Topics

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Price Discrimination with Varying Qualities of Information

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Abstract: Advances in information technology have greatly enhanced firms' ability to collect, market and utilize consumer information. As the market for consumer information expands rapidly, businesses are armed with unprecedented means to target any group of consumers they desire. This has important and far-reaching impacts on consumer welfare. In this paper we analyze the welfare impacts of price discrimination facilitated by increasing qualities of consumer information. We employ a two-dimensional spatial differentiation model where consumer information is available on one dimension, and better information leads to more refined price discrimination. We find that as information quality improves, equilibrium prices and profits monotonically increase while consumer surplus and social surplus monotonically decrease. Price discrimination has a reduced demand elasticity effect which becomes stronger when consumer information becomes more precise. Our results suggest that regulators need to pay more attention to the potential damage to consumer welfare by the increasing collection and utilization of consumer information. We also endogenize firms' information acquisition decisions.

Keywords: multi-dimension, price discrimination, information quality, reduced demand elasticity effect

JEL Classification Codes: D43, L13, L40

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1 Introduction

Advances in information technology have significantly enhanced firms' ability to collect ever increasing amount of customer-specific information.¹ Much of the early attention on this matter was centered on privacy. Since then, there has been increasing focus on economic impacts such as how the abundance of consumer information affects firm behavior and in turn consumer welfare. The usage of consumer information certainly makes economic sense for businesses – the success of companies such as Facebook has shown how valuable user-generated information can be. But how about consumers? In this paper, we analyze a setting where businesses can use consumer information to segment consumers into different groups and price discriminate accordingly. We investigate how improved consumer information affects consumer welfare – a question of interest to both academics and regulators.

There is an extensive literature devoted to this question with the common assumption of one-dimensional product differentiation.² Many studies find that price discrimination leads to a prisoners' dilemma game, benefiting consumers at the cost of firms. The corresponding policy implication then is that regulators should not intervene when firms collect consumer information and use it to price discriminate (see, for example, *Pros and Cons of Price Discrimination*, Swedish Competition Authority, Stockholm 2005). Allowing information quality to vary, Liu and Serfes (2004) show that while price discrimination always benefits consumers relative to uniform pricing, consumer welfare exhibits an inverse U-shaped relationship with information quality. This suggests that moderate consumer information is the best for consumers. However, as pointed out in several studies (e. g., Irmen and Thisse 1998; Liu and Shuai 2013a), consumer characteristics are often multi-dimensional. Will the results in one-dimensional models carry through when consumer characteristics are multi-dimensional?³ What would the policy implications of price discrimination be? Should the

1 A prominent example is Big Data and the opportunity/challenge it brings. A quick search of "Big Data" on Google returns 757 million results. See, for example, McAfee and Brynjolfsson (2012) for an excellent introduction.

2 Most existing studies focus on either two-group or perfect price discrimination. Exceptions include Liu and Serfes (2004), Colombo (2011) and Sonderegger (2011) who consider varying qualities of consumer information.

3 Note that even when consumer characteristics are multi-dimensional, firms may choose to price discriminate on one-dimension only. Correspondingly, an observation of one-dimensional price discrimination does not necessarily imply that the underlying model is one-dimensional. See Section 3.3 for more details.

regulator allow no or moderate information quality to be utilized, or should it refrain from interfering altogether?

In this paper, we analyze the welfare impacts of price discrimination with varying qualities of information in a multi-dimensional setting. In particular, we employ a two-dimensional model and assume that consumer information (which facilitates price discrimination) is available only on one dimension.⁴ This assumption is made mainly for tractability. One interpretation is that some consumer characteristics are easier to collect and utilize for price discrimination purpose (e. g., age for movie admission pricing) relative to others (e. g., what kind of movie a viewer likes). Alternatively, it may be that some consumer characteristics are binary (e. g., gender) while others may be continuous (e. g., age and location). For both interpretations, varying qualities of consumer information are possible on some dimensions but not others, as in our setup. We are interested in how improvement in the quality of consumer information (i. e., finer price discrimination) affects profits, consumer welfare and social welfare.

We first restrict information quality to be exogenous so firms have the same quality of information. Our results show that equilibrium prices and profits monotonically increase with the quality of consumer information. This is in sharp contrast to common findings in the existing literature. Most studies show that oligopolistic third-degree price discrimination reduces profits relative to uniform pricing when there is best-response asymmetry.⁵ For example, in Liu and Serfes (2004), discriminatory profits exhibit a U-shaped pattern in information quality but always lower than the profit under uniform pricing. The non-monotonic result comes from the interplay of two opposite effects when information quality improves. In markets served by both firms, better consumer information intensifies competition, the intensified competition effect. However, in markets where one firm enjoys significant advantage over its rival and serves the markets alone, better information allows the firm to extract more surplus, the surplus extraction effect. The intensified competition effect is also present in our model. However, firms share all markets in the equilibrium so there is no surplus extraction effect in our model. Instead, improvement in consumer information has a *reduced demand elasticity effect*. When price discrimination is more refined (better consumer information), the measure of consumers who view both firms as viable competitors (“marginal consumers”) shrinks so the same price cut can only steal fewer customers. This reduces

⁴ In Section 3.3, we check whether firms have an incentive to deviate from this and price discriminate on both dimensions.

⁵ Best-response asymmetry refers to the case where one firm’s strong market is the other’s weak market and vice versa.

firms' incentive to undercut each other's prices, sustaining higher equilibrium prices and profits.⁶ While the exact magnitude of the reduced demand elasticity effect is specific to our model's assumptions, the qualitative result and the underlying intuition are likely to be more general – as the consumer segmentation becomes finer, consumer demand becomes less sensitive to price changes, resulting in higher equilibrium prices.

While better consumer information helps firms, its impact on consumer surplus and social surplus is exactly the opposite. This should not be surprising. When consumer information quality improves, there is more brand switching which is inefficient and leads to lower social surplus. In the meantime, firms' profits increase with information quality. Increasing profit combined with lower social surplus suggests that consumer surplus monotonically decreases with information quality. If regulators are concerned with consumer surplus, they should aim to curb the increasing collection and usage of consumer information.

Next, we endogenize firms' information acquisition decisions in a two-stage information-then-price game. The low-quality firm can segment the consumers into different segments. The high-quality firm has better information which allows it to divide each of these segments into multiple sub-segments. We find that each firm's profit increases with its own information quality, holding the other firm's information quality fixed. On the other hand, fixing its own information quality, high-(low) quality firm's profit decreases with (is independent of) the other firm's information quality. Overall, better information continues to benefit firms at the cost of consumers. We then characterize the subgame perfect Nash equilibrium for two cases of information acquisition: All or no information or allowing intermediate qualities information.⁷

1.1 Literature Review

Our paper adds to the extensive literature on oligopolistic third-degree price discrimination. One strand of this literature assumes that firms have perfect information about consumers' location, i. e. firms can identify the location of each consumer and practice perfect price discrimination (e. g. Lederer and

⁶ Liu and Shuai (2013a) show that two-group price discrimination raises profits relative to uniform pricing in a two-dimensional model, due to reduced demand elasticity effect. However, they do not consider what happens when consumer information becomes more precise which is what this paper analyzes. The result is ex-ante unclear with two opposite effects – it leads to non-monotonic results in one-dimensional models but monotonic results here.

⁷ Please see Section 4 for more details.

Hurter 1986; Anderson and de Palma 1988; Thisse and Vives 1988; Shaffer and Zhang 2002; Bhaskar and To 2004; Ghose and Huang 2009). Another strand of this literature assumes that firms can segment consumers into two groups and price discriminate (e. g. Bester and Petrakis 1996; Chen 1997; Corts 1998; Shaffer and Zhang 1995; Fudenberg and Tirole 2000; Shaffer and Zhang 2000).⁸ Between the two extremes of two-group and perfect price discrimination, there are studies which assume firms can segment consumers into various (more than two) groups and price discriminate accordingly (e. g. Liu and Serfes 2004; Colombo 2011; Oukssel and Eruysal 2011).

