in the presence of technology spillover within the framework of a third-country trade model, in which a high-quality firm from H-country and a low-quality firm from L-country compete in the third market under Cournot and Bertrand competition.

The leading motive for departing from the previous studies on product R&D policy was to take account of unilateral spillover from a high-quality firm to a low-quality firm and to take account of a consumer in the third country who consumes multiple differentiated products, namely to employ different types of demand function with Hotelling-type demand function.

The new findings are as follows: If each government determines its optimal R&D policy non-cooperatively, both L-country and H-country subsidize (tax) their domestic firms' R&D activities if spillover is small (large) irrespective of the mode of competition. However, we also demonstrated that there is a range of spillover for which the optimal R&D policy is subsidy under Bertrand but tax under Cournot for both countries. The optimal policy is a tax for a wider range of spillover effects in the case of Cournot, compared to the case of Bertrand. If both governments decide their policies so as to maximize their joint social surplus, the low-quality firm can gain from an R&D tax irrespective of the degree of spillover, and the high-quality firm can benefit from an R&D subsidy if spillover is large and from a tax if spillover is small. These results hold irrespective of the mode of competition and are in sharp contrast with those of the previous studies analyzed under the Hotelling vertically differentiated model. Even if we assume general functions on spillover effects and product R&D investment costs and consider spillover effects to be more than 1, we can obtain similar results, but the analysis becomes more complicated.

Moreover, we suggest that governments should implement different policies depending on the characteristics of the industries involved when deciding their industrial policy for domestic firms. For example, in high-tech industries, the policies regarding firm's productivity or quality levels differ from those presented in previous studies, whereas in industries related to daily necessities such as food and clothing, even if the productivity or qualities are different, the government should implement the same policy across firms.

These results, however, are sensitive to the strategic nature of the quantity- and quality-setting stages. As indicated in the analysis, the decisive factors in determining the optimal R&D policies are the slopes of reaction functions in the third and second stages. In fact, the strategies in the second stage are confined

to those in the first stage. Although this study limits itself to the case of strategic substitutes in the quantity-setting third stage, we can consider the case of a strategic complement. This is derived from the substitutability between products, denoted as n in the utility function in eq. [1]. Expanding the feasible area to $n \in [-m, m)$, allows us to analyze the case of strategic complements in the quantity-setting third stage. As such, it is possible to change the results. On a related note, this study reveals that the results of studies using this model to consider process R&D policies, such as those by Liao (2007), are robust as long as the endogenous quality choices do not affect the firms' strategic nature in the stage of quantity or price competition.

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