Robert Baumann* and Justin Svec

The Impact of Political Uncertainty: A Robust Control Approach

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Abstract: We examine how candidate uncertainty affects the policy platforms chosen in a unidimensional, two-candidate Downsian spatial model. The candidates, we assume, do not know the true distribution of voters. Following the robust control literature, candidates respond to this uncertainty by applying a max–min operator to their optimization problem. This approach, consistent with findings within the behavioral economics literature, protects the candidate by ensuring that her expected utility never falls too far, regardless of the true voter distribution. We show that this framework produces a continuum of equilibria upon which the candidates can converge and that the size of this continuum is weakly increasing in each candidate's uncertainty. We argue that our model can explain movements in political platforms over time. That is, the mere presence of candidate uncertainty, in addition to shifts in attitudes or demographics, can cause political candidates to change their policy positions across elections.

Keywords: robust control, candidate uncertainty, voting, spatial model

JEL Classification: H00, D78, D84

1 Introduction

A common approach to modeling the behavior of political candidates comes from the spatial model developed by Downs (1957) and Black (1958). In that model, voters are assumed to have single-peaked preferences over a unidimensional policy space, candidates can commit to their policy platforms, and both candidates and voters have perfect information. Given these assumptions, then in an election with two candidates, both candidates announce policy platforms that reflect the median voter's preferred policy. However, anecdotal evidence from elections seems inconsistent with this intuitive result. Consequently, researchers soon began modifying the theory in an attempt to better reflect reality.

***Corresponding author: Robert Baumann**, Department of Economics, College of the Holy Cross, PO Box 87A, Worcester, MA 01610, USA, E-mail: rbau-mann@holycross.edu

Justin Svec, Department of Economics, College of the Holy Cross, PO Box 87A, Worcester, MA 01610, USA, E-mail: jsvec@holycross.edu

An early criticism of the model was that the assumption of perfect information was unrealistic. Uncertainty, it was argued, could arise for multiple reasons. Candidates could be unsure of the distribution of voters along the policy space or voters could be unsure about the policy positions of the candidates. These factors make it difficult for the political candidate to know the median voter's preferred policy and therefore she might choose a platform that does not maximize the likelihood of winning the election. Another type of uncertainty often reflects a distrust among voters that a candidate will be willing or able to keep her campaign promises once in office.¹

In studies that incorporate uncertainty, candidates or voters are typically assumed to know the probability model that characterizes this randomness.² For example, in Enelow and Hinich (1989), candidates face uncertainty about the preferences of the voters. That is, the candidates only know the preferences of the voters with error. Importantly, the authors assume that the candidates know the true distribution of this error term, which then allows a straightforward calculation of the candidates' expected vote shares. While the intention of this approach is to model uncertainty, we believe these models require more information than a candidate is likely to have.

In our paper, we take a different approach. As we detail below, one merit of this alternative approach is that there is a long line of behavioral research that is consistent with the assumed behavior. Specifically, we assume that the political candidates do not know the true voter distribution, similar to the assumption made in Enelow and Hinich (1989). Rather, the candidates are endowed with a set of alternative voter distributions that they believe could characterize the true distribution. Following the robust control literature, we assume that the candidates do not place a prior distribution over these alternative voter distributions. That is, the candidates do not assign particular weights reflecting their subjective belief that each alternative voter distribution is true. Assuming the candidates are uncertainty averse and have a preference

1 One additional type of uncertainty is when the voters are unsure of their own preferences.

2 The behavioral literature distinguishes between two types of uncertainty. The first type, called "risk," refers to a situation in which the agent does not know how the environment's randomness will be resolved, but does know the distribution over this randomness. Thus, risk refers to a situation corresponding to the "known unknown." The second type, called "uncertainty," refers to a situation in which the agent knows neither how the randomness will be resolved nor the distribution over the randomness. Thus, uncertainty refers to a situation corresponding to the "unknown unknown."

This distinction is important because, as discussed in Klibanoff (1996), "there is strong evidence which suggests that thoughtful decision-makers react to uncertainty differently than they react to risk."

for robustness, the candidates apply a max–min operator to their optimization problem. This operator induces each candidate to choose the policy platform that maximizes her subjective expected utility, where the expectation is taken with respect to the worst-case voter distribution within the set of alternative voter distributions. This behavior protects the candidate by ensuring that her subjective expected utility never falls too far, regardless of the true voter distribution. Our goal in this paper, then, is to determine how this behaviorally consistent response to uncertainty impacts the results of an otherwise standard spatial model of political competition.