Our paper is closely related to Liu and Serfes (2004). They adopt a one-dimensional Hotelling model where exogenous consumer information allows firms to divide the Hotelling line into N equal-sized segments and practice price discrimination. They find that both firms have an incentive to price discriminate and equilibrium profit is U-shaped in information quality (N). In this paper, we also conduct comparative statics regarding information quality, but in a multi-dimensional setting. This change leads to very different results: When consumer information quality improves, firms are always better off at the loss of consumer surplus and social surplus.

Another closely related paper is Lewis and Sappington (1994) who consider how much information a monopoly supplier wants to provide to heterogeneous buyers about their match values with the product. Better information allows the supplier to segment the market and charge higher prices to higher value buyers. In the meantime, improved information generates more accurate private signals which are unobserved to the supplier. Correspondingly, the supplier has to provide more information rents to induce buyers to reveal their signals truthfully. Optimal information is obtained by balancing the two opposite effects of segmentation and information rents. Assuming that information can be conveyed by the supplier to consumers at no cost, their key finding is that optimal information always takes one of the two extremes: The supplier provides either the best available information or no information at all. In our paper, optimal choice of information quality of a firm depends on the other firm's choice. A firm's profit always increases with its own information quality and (weakly) decreases with the other firm's information quality. Equilibrium information quality is obtained by balancing the benefit of better information and the corresponding higher information cost. The question analyzed in Lewis and Sappington (1994) is further investigated and generalized in Johnson and

⁸ Sonderegger (2011) assumes that firms to segment consumers into two groups: loyal and competitive, and allow firms to choose non-linear price for each segment, a third-degree price discrimination combined with second-degree price discrimination.

Myatt (2006). They propose a general framework to analyze various monopoly decisions such as product design, advertising and marketing as well as product-line choices. They show that these decisions affect demand dispersion (or demand rotation) and that firms' profits are usually a U-shaped function of such dispersion. Therefore, firms have a preference for extremes, favoring either very high or very low levels of dispersion.

Our paper is also related to the literature analyzing multi-dimensional product differentiation. Tabuchi (1994) discusses firms' location choice in a two-dimensional model. Irmen and Thisse (1998) extend the analysis to a general n -dimensional setting. While consumer distribution is assumed to be uniform in these studies, Liu and Shuai (2013b) allow non-uniform distribution. Our paper is closely related to Liu and Shuai (2013a) who consider a two-dimensional model but focus on the extreme forms of price discrimination: either two-group or perfect price discrimination. They find that both forms of price discrimination raise firm profits relative to uniform pricing, because price discrimination has a reduced demand elasticity effect as in this paper. However, they do not consider intermediate levels of information quality and analyze the corresponding comparative statics results.⁹ We extend their analysis by allowing information quality to vary and investigate how welfare results change with information quality.¹⁰

The rest of the paper is organized as follows. We present the model in Section 2, and analyze the exogenous information case in Section 3. Section 4 endogenizes information acquisition decisions and we conclude in Section 5. Proofs of Lemmas and Propositions are relegated to the Appendix.

2 The Model

We employ a Hotelling model where product differentiation occurs on two dimensions. A continuum of consumers of measure 1 is uniformly distributed over the square with length of each side being L . For simplicity, we normalize $L=1$ and assume that consumer distribution on different dimensions is

⁹ As shown by Liu and Serfes (2004), the impact of information quality on firm profits and consumer welfare is non-monotonic in a one-dimensional model. Ex-ante it is unclear whether the impact will remain non-monotonic or become monotonic in a two-dimensional model.

¹⁰ Liu and Shuai (2013a) allows price discrimination on either and both dimensions. With varying levels of information quality, it is intractable to consider price discrimination on both dimensions so in the main model we restrict price discrimination to be on one dimension only. We analyze firms' incentive to price discriminate on the other dimension in Section 3.3.

independent. Two firms – A and B – are located at two opposite corners of the unit square with firm A at $(0, 0)$ and firm B at $(1, 1)$ respectively. Both firms have constant marginal costs which we normalize to zero. As is standard in the literature, transport cost is assumed to be quadratic in the distance traveled.¹¹ For example, consider a consumer located at (x, y) . If she buys from A , she would enjoy an indirect utility of

$$u_A = V - p_A - t_1 x^2 - t_2 y^2,$$

where p_A is firm A 's price and t_l is the unit transport cost on dimension $l = 1, 2$. Without loss of generality, assume that $t_1 \geq t_2$. Similarly, if she buys from firm B , her utility will be

$$u_B = V - p_B - t_1(1-x)^2 - t_2(1-y)^2.$$

Each consumer buys at most one unit from the firm which maximizes her utility, conditional on the utility being nonnegative. We assume that V is sufficiently large so all consumers buy in the equilibrium (covered market).¹²

We allow firms to obtain information about consumers which facilitates price discrimination. In practice, firms can obtain such information from a variety of sources including past transactions, surveys, credit reports (see Liu and Serfes [2004] and papers cited there for more details). We assume that such information can be of varying qualities and we are interested in the comparative statics with respect to the quality of consumer information. In particular, we assume that there is consumer information on one dimension which allows firms to segment consumers into N equal-sized groups on that dimension. Without loss of generality, assume that this is dimension 1, and we rule out the possibility of consumer information on dimension 2 (see Figure 1).¹³ If a firm price

11 In our setup, linear and quadratic transport cost lead to the same equilibrium prices and profits. This is because the difference in transport cost is the same under either case, i. e., $[t_1 x + t_2 y] - [t_1(1-x) + t_2(1-y)] = [t_1 x^2 + t_2 y^2] - [t_1(1-x)^2 + t_2(1-y)^2]$.

12 Each consumer has two transport costs T_A and T_B , to firm A and B respectively. Their difference, say $\Delta T = T_A - T_B$, is only one-dimensional and distributed on $[-2, 2]$. However, the density is not uniform, but rather triangular. Moreover, in our setting, information allows a firm to segment consumers into equal sized segments. Correspondingly, consumers with the same ΔT can end up in different groups and face different prices. In this sense, our two-dimensional model is qualitatively different from one-dimensional models.

13 It is interesting but intractable to consider varying qualities of information on both dimensions. Our analysis fits the case where consumer information is either difficult to obtain on some dimensions or the consumer information is binary so there is no room for improvement. The former is in our main model. The latter is considered in Section 3.3 where we assume that information of certain quality is available on the second dimension, and we check whether firms have an incentive to price discriminate on the second dimension as well.

Figure 1: Consumer segmentation.

discriminates, then it can choose N different prices (one for each of the N segments). We restrict N to be a positive integer.

We assume that the information quality – captured by N , is dictated by technology level and thus exogenous.¹⁴ Let $\Gamma_i \in \{D, U\}$ denote firm i 's decision of whether or not to price discriminate and let $(\Gamma_A - \Gamma_B)$ denote the corresponding subgame. There are three subgames depending on the two firms' price discrimination decisions. In $(D-D)$, both firms price discriminate. Only one firm price discriminates in $(D-U)$ and $(U-D)$. In $(U-U)$, neither firm price discriminates. Our objective in the main model is to analyze how welfare results vary with information quality when firms price discriminate $(D-D)$ and how the results compare with those in one-dimensional models.

We first establish several results which will be used repeatedly throughout the paper. The first one deals with the derivation of the marginal consumers – those who are indifferent between buying from either firm. The second one shows that firms' profit functions are differentiable in their own prices, even at corner points (e. g., $(0, 1)$). The last one establishes the existence and uniqueness of pure strategy equilibrium for each subgame.

¹⁴ Later on, in Section 4, we relax this assumption and allow firms to acquire different qualities of information. Therefore, the decision is not just whether or not to acquire information, but of what quality as well.

Generic Marginal Consumer

Let $(x, \bar{y})$ denote a generic marginal consumer, characterized by

$$u_A = u_B \Rightarrow p_A + t_1 x^2 + t_2 y^2 = p_B + t_1 (1-x)^2 + t_2 (1-y)^2.$$

Solving for y , we can obtain

$$\bar{y} = \frac{p_B - p_A + t_1 + t_2 - 2t_1 x}{2t_2}. \quad [1]$$

This formula holds under uniform pricing and for each group of consumers under price discrimination. The corresponding marginal consumers line is the set of $(x, \bar{y})$ with the appropriate range of x and satisfying eq. [1] (see Figure 1 for example).