With this setup, we show that the Nash equilibrium in the candidate game involves both candidates choosing the same policy platform, i. e. convergence is the equilibrium outcome. However, the novel aspect of this convergence is that there is a multiplicity of possible policy platforms upon which the candidates could settle. That is, the candidates are no longer tethered to the median voter's preferred policy because the candidates are unsure as to the location of the median voter. Further, the set of possible equilibria weakly grows with a candidate's uncertainty level. Thus, model uncertainty combined with uncertainty aversion leads to a multiplicity of equilibria, all of which involve the candidates announcing the same policy platform, and some of which could be quite distant from the median voter's ideal point. We can show that these results are robust to both the timing of the game and the level of the candidates' uncertainty.

The logic behind this result can be understood using the following thought experiment. Suppose candidate l has announced a particular policy platform, and candidate r is sufficiently uncertain as to believe that the median voter could be on either side of this announced platform. Candidate r then has three options: choose a platform to the left of l , to the right of l , or announce the same platform as l . If r chooses a platform to either side of l , then candidate r 's uncertainty aversion leads her to fear that the true voter distribution is one in which the majority of the voters favor candidate l . But, if candidate r announces the same platform as candidate l , then candidate r guarantees that she receives half of all votes, leading the candidate to believe that she will receive a higher vote share for this choice than any alternative platform. Effectively, uncertainty and uncertainty aversion lead the candidates to behave in a way that insures themselves against detrimental voter distributions, a behavior that is consistent with the introduction of uncertainty aversion in other literatures; see Bose, Ozdenoren, and Pape (2006), for example. This thought experiment also reveals why there is a multiplicity of equilibria in this political game: the above logic holds for any policy platform for which both candidates believe that the median voter could be on either side of the platform. If the candidates face a large

degree of uncertainty about the true voter distribution, this set of policy platforms could be quite large, meaning that the set of possible equilibria could also be large.

Because of the presence of multiple equilibria, our model can be used to explain movements in political party positions over time. In our approach, changing political positions could be the result of candidates responding to uncertainty even if the electorate does not change its views or demographic composition.

Our paper proceeds as follows. In Section 2, we review the pertinent literature. Section 3 provides some justification for our modeling choices. The theoretical model and our key results are contained in Section 4, while Section 5 further explores our results using three simple numerical examples. We conclude in Section 6.

2 Literature Review

As stated in the introduction, Downs (1957) and Black (1958) are the focal points for studies of candidate policy choice. Their application of Hotelling's (1929) spatial model assumes single-peaked preferences over a one-dimensional policy space, candidate commitment to policy choices and full information of the distribution of voter-preferred policies. In an election with two candidates, both choose policy platforms that reflect the median voter's preferred policy.

Several theoretical extensions have been proposed to this baseline spatial voting model. As it is the subject of this paper, we begin with the literature that incorporates model uncertainty. To our knowledge, there is only one other formal analysis that assumes, as we do, that the political candidates face model uncertainty: Bade (2011).³ In that paper, the author assumes that the political candidates are uncertain about the distribution of the voters' ideal points and have uncertainty-averse preferences.⁴ In effect, these candidates worry that, for any announced platform, the actual distribution of voters' ideal points is one that minimizes the candidate's preferences. Given this type of uncertainty, the author shows the conditions under which a political equilibrium is assured: in a multidimensional setting

³ Although Berliant and Konishi (2005) implement uncertainty aversion to potentially justify why candidates would not announce a position on certain issues, the authors do not provide a formal exposition of candidate behavior or equilibrium under uncertainty aversion.

⁴ In a second model within the same paper, the author additionally assumes that the candidates face uncertainty about the shape of the voters' indifference curves.

with two uncertainty-averse candidates, a political equilibrium exists if either the candidates' uncertainty is sufficiently large or if the candidates are sufficiently averse to uncertainty. In addition to these sufficient conditions, the author is able to show that in any political equilibrium, both candidates receive equal vote shares and that as a candidate's uncertainty rises, the set of possible political equilibria weakly rises.