Differentiability of Profit Functions at Corner Points

Consider uniform pricing for example. Suppose that the marginal consumers line goes through the top left corner point $(0, 1)$ in segment 1. Let $\tilde{p}_A$ and $\tilde{p}_B$ denote the two firms' prices. If firm B raises p_B slightly above $\tilde{p}_B$, the marginal consumers line will shift up and firm B 's demand q_B in segment 1 will be the triangular area above the marginal consumer line. However, if it lowers p_B slightly below $\tilde{p}_B$, the marginal consumers line will shift down and firm B 's demand q_B will be a trapezoid area. We can see that firms' profit functions take different forms when the marginal consumers line is slightly above vs. slightly below the corner point $(0, 1)$. Nevertheless, it can be easily verified that

$$\left. \frac{\partial \pi_B}{\partial p_B^-} \right|_{p_B = \tilde{p}_B} = \left. \frac{\partial \pi_B}{\partial p_B^+} \right|_{p_B = \tilde{p}_B}.$$

That is, firms' profit functions are differentiable in their own prices, even when marginal consumers line crosses a corner point. This indicates that we can use FOCs as equalities (so long as both firms have positive sales). This holds for uniform pricing and for each segment under price discrimination.

Existence and Uniqueness of Pure Strategy Equilibrium

Our utility function (quadratic transport cost) satisfies Assumption A1 in Caplin and Nalebuff (1991) which requires consumer preferences to be linear in consumer characteristics (p. 29). Moreover, uniform distribution is log-concave, satisfying their Assumption A2 (p. 30). Then there must exist a pure strategy equilibrium in prices which is also unique based on Caplin and Nalebuff (Theorem 2, p. 39 for existence; Proposition 6, p. 42 for uniqueness). Although Caplin and Nalebuff consider uniform pricing, their existence and uniqueness

results still apply in our model with price discrimination. This is obvious for the $(D-D)$ subgame since both firms treat each segment as a separate market. How about $(D-U)$? Suppose that firm A price discriminates on dimension 1 while firm B chooses uniform pricing. Correspondingly, firm A chooses p_{Am} to maximize its profit from segment m , $m = 1, \dots, N$. Since the choices of p_{Am} 's are independent from each other, we can reinterpret firm A as N independent firms: firm Am chooses price p_{Am} . This interpretation transforms our model of 2 firms with price discrimination into $N + 1$ firms with uniform pricing, and the existence and uniqueness results in Caplin and Nalebuff apply. One can write down the $N + 1$ first-order conditions by the two firms, and see that they are the same as the $N + 1$ first-order conditions by the hypothetical " $N + 1$ firms", which would lead to the same solutions.

3 Analysis

3.1 Uniform Pricing ($U-U$)

This is the standard setup in the linear city model and has been analyzed in several studies (e.g., Tabuchi 1994 and Liu and Shuai 2013b). The results are summarized in the next Lemma. We refer interested readers to these studies for more details.

Lemma 1: ($U-U$) *When both firms choose uniform pricing, in the unique pure strategy equilibrium, each firm's price and profit are*

$$p^{U-U} = t_1, \quad \pi^{U-U} = \frac{t_1}{2}.$$

3.2 Both Firms Price Discriminate ($D-D$)

Now both firms price discriminate on dimension 1. The whole unit square is divided into N equal-sized segments along dimension 1 (see Figure 1). Segment m is represented by $\left[\frac{m-1}{N}, \frac{m}{N}\right] \times [0, 1]$, a rectangular area bounded by two horizontal lines (top and bottom) and two vertical lines (left and right). Let p_{im} denote firm $i = A, B$'s price in segment $m = 1, \dots, N$. Depending on the prices, there are four possible demand structures in segment m . The marginal consumers line can (i) lie above the top left corner point $\left(\frac{m-1}{N}, 1\right)$; (ii) lie below the

bottom right corner point $(\frac{m}{N}, 0)$, (iii) cross the top and bottom horizontal lines, or (iv) cross the left and right vertical lines. To simplify our analysis, we assume that $\frac{t_2}{t_1} \geq \frac{N+2}{3N}$.¹⁵ Given this assumption, we can rule out the possibility of (iii) immediately, because the marginal consumers line (with a slope of $-\frac{t_1}{t_2}$) must be flatter than the diagonal connecting the top left and bottom right corner (with a slope of $-N$). The next lemma shows that only (iv) can be supported as an equilibrium, so the marginal consumers line will cross the two vertical lines.

Lemma 2: (*D – D*) When both firms price discriminate, marginal consumers line must cross both vertical lines in all segments in the equilibrium.

Proof: See the Appendix. ■

Based on Lemma 2, there is only one demand structure in all segments. Solving firms' FOCs in each segment, we can obtain the unique pure strategy equilibrium which is summarized in the next Proposition.

Proposition 1: (*D – D*) When both firms price discriminate, in the unique pure strategy equilibrium,

(i) In segment $m = 1, \dots, N$, firms' prices are

$$p_{Am} = \frac{t_1 N + 3t_2 N + t_1 - 2t_1 m}{3N}, \quad p_{Bm} = \frac{3t_2 N - t_1 N + 2t_1 m - t_1}{3N}.$$

(ii) Each firm makes a profit of $\pi^{D-D} = \frac{27t_2^2 N^2 + t_1^2 N^2 - t_1^2}{54N^2 t_2}$.

Proof: See the Appendix. ■

Note that if we set $N = 1$, the equilibrium characterized in the above proposition becomes the same as that under uniform pricing. It can be easily verified that $\frac{\partial \pi^{D-D}}{\partial N} > 0$. Therefore, Proposition 1 tells us that the discriminatory profit monotonically increases with information quality N , including from $N = 1$ (uniform pricing) to $N = 2$ (2-group price discrimination).

Corollary 1: (*Comparative statics: profits*) When both firms price discriminate, equilibrium profits monotonically increase with the information quality N .

¹⁵ This assumption also simplifies the asymmetric information case in Section 4.1.

Discussion: In a one-dimensional model, Liu and Serfes (2004) find that discriminatory profit is *U*-shaped in information quality and is always below the profit under uniform pricing. The *U*-shape pattern comes from the interplay of two opposite forces. As is well known now, in the case of best-response asymmetry, better information generates an *intensified competition effect*. On the other hand, both firms make positive sales in the middle segments only. In markets where the disadvantaged firm is driven out of market and chooses marginal cost pricing, better information allows the advantageous firm to extract more surplus from consumers, the *surplus extraction effect*.

In our two-dimensional model, both firms make positive sales in all segments so price discrimination does NOT have a surplus extraction effect. Instead, it has a reduced *demand elasticity effect*. Liu and Shuai (2013a) identify the reduced demand elasticity effect when information quality goes from $N = 1$ to $N = 2$ (a special case of our model here).¹⁶ Next, we illustrate that this reduced demand elasticity effect continues to be present, in fact, becomes stronger as N increases. Combined, equilibrium profits (consumer surplus) monotonically increase (decreases) with N , in contrast to the non-monotonic relationships found in one-dimensional models. For simplicity, assume that $t_1 = t_2 = t$.

In a one-dimensional model, it is easy to verify that the marginal consumer satisfies

$$\frac{\partial \bar{x}}{\partial p_A} = -\frac{1}{2t},$$

under uniform pricing (UP) and

$$\frac{\partial \bar{x}_1}{\partial p_{A1}} = -\frac{1}{2t}$$

under price discrimination (PD) in segment 1. See Figure 2.

With only one dimension, firm A 's demands are $q_A = \bar{x}$ and $q_{A1} = \bar{x}_1$. Then

$$\frac{\partial q_A}{\partial p_A}(\text{UP}) = -\frac{1}{2t} = \frac{\partial q_{A1}}{\partial p_{A1}}(\text{PD}), \quad [2]$$

so there is no reduced demand elasticity effect.

In our two-dimensional model, using the previous expression for marginal consumers, we can obtain

¹⁶ Liu and Shuai (2013a) show that price discrimination has no reduced demand elasticity effect in a one-dimensional model, going from $N = 1$ to $N = 2$. We find that the same holds going from $N = 1$ to $N > 2$. The ideas are similar and we include it in this paper for the sake of completeness and for ease of comparison between one- and two-dimensional models.

Figure 2: One-dimensional Hotelling model.

$$\frac{\partial \bar{y}(x)}{\partial p_A}(\text{UP}) = -\frac{1}{2t} = \frac{\partial \bar{y}(x)}{\partial p_{A1}}(\text{PD}),$$

which is no different from eq. [2]. However, with two dimensions, to calculate the area for demand, we need to multiply the length of the base which is 1 under uniform pricing but $\frac{1}{N}$ under price discrimination.¹⁷ See Figure 3. Therefore,

$$\frac{\partial q_A}{\partial p_A}(\text{UP}) = -\frac{1}{2t} < \frac{\partial q_{A1}}{\partial p_{A1}}(\text{PD}) = -\frac{1}{2Nt}.$$

We can see that demand is less responsive to price changes under price discrimination than under uniform pricing. Correspondingly, firms have less incentive to lower their prices, supporting higher equilibrium prices. Since $\frac{1}{2Nt}$ decreases with N , the measure of consumers who view both firms as viable competitors shrinks with N and the reduced demand elasticity effect increases with N .¹⁸ It is this reduced demand elasticity effect which makes discriminatory

¹⁷ One can see that the same price change leads to the same-distance parallel shift of marginal consumers line. However, the measure of consumers covered by the shift (those who switch firms after the price change) goes down when N increases. That is, as price discrimination becomes more refined, the measure of consumers who view firms as viable competitors shrinks.

¹⁸ Our results complement the findings in Liu and Shuai (2013a) who show that both 2-group and perfect price discrimination improve profit relative to uniform pricing. We not only show that price discrimination of intermediate level raises profit, but establish the monotonic relationship between discriminatory profit and information quality. On the other hand, we restrict price discrimination to occur on only one dimension, while in Liu and Shuai (2013a) firms choose whether or not to price discriminate on each dimension.

profit monotonically increasing with information quality and always above the profit under uniform pricing.

Next, we examine the effect of price discrimination on consumer surplus and social surplus. The results are the following.

Proposition 2: (*Comparative statics: consumer and social surplus*) *Consumer surplus and social surplus monotonically decrease with information quality N .*

Proof: See the Appendix. ■

It is not surprising that finer price discrimination lowers social welfare. Finer price discrimination carves the market into more asymmetric segments which raises total transportation costs paid by the consumers. Lower social surplus, combined with higher profit, immediately implies lower consumer surplus.¹⁹

One can also analyze the asymmetric subgame ($D-U$) (and by symmetry the subgame ($U-D$)). This subgame is a special case of what we will analyze in the next section (by setting firm B 's information quality to be $N_B = 1$). To avoid lengthy repetition, we do not analyze ($D-U$) separately here.

3.3 Do Firms Have an Incentive to Price Discriminate on Both Dimensions?

We have restricted that consumer information is available on only one dimension which limits price discrimination to be on that dimension only. A natural question is what happens if consumer information on both dimensions is available. In particular, would firms have an incentive to deviate and price discriminate on the other dimension as well? Suppose that both firms have information on dimension 1 which would facilitate N -group price discrimination on dimension 1. Next, we investigate whether a firm would have an incentive to acquire information on dimension 2 as well.²⁰ To simplify the analysis, we assume that $t_1 = t_2 = t$ and information on dimension 2 enables only 2-group price

¹⁹ Caution is needed when interpreting our welfare results. Since our model assumes inelastic demand, total output is invariant to pricing. If total output varies with pricing, then price discrimination can potentially increase consumer and social welfare when it results an increase in total output.

²⁰ There are other types of deviation which we are not considering here. For example, a firm may choose to deviate and price discriminate on dimension 2 only (instead of on dimension 1 only).

discrimination (e. g., binary consumer characteristic) so the choice on dimension 2 is either uniform pricing or 2-group price discrimination.²¹

Without loss of generality, suppose that firm A deviates and acquires information on dimension 2 as well. Let $(DD-D)$ denote this subgame. Consumers are divided into N equal-sized groups along dimension 1. Firm B chooses a price for each group. For firm A , however, each of these N groups is further divided into two equal-sized groups along dimension 2. We are able to obtain closed form expression for π_A^{DD-D} but it is very complicated for us to compare with π^{D-D} analytically. To circumvent this problem, we checked for all N 's between 2 and 10^{10} and verified that $\pi_A^{DD-D}(m, N) < \pi_A^{D-D}(m, N)$, $\forall m, N$. We also consider the limiting case of $N \rightarrow +\infty$ (perfect price discrimination) and show that firm A has no incentive to deviate to and price discriminate on the second dimension as well.

4 Endogenous Information

So far we have assumed that both firms have the same quality of consumer information, and our results show that better information benefits firms at the cost of consumers. In this section, we analyze firms' information acquisition decisions.

There are multiple ways to model information acquisition. In the first case (*All or no information*), it may be that information is generated by a third-party (e. g., marketing company) and its quality (say N) is exogenous and dictated by the technology. Firm i 's ($i = A, B$) decisions is either to acquire information of this exogenous quality N at cost C , or not to acquire information at all ($N_i = 1$). In the second case (*Intermediate information quality*), information of different qualities (at different costs) is available and firms choose what qualities of information to acquire. Note that in this case both firms may acquire information, but of different qualities ($N_A \neq N_B$ and $\min\{N_A, N_B\} > 1$). Cost of information $C_i(N_i)$ is strictly increasing and convex in information quality N_i , $i = A, B$. To obtain closed form solution, we assume that $C_i = \frac{a}{2}(N_i)^2$, $a > 0$.

We endogenize firms' information acquisition decisions by analyzing the following two-stage information-then-price game. In stage 1, firms simultaneously decide whether or not to acquire information. In the case of intermediate information quality, firms also choose information qualities. After observing each other's information quality choices, firms choose prices simultaneously in stage 2. We solve the game backwards, starting with stage 2.

²¹ All derivations were performed using Maple software. All Maple files are available upon request. Our earlier assumption of $\frac{b_2}{b_1} \geq \frac{N+2}{3N}$ is insufficient to ensure that marginal consumers line always crosses both vertical lines in any segment.

4.1 Stage 2: Pricing Decision

Without loss of generality, assume that firm B has acquired information of quality $N_B = N$. This information allows firm B to segment the unit square into N equal-sized segments along dimension 1. However, firm A can further divide each of these N segments into K equal-sized sub-segments. That is, firm A 's information quality is $N_A = N \cdot K$. Pick a representative segment m of firm B . Denote B 's price by p_{Bm} . Let p_{Amk} denote firm A 's price in sub-segment $k = 1, \dots, K$ within the segment m . In the case of asymmetric price discrimination, sequential pricing (uniform price followed by discriminatory prices) rather than simultaneous pricing is commonly assumed in the existing literature on spatial price discrimination. Part of the reason is due to the nonexistence of pure strategy equilibrium in prices under simultaneous pricing. There is also support for this approach from the managerial perspective (see Shaffer and Zhang [1995] for more details). In our two-dimensional model, there is always a unique pure strategy equilibrium in prices under simultaneous pricing. To make the results across scenarios more comparable, we assume that all prices are chosen simultaneously.²²

In each of the $N \cdot K$ sub-segments, there are four possible demand structures (as in $(D-D)$ earlier). Since the marginal consumers line (with a slope of $-\frac{t_1}{t_2}$) must be flatter than the diagonal connecting the top left and bottom right corners (with a slope of $-NK$), it cannot cross the two horizontal lines. We can also rule out marginal consumers line crossing top or bottom horizontal line respectively. The only possible demand structure in equilibrium is for marginal consumers line to cross both vertical lines in all sub-segments.

Lemma 3: *In the equilibrium, the marginal consumers line must cross both vertical lines in all sub-segments.*

Proof: See the Appendix. ■

Lemma 3 pins down the demand structure in all segments. We can then solve for the equilibrium prices and profits, which are summarized in the next Proposition.