There are numerous salient differences between the current paper and Bade (2011). First, whereas Bade (2011) assumes a multidimensional policy space, we assume a unidimensional policy space. The modification is appealing both because the unidimensional case is the canonical representation of the Downsian voting model and because the assumption of a unidimensional policy space allows us to derive more specific results about the political equilibria than in Bade (2011). Second, and most important, the emphasis in Bade (2011) is to determine whether model uncertainty and uncertainty-averse preferences alleviate the issue of nonexistence of political equilibria in a multidimensional policy space. As such, the author's goal is to derive the sufficient conditions under which an equilibrium exists. In our paper, though, we go further than this. In addition to showing that an equilibrium exists, we can explicitly derive the set of political equilibria for any combination of candidates' uncertainty. Finally, one last difference between Bade (2011) and the current paper is that we choose to model the candidates' uncertainty aversion using the robust control approach of Hansen and Sargent (2008), while in Bade (2011), the author follows Gilboa and Schmeidler (1989). This last difference does not affect the theoretical results, but as discussed in Hansen, Sargent, Turmuhambetova, and Williams (2006), it would offer an econometrician an advantage when trying to test the model's implications. This is because the approximating model of robust control [which has no counterpart in the max–min expected utility theory of Gilboa and Schmeidler (1989)] offers an econometrician a simple tool to deduce the probability models over which an agent is uncertain, rather than imputing those models in an ad hoc fashion.

A larger literature exists that introduces risk into the Downsian voting model. This literature focuses on multiple types of risk, including voter uncertainty of candidate policies and candidate uncertainty of voter preferences. Voter uncertainty can occur in several ways. The easiest to imagine is voter skepticism that a candidate will keep her campaign promises. Alternatively, the candidate may be unable to achieve her campaign promises because of resistance from another branch of government [e.g. Banks (1990) is flexible to either manifestation]. Another common way to model this type of uncertainty is to assume that candidates have private ideological policy preferences (Wittman 1983). In these models, candidates are maximizing a utility function that includes ideological policy preferences

and winning the election. Candidate uncertainty is typically modeled by giving candidates either a probability distribution over an error term (Enelow and Hinich 1989), a probability distribution of the median voter's location (Aragones and Palfrey 2002, 2005) or a subjective probability of winning (Calvert 1985); see Roemer (2001) for a more thorough review of how candidates respond to different types of risk. Importantly, unlike in our paper, these papers assume that the candidates have complete confidence in these probability distributions and face no doubt about the correctness of their model.

A second common extension is to incorporate intangible characteristics of the candidates (termed “valence”; see Stokes 1963; Groseclose 2001; Aragones and Palfrey 2002; Kartik and McAfee 2007; Schofield 2007, among others). In these studies, it is possible for candidates to choose positions away from each other and/or the median voter in order to exploit a valence advantage or avoid an opponent who has one.

Although policy choices are difficult to measure numerically, several empirical tests of convergence exist (Fiorina 1974; Poole and Rosenthal 1984; Adams and Merrill 1999, and a series of studies by Erikson and Wright 1985, 1993, 2005). Most of these studies find that candidate policy choices often diverge from their opponent and/or the likely position of the median voter.

In addition to the spatial voting literature, our paper is related to the burgeoning robust control literature. In this literature, mostly concentrated within macroeconomics and finance, researchers relax the assumption of rational expectations and examine whether the same theoretical conclusions hold under uncertainty and uncertainty aversion. One of the most vibrant applications of robust control has been to reanalyze optimal monetary policy; some examples are Dennis (2010), Dennis, Leitemo, and Soderstrom (2009), Hansen and Sargent (2008), Leitemo and Soderstrom (2008), and Levin and Williams (2003). Subsequent research, including Karantounias (2013) and Svec (2012, 2014), has applied the same framework to fiscal policy models. Robust control has also been used to explain the equity premium; see Hanson, Barillas, and Sargent (2009) for an example. Finally, there has been a recent push to explore the impact of model uncertainty and uncertainty aversion in other fields. For one example in the field of education, see Congdon-Hohman, Nathan, and Svec (2014).

3 Uncertainty Aversion

The key assumption of our model is that political candidates respond to uncertainty about the distribution of voters' preferred policies as if they

are uncertainty averse.⁵ So, before we describe the structure of our model and our results, it would be helpful to pause in order to provide support for this underlying assumption.