Proposition 3: *Suppose that firms' information qualities are $N_A = N \cdot K$ and $N_B = N$ respectively. Let $m = 1, \dots, N$ denote a segment of firm B and let $k = 1, \dots, K$ denote a sub-segment.*

²² Since prices are strategic complements, assuming sequential pricing will raise industry profits. One can assume sequential pricing for the $(D-U)$ scenario and verify that π_A increases with N while π_B is independent of N . These results are in the same spirit as those in Corollary 1.

(1) *Equilibrium prices are*

$$p_{Amk} = \frac{5t_1K + 6t_2KN + 2t_1KN - 4t_1mK - 6t_1k - 3t_1}{6KN}, \quad p_{Bm} = \frac{3t_2N - t_1 - t_1N + 2mt_1}{3N}.$$

(2) *Equilibrium profits (before subtracting information costs) are*

$$\pi_A = \frac{4t_1^2K^2N^2 + 5t_1^2K^2 - 9t_1^2 + 108t_2^2K^2N^2}{216N^2K^2t_2}, \quad \pi_B = \frac{t_1^2N^2 + 27t_2^2N^2 - t_1^2}{54N^2t_2}.$$

Proof: See the Appendix. ■

Corollary 2: *Each firm's profit increases in its own information quality, holding the other firm's information quality fixed. Holding its own information quality fixed, π_A decreases with $N_B = N$ while π_B is independent of $N_A = N \cdot K$.*

Let us start by verifying the first result. It is easy to see that π_B increases in $N_A = N$. Similarly, holding $N_B = N$ fixed, an increase in $N_A = N \cdot K$ (through an increase in K) also raises π_A . Next, let us check how a firm's profit changes with rival's information quality, holding its own quality fixed. Holding $N_A = N \cdot K$ fixed, an increase in $N_B = N$ means a decrease in K which means firm A 's advantage over firm B shrinks. Correspondingly, π_A decreases with N . Interestingly, however, holding $N_B = N$ fixed, π_B is independent of $N_A = N \cdot K$.

4.2 Stage 1: Information Acquisition Decision

In the previous section, we have derived the equilibrium profit $\pi_i(N_A, N_B)$ for firm $i = A, B$ with information qualities N_A and N_B respectively.²³

4.2.1 All or No Information

In this case, information quality N is exogenous and dictated by technology level. Firms can either acquire information of quality N or acquire no information. Let $C \geq 0$ denote the cost of information. Firms' profits net of information acquisition cost are illustrated in Table 1.²⁴ Each cell contains a pair of profits with the first (second) being firm A 's (B 's) profit.

²³ Note that if we set $K = 1$, we recover the $(D-D)$ subgame in Section 3.2. If $N = 1$ and $\kappa > 1$, then we recover the $(D-U)$ subgame where only firm A price discriminates. It can be shown that each firm earns a profit of $\frac{t_1}{2}$ under $(U-U)$. Note that we cannot recover $(U-U)$ by setting $N = K = 1$ in the profit expressions. This is because under $(U-U)$ marginal consumers line crosses the two horizontal lines, not the two vertical lines.

²⁴ In Proposition 3, firms' information qualities are $(N_A, N_B) = (NK, N)$. We set $N = 1$ to get $(N_A, N_B) = (K, 1)$. We then replace K by N in the resulting profit expressions to put in Table 1.

Table 1: All or no information.

Firm A \ B	$N_B = N$	$N_B = 1$
$N_A = N$	$\left(\frac{27t_2^2N^2 + t_1^2N^2 - t_1^2}{54N^2t_2} - C, \frac{27t_2^2N^2 + t_1^2N^2 - t_1^2}{54N^2t_2} - C \right)$	$\left(\frac{t_1^2N^2 + 12t_2^2N^2 - t_1^2}{24t_2N^2} - C, \frac{t_2}{2} \right)$
$N_A = 1$	$\left(\frac{t_2}{2}, \frac{t_1^2N^2 + 12t_2^2N^2 - t_1^2}{24t_2N^2} - C \right)$	$\left(\frac{t_1}{2}, \frac{t_1}{2} \right)$

Using the profit expressions in the table above, one can easily derive the equilibrium information qualities which are presented in the next Proposition.

Proposition 4: (All or no information) When firms choose whether to acquire information of exogenous quality or to acquire no information all,

- (i) (D–D) is an equilibrium if and only if $C \leq C_1 \equiv \frac{t_1^2(N^2 - 1)}{54t_2N^2}$;
- (ii) Neither firm price discriminating (U–U) is an equilibrium if and only if $C \geq C_2 \equiv \frac{t_1^2(N^2 - 1)}{24t_2N^2} - \frac{t_1 - t_2}{2}$;
- (iii) There are two asymmetric equilibria: (D–U) and (U–D), if and only if $C \in [C_1, C_2]$;
- (iv) Equilibrium prices and profits can be obtained by substituting appropriate N and K values into the expressions in Proposition 3.

The results above are straightforward using Table 1 and the proof is skipped. Two observations worth pointing out. First, the threshold value C_1 increases with N , so the equilibrium is more likely to feature price discrimination when N increases. Second, even if only one firm price discriminates in the equilibrium, no firm's profit decreases with N and the industry profit increases with N .

4.2.2 Allowing Intermediate Qualities of Information

Following Liu and Serfes (2004), we restrict information quality to be such that it allows the firm to segment the whole market into $N = 2^s$ equal-sized segments along dimension 1, where s is a nonnegative integer.²⁵ Information acquisition cost now becomes $C_i(N_i) = \frac{a}{2}(N_i)^2 = \frac{a}{2}(2^{s_i})^2$. Let $\pi_i(s_A, s_B)$ denote firm i 's profit,

²⁵ This allow us to avoid situations where firms' segments partially overlap with each other, for example, when $(N_A, N_B) = (6, 4)$. Such a case would be extremely tedious to analyze generally.

before information acquisition cost is subtracted. Without loss of generality, assume that $s_A \geq s_B$ (if $s_A < s_B$ we can switch firm labels of A and B). We look for subgame perfect Nash equilibrium in pure strategies. The results are summarized in the next Proposition.

Proposition 5: (*Intermediate levels of information*) When firms can acquire intermediate qualities of information,

- (i) $(s_A, s_B) = (s, s)$ is an equilibrium, if and only if there exists an integer $s \in [s_{A1}, s_{B1}]$ where

$$s_{A1} = \frac{\ln\left(\frac{t_1^2}{12at_2}\right)}{2 \cdot \ln 4} - \frac{1}{2}, \quad s_{B1} = \frac{\ln\left(\frac{t_1^2}{27at_2}\right)}{2 \cdot \ln 4} + \frac{1}{2}.$$

- (ii) $(s_A, s_B) = (s + 1, s)$ is an equilibrium if and only if there exists an integer $s \in [s_{B2}, s_{A2}]$ where

$$s_{A2} = \frac{\ln\left(\frac{t_1^2}{12at_2}\right)}{2 \cdot \ln 4} - \frac{1}{2}, \quad s_{B2} = \frac{\ln\left(\frac{t_1^2}{27at_2}\right)}{2 \cdot \ln 4} - \frac{1}{2}.$$

- (iii) Equilibrium prices and profits can be obtained by substituting $N_i = 2^{s_i}$ into the expressions in Proposition 3.

Note that $s_{A1} = s_{A2}$. It can be verified that $s_{B1} - s_{A1} \in (0, 1)$ and $s_{A2} - s_{B2} \in (0, 1)$. Correspondingly, there is at most one integer in each of the two intervals, and thus at most one pure strategy equilibrium of each type

(symmetric or asymmetric). When $s_{A1} = \frac{\ln\left(\frac{t_1^2}{12at_2}\right)}{2 \cdot \ln 4} - \frac{1}{2}$ happens to be a nonnegative integer, we have two equilibria: (i) symmetric: $s_A = s_B = s_{A1}$; and (ii) asymmetric: $s_A = s_{A1} + 1$, $s_B = s_{A1}$. Otherwise, there is a unique equilibrium. Next, we assign parameter values of t_1 , t_2 and a and provide numerical examples.

Numerical examples: For simplicity, we set $t_1 = t_2 = 1$.