Uncertainty aversion is consistent with both a long line of behavioral research and the robust control literature. In one of the first papers documenting uncertainty aversion, Ellsberg (1961) runs a series of experiments in which subjects must choose between a gamble with unknown odds and a gamble with known odds. The author shows that most subjects reject gambles with unknown odds in favor of gambles with known odds. This result is particularly surprising because, if we assume that the subjects had any fixed prior over the unknown gamble as would be assumed in a Bayesian approach, the subjects would have been weakly better off by choosing the unknown-odds gamble. Ellsberg rationalizes this behavior by modifying standard expected utility theory in two ways: first, he assumes the agent believes that there is a set of probability models that could characterize the unknown-odds gamble and second, for any bet made, the agent worries that the worst-case probability model will result, ensuring that she would lose the bet. These modifications have helped inform the theoretical formalization of behavior under model uncertainty.

Subsequent research further explored this finding of uncertainty aversion. In an early review of the literature, Camerer and Weber (1992) discuss papers that show that uncertainty aversion characterizes people's behavior in a number of different environments, including betting on natural events and trading in markets. Also, there is considerable recent experimental evidence that supports the finding of uncertainty aversion; examples include Halevy (2007), Bossaerts et al. (2010), Ahn et al. (2008) and Abdellaoui et al. (2011). Thus, given the extensive literature documenting people's uncertainty aversion, we believe that our assumption is particularly relevant. Finally, in the context of voting, Ghirardato and Katz (2006) suggest that uncertainty aversion could justify why voters choose not to fill out the entire voting ballot. This is curious behavior under rational expectations, as this voting is essentially costless because the voters are already at the ballot box.

Finally, we believe that our application of uncertainty and uncertainty aversion to electoral competition is compelling because candidates likely face uncertainty about both the electorate's preferences and the likelihood of specific types of voters to vote. Polling and campaign donations can certainly be used to assuage the former type of uncertainty and, to a limited degree, the second type as well. However, variation in voter turnout increases the noise from these

5 Uncertainty aversion is also known as ambiguity aversion.

signals, and unfortunately for candidates voter turnout can vary considerably over time. The United States Election Project⁶ has tracked voter turnout in presidential elections since 1948. This project shows that the range of US voter turnout rates across elections is large and that there seems to be little predictive power of turnout rates from election to election. Delving deeper, one could argue that presidential elections are determined by the electoral college, which shifts the focus of candidates to a handful of swing states. This could mean that changes in voter turnout at the national level are less important than the volatility in voter turnout in the swing states. Nevertheless, even focusing on swing states, a significant amount of variation in voter turnout rates remains. Using Ohio as an example, the state's near-average mix of demographics along with the sixth highest number of electoral votes has made this a hotly contested state in presidential elections. Over the five presidential elections from 1996 to 2012, the turnout rate in Ohio (relative to registered voters) varied from 63.65% (2000) to 71.77% (2008).⁷ This translates to a large swing in the number of votes given an average of 7.7 million registered voters over this time period. For candidates not to face uncertainty, they would have to correctly forecast these changes in voter turnout, swings that are likely products of national, state and local conditions.

4 Theoretical Model

In this section, we formulate our model of candidate competition under model uncertainty. This model is a modification of the spatial model of Downs (1957) and Black (1958).

Assume that there is a continuum of voters who have continuous, symmetric and single-peaked policy preferences on a unidimensional line from $[0, 1]$.⁸ Each voter cares only about the policies that will be enacted by the

⁶ McDonald (2013).

⁷ The source of these data is the Ohio Secretary of State. These voter turnout rates are taken relative to registered voters, which some (such as the United States Election Project) warn against since they usually include registered voters who have moved. However, most of these warnings are for comparisons across states and we have no reason to believe that any fix would significantly diminish the variation of voter turnout.

⁸ We assume that the political candidates know that the voters' preferences are symmetric and single peaked. If the voters' preferences are not symmetric, then we could use a similar approach to our current one to identify the range of equilibrium outcomes, as long as the candidates know the form of the asymmetry.

winning candidates.⁹ Also, assume that there are two candidates for political office. The sole goal of the candidates is to get elected, and so they choose their policy platforms in order to maximize the number of votes they receive. The candidates are assumed to have access to some commitment technology, implying that the winner of the election enacts the policy that she announces as candidate. Finally, the politician that wins the majority of votes gets elected; if there is a tie, then a coin is flipped to determine which politician is elected.