When $a = 0.01$, we have $s_{A1} \approx 0.2647$, $s_{B1} \approx 0.7722$. There is no integer s in between s_{A1} and s_{B1} so there is no symmetric equilibrium. However, there is an integer $s = 0$ in between $s_{B2} \approx -0.0278$ and $s_{A2} \approx 0.2647$. Therefore, the unique equilibrium involves $s_A = 1$ and $s_B = 0$. This leads to $N_A = 2$ and $N_B = 1$.

When $a \approx 0.0089$, there is no integer s in between $s_{B2} \approx 0.0147$ and $s_{A2} \approx 0.3072$ so there is no asymmetric equilibrium. However, there is an integer $s = 1$ in between $s_{A1} \approx 0.3072$, $s_{B1} \approx 1.0147$. Therefore, the unique equilibrium is $s_A = s_B = 1$. This leads to $N_A = N_B = 2$.

5 Conclusion

The rapid development of information technology improves firms' ability to acquire consumer information. Better information allows refinement of market segmentation based on which firms can price discriminate across consumers. Since consumer characteristics are often multi-dimensional, we employ a two-dimensional product differentiation model to examine comparative statics results with respect to varying information quality, captured by a positive integer N . When both firms have the same information quality N , discriminatory profits monotonically increase with N , and are always above the profits under uniform pricing. We find that (refined) price discrimination has a reduced demand elasticity effect. This is because as market segmentation is refined, the measure of marginal consumers who consider both firms as viable competitors shrinks so the same price cut can only steal fewer consumers. This reduces firms' incentive to undercut each other's prices, allowing firms to enjoy higher equilibrium prices and profits. We then allow firms to acquire information of different qualities (at different costs). In particular, we assume that the higher quality firm can further divide each of the low quality firm's segments into multiple sub-segments. We are able to characterize the equilibrium for the pricing game, and then endogenize firms' information acquisition decisions.

Price discrimination is widely observed in practice. A common finding and justification in the literature is that while price discrimination hurts firm profits, each firm has an incentive to unilaterally price discriminate (prisoners' dilemma). Our analysis provides an alternative and potentially more plausible explanation as to why firms engage in price discrimination and consumer information collection. That is, price discrimination raises firms' profits and the more refined price discrimination is, the higher the profits will be. Our results suggest that regulators should use more caution regarding the collection and usage of consumer information since better information can potentially reduce competition and raise prices, benefiting firms at the cost of consumers.

In this paper we have only considered two-dimensional consumer characteristics. An interesting extension with potentially important policy implications is to consider more than two dimensions. Think of a simple setting with the following consumer characteristics: age, gender and an aggregate dimension which is the sum of other dimensions. Information quality varies on the aggregate dimension. Should the regulators allow discrimination on some dimensions but not on others? What information quality on the aggregate dimension maximizes consumer welfare?

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Appendix: Proofs of Lemmas and Propositions

Proof of Lemma 2

Pick any segment $m = 1, \dots, N$. Firms' profits in segment m are

$$\pi_{Am} = p_{Am}q_{Am}, \quad \pi_{Bm} = p_{Bm}q_{Bm}.$$

Firms' FOCs are²⁶

$$q_{Am} + p_{Am} \frac{\partial q_{Am}}{\partial p_{Am}} = 0, \quad q_{Bm} + p_{Bm} \frac{\partial q_{Bm}}{\partial p_{Bm}} = 0. \quad [3]$$

Note that a general marginal consumer $(x, \bar{y})$ (characterized in eq. [1]) always satisfies

$$\frac{\partial \bar{y}}{\partial p_A} = - \frac{\partial \bar{y}}{\partial p_B} = - \frac{1}{2t_2},$$

which leads to

$$\frac{\partial q_{im}}{\partial p_{im}} = \frac{\partial q_{jm}}{\partial p_{jm}} = - \frac{\partial q_{im}}{\partial p_{jm}}, \quad i \neq j.$$

This holds regardless of demand structure and consumer distribution. Substituting it into eq. [3], we can obtain

$$\frac{q_{Am}}{q_{Bm}} = \frac{p_{Am}}{p_{Bm}}. \quad [4]$$

Next, we show that demand structures (i) and (ii) cannot be supported as an equilibrium.

- (i) Marginal consumers line crosses top horizontal and right vertical line.
In this case, it must be that $p_{Am} \leq p_{Bm}$ and $q_{Am} > q_{Bm}$. Equation [4] cannot hold.
- (ii) Marginal consumers line crosses bottom horizontal and left vertical line
In this case, it must be that $p_{Am} \geq p_{Bm}$ and $q_{Am} < q_{Bm}$. Equation [4] cannot hold.

Therefore, the equilibrium demand structure must be (iii), i.e., the marginal consumers line must cross both vertical lines. ■

²⁶ Recall that profit functions are differentiable with respect to prices even when the marginal consumers line crosses a corner point, for example, $(0, 1)$. Correspondingly, the FOCs allow *corner solutions* as well.

Proof of Proposition 1

Pick a segment $m=1, \dots, N$. Suppose that the marginal consumers line crosses left vertical line at $y=y_l$ and crosses right vertical line at $y=y_r$. Substituting $x = \frac{m-1}{N}$ and $x = \frac{m}{N}$ into eq. [1], we have

$$y_l = \frac{p_{Bm}N - p_{Am}N + t_1N - 2t_1m + 2t_1 + t_2N}{2t_2N}, \quad y_r = \frac{p_{Bm}N - p_{Am}N + t_1N - 2t_1m + t_2N}{2t_2N}.$$

Firms' demands from segment m are

$$q_{Am} = \frac{y_l + y_r}{2N}, \quad q_{Bm} = \frac{2 - (y_l + y_r)}{2N}.$$

Firm i 's problem is

$$\max_{p_{im}} \pi_{im} = p_{im} q_{im}, \quad i = A, B.$$

Solving the first-order conditions, we can obtain the following equilibrium prices

$$p_{Am} = \frac{3t_2N + t_1 + t_1N - 2t_1m}{3N}, \quad p_{Bm} = \frac{3t_2N - t_1N + 2t_1m - t_1}{3N},$$

which allow us to calculate firms' profits in segment m (π_{im}). Aggregating profits over all segments, each firm earns a total profit of

$$\pi^{D-D} = \sum_{m=1}^N \pi_{im}, \quad i = A, B \Rightarrow \pi^{D-D} = \frac{27t_2^2N^2 + t_1^2N^2 - t_1^2}{54N^2t_2}. \quad \blacksquare$$

Proof of Proposition 2

Pick a segment m . Consumers' travel costs are

$$\begin{aligned} TC_m^{D-D} &= \int_{x=\frac{m-1}{N}}^{x=\frac{m}{N}} \left(\int_{y=0}^{y=\bar{y}} (t_1x^2 + t_2y^2) dy + \int_{y=\bar{y}}^{y=1} (t_1(1-x)^2 + t_2(1-y)^2) dy \right) dx \\ &= \frac{20t_1^2m - 20t_1^2m^2 - 8t_1^2 + 9t_2^2N^2 + 18t_1t_2N^2 - 10t_1^2N - 5t_1^2N^2 + 20t_1^2Nm}{36N^3t_2} \end{aligned}$$

The aggregate travel costs over all segments are,

$$TC^{D-D} = \sum_{m=1}^N TC_m^{D-D} = \frac{27t_2^2N^2 - 4t_1^2 - 5t_1^2N^2 + 54t_1t_2N^2}{108t_2N^2}.$$

It can be easily verified that $\frac{\partial TC^{D-D}}{\partial N} > 0$, so aggregate total travel costs increase with information quality N . Correspondingly, social surplus decreases with N . Combined with the fact that industry profits increase with N , consumer surplus must decrease with N . $\blacksquare$

Proof of Lemma 3

This proof is lengthy and thus divided into several steps:

Step 1: Show that in equilibrium both firms make positive sales in all sub-segments.

Step 2: Characterize the conditions for marginal consumers line (MCL) to cross top or bottom horizontal line in any sub-segment.

Step 3: Show that these conditions are always violated in the equilibrium. Therefore, in equilibrium, MCL must cross the two vertical lines in all sub-segments.

Step 1: Show that in equilibrium both firms make positive sales in all sub-segments

We consider a typical segment $m = 1, \dots, N$ of firm B . Denote firm B 's price by p_B . This segment is further divided into K sub-segments. In each sub-segment, let p_{Ak} denote firm A 's price and let π_{Bk} denote firm B 's profit. For firm A to make zero sales in any sub-segment, it must be that firm B 's price is zero or negative, which cannot be optimal for firm B . Therefore, firm A must make positive sales in all sub-segments. Next, we show that firm B must make positive sales in all sub-segments as well. Suppose not. Let S denote the set of sub-segments where firm B makes no sale. Consider any sub-segment $k \in S$. p_{Ak} must be such that it drives firm B exactly out of market. Fixing this p_{Ak} , firm B will still make no sale in sub-segment k if p_B increases, but will make positive sales if p_B decreases slightly.

- For all sub-segments in S , if p_B increases, firm B still makes no sales, i. e., $\frac{\partial \sum_{k \in S} \pi_{Bk}}{\partial p_B^+} = 0$. However, if p_B decreases, firm B will make sales, i. e., $\frac{\partial \sum_{k \in S} \pi_{Bk}}{\partial p_B^-} < 0$.
- For sub-segments other than S , π_{Bk} is differentiable in p_B so $\frac{\partial \pi_{Bk}}{\partial p_B}$ (and thus $\frac{\partial \sum_{k \notin S} \pi_{Bk}}{\partial p_B}$) is the same whether p_B increases or decreases.
- For firm B not to have an incentive to increase p_B , we need

$$\frac{\partial \pi_B}{\partial p_B^+} = \underbrace{\frac{\partial \sum_{k \in S} \pi_{Bk}}{\partial p_B^+}}_{=0} + \frac{\partial \sum_{k \notin S} \pi_{Bk}}{\partial p_B} = \frac{\partial \sum_{k \notin S} \pi_{Bk}}{\partial p_B} \leq 0.$$

- For firm B not to have an incentive to decrease p_B , we need

$$\frac{\partial \pi_B}{\partial p_B^-} = \frac{\partial \sum_{k \in S} \pi_{Bk}}{\partial p_B^-} + \frac{\partial \sum_{k \notin S} \pi_{Bk}}{\partial p_B} \geq 0.$$

- The last two inequalities cannot hold at the same time since $\frac{\partial \sum_{k \in S} \pi_{Bk}}{\partial p_B^-} < 0$.

Therefore, in any pure strategy equilibrium, firm B must make positive sales in all sub-segments.

Step 2: Characterize the conditions for MCL to cross top or bottom horizontal line

Pick an arbitrary sub-segment $[a, b] \times [0, 1]$ with $0 \leq a < b \leq 1$. Let p_A and p_B denote firms' prices. Next, we characterize the conditions on p_B to have MCL crossing top or bottom horizontal line in this sub-segment.

- Claim 1: MCL crosses the top horizontal line if and only if

$$p_B \geq p_B^{high}(a, b) = 3at_1 + 3t_2 - t_1b - t_1;$$

Suppose that MCL crosses the top horizontal line at $(x^{top}, 1)$, where $x^{top} = \frac{p_B + t_1 - p_A - t_2}{2t_1}$. We first solve for p_A using firm A 's FOC, and then find that $x^{top} \geq a$ if and only if $p_B \geq p_B^{high} = 3at_1 + 3t_2 - t_1b - t_1$.

- Claim 2: MCL crosses the bottom horizontal line at segment N if and only if

$$p_B \leq p_B^{low}(a, b).$$

Suppose that MCL crosses the bottom horizontal line at $(x^{bottom}, 0)$, where $x^{bottom} = \frac{p_B - p_A + t_1 + t_2}{2t_1}$. Using first-order condition of firm A , we solve for p_A and then substitute it into the x^{bottom} expression. We find that $x^{bottom} \leq b$ if and only if $p_B \leq p_B^{low} = 3bt_1 - t_1 - t_1a - t_2$.

Step 3: Show that the conditions in Step 2 are always violated in equilibrium

Claim 3: In any segment for firm B , it cannot be that MCL crosses top horizontal line in some sub-segments while crosses bottom horizontal line in some other sub-segments.

Pick an arbitrary segment of firm B . Suppose that MCL crosses the top horizontal line at sub-segment $[a_1, b_1] \times [0, 1]$, and crosses the bottom horizontal line in sub-segment $[a_2, b_2] \times [0, 1]$. Then it must be that,

$$p_B \geq p_B^{high}(a_1, b_1) = 3a_1t_1 + 3t_2 - t_1b_1 - t_1, \quad \text{and} \quad p_B \leq p_B^{low}(a_2, b_2) = 3b_2t_1 - t_1 - t_1a_2 - t_2.$$

But this is impossible because

$$\begin{aligned} p_B^{high}(a_1, b_1) - p_B^{low}(a_2, b_2) &= 4t_2 + 3a_1t_1 - b_1t_1 + a_2t_1 - 3b_2t_1 \\ &\geq 4\left(\frac{N+2}{3N}t_1\right) + 3a_1t_1 - b_1t_1 + a_2t_1 - 3b_2t_1 \\ &= 4\left(\frac{N+2}{3N}t_1\right) - [(b_1 - a_1) + (b_2 - a_2)]t_1 - 2(b_2 - a_1)t_1 \\ &\geq \left(4\frac{N+2}{3N} - \frac{2}{NK} - \frac{2}{N}\right)t_1 > 0 \end{aligned}$$

The first inequality comes from the assumption $\frac{t_2}{t_1} \geq \frac{N+2}{3N}$, and the second in equality comes from $b_i - a_i = \frac{1}{NK}$, and $b_j - a_i \leq \frac{1}{N}$, $j \neq i = 1, 2$. The last inequality

comes from the fact that $N \geq 1$, $K \geq 1$ and $N \cdot K \geq 2$ (otherwise it becomes uniform pricing).

Next, we show that p_B must lie in between $p_B^{\text{low}}(a, b)$ and $p_B^{\text{high}}(a, b)$ for any sub-segment $[a, b]$ (To ease on notation, we drop off the second dimension $[0, 1]$). Therefore, MCL cannot cross either top or horizontal line in any sub-segment.

Consider any sub-segment k : $[a, b] \times [0, 1]$.

- Claim 4: If MCL crosses the top horizontal line, i. e., $p_B \geq p_B^{\text{high}}(a, b)$, then $\frac{\pi_{Bk}}{\partial p_B} < 0$.

We first calculate $\frac{\pi_{Bk}}{\partial p_B}$. Next, we fix p_B , and use firm A's FOC to solve for p_{Ak} .

We then substitute the optimal p_{Ak} into $\frac{\pi_{Bk}}{\partial p_B}$ and find that $\frac{\pi_{Bk}}{\partial p_B} < 0$ whenever

$$p_B \geq p_B^{\text{high}}(a, b).$$

- Claim 5: If MCL crosses the bottom horizontal line, i. e., $p_B \leq p_B^{\text{low}}(a, b)$, then $\frac{\pi_{Bk}}{\partial p_B} > 0$.

We first calculate $\frac{\pi_{Bk}}{\partial p_B}$. Next, we fix p_B , and use firm A's FOC to solve for p_{Ak} .

We then substitute the optimal p_{Ak} into $\frac{\pi_{Bk}}{\partial p_B}$ and find that $\frac{\pi_{Bk}}{\partial p_B} > 0$ whenever

$$p_B \leq p_B^{\text{low}}(a, b).$$

- Claim 6: Suppose that MCL crosses both vertical lines. If $p_B > p_B^{\text{high}}(a_1, b_1)$ for some sub-segment $[a_1, b_1]$, then $\frac{\pi_{Bk}}{\partial p_B} < 0$. On the other hand, if $p_B < p_B^{\text{low}}(a_2, b_2)$ for some sub-segment $[a_2, b_2]$, then $\frac{\pi_{Bk}}{\partial p_B} > 0$.

We first calculate $\frac{\pi_{Bk}}{\partial p_B}$. Next, we fix p_B , and use firm A's FOC to solve for p_{Ak} .

We then substitute the optimal p_{Ak} into $\frac{\pi_{Bk}}{\partial p_B}$. We find that $\frac{\pi_{Bk}}{\partial p_B} < 0$ whenever

$$p_B \geq p_B^{\text{high}}(a_1, b_1) \text{ for some sub-segment } [a_1, b_1], \text{ and } \frac{\pi_{Bk}}{\partial p_B} > 0 \text{ whenever}$$

$$p_B \leq p_B^{\text{low}}(a_2, b_2) \text{ for some sub-segment } [a_2, b_2].$$

Suppose that MCL crosses top horizontal line in a sub-segment, then Claim 4 applies and Claim 5 cannot (due to Claim 3). Based on Claims 4 and 6, a reduction in p_B raises π_{Bk} , $\forall k = 1, \dots, K$ so it cannot be an equilibrium. Similarly, suppose that MCL crosses bottom horizontal line for a sub-segment, then Claim 5 applies and Claim 4 cannot. Based on Claims 5 and 6, an increase in p_B raises π_{Bk} , $\forall k = 1, \dots, K$ so it cannot be an equilibrium either. Combined, MCL can never cross top or bottom horizontal line in any of the K sub-segments. That is, it must cross both vertical lines in all K sub-segments. Since we pick an arbitrary segment of firm B, this applies to all firm B's N segments, and all firm A's $N \cdot K$ sub-segments. ■

Proof of Proposition 3

This proof is similar to the proof for Proposition 1, except that firm A chooses a price and maximize its profit from each of the $N \cdot K$ sub-segments, while firm B chooses a price and maximizes its profit from each of the N segments.

In segment m , firms' problems are,

$$\max_{p_{Amk}} \pi_{Amk} = p_{Amk} \cdot q_{Amk}, \quad k = 1, \dots, K,$$

$$\max_{p_{Bm}} \sum_{k=1}^K p_{Bm} \cdot q_{Bmk}.$$

Firms' FOCs are:

$$\frac{\partial \pi_{Amk}}{\partial p_{Amk}} = \frac{p_{Bm}KN - 2p_{Amk}KN + t_1KN + t_2KN - 2t_1mK + 2t_1K - 2t_1k - t_1}{2t_2K^2N^2},$$

$$\frac{\partial \pi_{Bm}}{\partial p_{Bm}} = \sum_{k=1}^K \frac{\partial \pi_{Bmk}}{\partial p_{Bm}}, \quad \frac{\partial \pi_{Bmk}}{\partial p_{Bm}} = \frac{p_{Amk}KN - t_1KN + t_2KN - 2p_{Bm}KN + 2t_1mK - 2t_1K + 2t_1k + t_1}{2t_2K^2N^2}.$$

Solving the FOCs, we can obtain the following equilibrium prices,

$$p_{Amk} = \frac{5t_1K + 6t_2KN + 2t_1KN - 4t_1mK - 6t_1k - 3t_1}{6KN}, \quad p_{Bm} = \frac{3t_2N - t_1 - t_1N + 2t_1m}{3N}.$$

Firms' profits are then

$$\pi_A = \frac{4t_1^2K^2N^2 + 5t_1^2K^2 - 9t_1^2 + 108t_2^2K^2N^2}{216N^2K^2t_2}, \quad \pi_B = \frac{t_1^2N^2 + 27t_2^2N^2 - t_1^2}{54N^2t_2}. \quad \blacksquare$$

Proof of Proposition 5

Note that s_i must be a nonnegative integer. We divide the proof into the following steps.

Step 1: Show that if firm $i = A, B$ has no incentive to raise (lower) s_i by 1, it has no incentive to raise (lower) s_i by more than 1.

Step 2: Characterize exact conditions for symmetric equilibrium.

Step 3: Characterize exact conditions for asymmetric equilibrium.

Step 1: Show that if a firm has no incentive to raise (lower) s_i by 1, it has no incentive to raise (lower) s_i by more than 1.

Let $\pi_i(s_A, s_B)$ denote firm i 's profit from Proposition 3. Let $\Pi_i(s_A, s_B)$ denote firm i ' net profit, after subtracting its information acquisition cost. It is straightforward to show that π_i and Π_i are both strictly concave in s_i , holding s_j , $j \neq i$ fixed. Therefore, If increasing s_i by 1 lowers firm i 's net profit, increasing s_i by more than 1 will further lower firm i 's profit. Similarly for the incentive to lower s_i .

Step 2: Characterize exact conditions for symmetric equilibrium.

We consider a symmetric equilibrium candidate where $s_A = s_B = s$. We characterize conditions such that neither firm has an incentive to raise or lower s_i by 1. This is done by checking the following deviations (i) s_A goes up by 1, and (ii) s_B goes down by 1. We calculate the profit differences:

$$\Delta\Pi_{A1} = \Pi_A(s, s) - \Pi_A(s+1, s), \quad \Delta\Pi_{B1} = \Pi_B(s, s) - \Pi_B(s, s-1).$$

For neither firm to have an incentive to deviate, we need $\Delta\Pi_{A1} \geq 0$ and $\Delta\Pi_{B1} \geq 0$. We find that $\Delta\Pi_{A1} \geq 0$ if and only if $s \geq s_{A1}$ and $\Delta\Pi_{B1} \geq 0$ if and only if $s \leq s_{B1}$, where

$$s_{A1} = \frac{\ln\left(\frac{t_1^2}{12at_2}\right)}{2 \cdot \ln 4} - \frac{1}{2}, \quad s_{B1} = \frac{\ln\left(\frac{t_1^2}{27at_2}\right)}{2 \cdot \ln 4} + \frac{1}{2}.$$

Correspondingly, $s_A = s_B = s$ is an equilibrium if and only if $s \in [s_{A1}, s_{B1}]$. It can be verified that $s_{B1} - s_{A1} \in (0, 1)$ so there is at most one integer s in this interval, and thus at most one symmetric equilibrium.

Step 3: Characterize exact conditions for asymmetric equilibrium.

Consider any asymmetric equilibrium candidate (s_A, s_B) with $s_A > s_B$. We first show that $s_A \geq s_B + 2$ can never be supported as an equilibrium. In particular, when $s_A \geq s_B + 2$, we find that if firm A has no incentive to decrease s_A by 1, then firm B must have an incentive to raise s_B by 1.

We then show that one only needs to check firm A 's incentive to lower s_A by 1 and firm B 's incentive to raise s_B by 1. In particular, we show that if firm B has no incentive to raise s_B by 1, then firm A will have no incentive to raise s_A by 1 either. Similarly, if firm A has no incentive to lower s_A by 1, then firm B has no incentive to lower s_B by 1.

Next, consider an asymmetric equilibrium candidate $(s_A, s_B) = (s+1, s)$. We check firms' deviation incentives by calculating

$$\Delta\Pi_{A2} = \Pi_A(s+1, s) - \Pi_A(s, s), \quad \Delta\Pi_{B2} = \Pi_B(s+1, s) - \Pi_B(s+1, s+1).$$

For neither firm to have an incentive to deviate, we need $\Delta\Pi_{A2} \geq 0$ and $\Delta\Pi_{B2} \geq 0$. We find that $\Delta\Pi_{A2} \geq 0$ if and only if $s \leq s_{A2}$ and $\Delta\Pi_{B2} \geq 0$ if and only if $s \geq s_{B2}$, where

$$s_{A2} = \frac{\ln\left(\frac{t_1^2}{12at_2}\right)}{2 \cdot \ln 4} - \frac{1}{2}, \quad s_{B2} = \frac{\ln\left(\frac{t_1^2}{27at_2}\right)}{2 \cdot \ln 4} - \frac{1}{2}.$$

Correspondingly, $(s_A, s_B) = (s + 1, s)$ is an equilibrium if and only if $s \in [s_{B2}, s_{A2}]$. It can be verified that $s_{A2} - s_{B2} \in (0, 1)$ so there is at most one symmetric equilibrium.

Note that $s_{A1} = s_{A2}$. If s_{A1} happens to be an integer, then both $s_A = s_B = s_{A1}$ and $(s_A, s_B) = (s_{A1} + 1, s_{A1})$ can be supported as equilibria. Otherwise, there is a unique equilibrium. ■

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