If we assume that both candidates have full information as to the true distribution of voters' preferred policies on the spectrum $[0, 1]$, then the unique Nash equilibrium in the candidate game is for both candidates to announce the policy that conforms to the median voter's preferred policy. This is the result described in the Median Voter Theorem.

In this paper, though, our goal is to analyze how model uncertainty influences the equilibrium in the candidate game. The type of model uncertainty we consider is one in which the political candidates have a limited degree of uncertainty about the true voter distribution. That is, even though they do not know the true distribution, they believe it lies within some reasonably narrow band of possible alternative distributions.

To this end, we assume that both politicians are endowed with an approximating probability model that specifies the distribution of voters' preferred points along the unidimensional spectrum from $[0, 1]$. For simplicity, both politicians are endowed with the same approximating probability model¹⁰, labeled $f(x)$, where x represents a particular policy and $f(x)$ represents the mass of voters with that policy as their ideal policy. The politicians, however, are not confident that this approximating probability model correctly characterizes the true probability distribution of the voters.¹¹ They worry that other probability models could potentially characterize the distribution of voters' preferred policies. In order to ensure that these alternative models conform to

⁹ We abstract away from any difference in candidate valence, a characteristic that is at the heart of Groseclose (2001), Kartik and McAfee (2007) and Schofield (2007).

¹⁰ At the end of this section, we briefly discuss the consequences of the candidates' being endowed with different approximating probability models.

¹¹ To be clear, this type of uncertainty is not meant to represent the possibility that the political candidates do not know their own beliefs about the distribution of voters. Rather, this type of uncertainty reflects the fact that there are potentially an uncountable number of possible voter distributions that could be the true one and that the candidate is unwilling or unable to assign a prior distribution over all of these possible voter distributions.

some degree with the approximating probability model, we place restrictions on what types of alternative models are allowed. To do so, we follow the robust control literature, and in particular, Hansen and Sargent (2005, 2007).

We assume that each member of the set of alternative models is absolutely continuous with respect to the politician's approximating probability model. This implies that the politicians only fear models that correctly put no weight on types of voters that do not exist. That is, the alternative models could indicate different proportions of voters at each $x \in [0, 1]$, as long as the mass of each type of voter under the approximating probability model is between zero and one. Further, the assumption of absolute continuity implies that the Radon–Nikodym theorem holds, which indicates that there exists a measurable function m such that the expectation of a random variable X under the alternative models can be rewritten in terms of the approximating probability model:

$$\tilde{E}[X] = E[mX],$$

where $\tilde{E}$ is the subjective expectations operator. To guarantee that each alternative model is a legitimate probability model, we assume $E[m] = 1$.

Using the function m , we can now define the distance between the alternative and approximating probability models to be the entropy:

$$\Xi(m) \equiv E[m \log m],$$

a measure that is convex and grounded. While common in the robust control literature¹², this entropy measure to our knowledge is novel in the political economy literature. It is useful because it allows us to consider the objective functions of agents with a limited degree of uncertainty. That is, the entropy term allows us to model an environment in which agents can narrow down the set of alternative probability models to ones that are reasonably close to the approximating probability model. Following the robust control literature, we will use this distance measure to define the politicians' multiplier preferences. The multiplier preferences characterize how the politicians value each possible position along the policy spectrum.

Without loss of generality, let us focus on candidate r , taking as given the location of candidate l for the moment. For a given platform of the opponent l , candidate r has three possible positions: she can choose $r > l$, $r < l$ or $r = l$. Each of these possibilities leads to different objective functions for candidate r .

¹² See Svec (2012, 2014) for examples.

We will go through each of these in turn, but initially we focus on the first possibility. Assuming that r chooses a policy platform to the right of l and that the voters have symmetric preferences, her goal is to choose the policy platform that maximizes the following objective function:

$$\min_{m(x)} \left\{ \int_{\frac{r+1}{2}}^1 m(x)f(x)dx + \theta_r \int_0^1 m(x) \log m(x)f(x)dx - \theta_r \lambda \left[\int_0^1 m(x)f(x)dx - 1 \right] \right\}$$

The first term in this objective function describes the mass of voters that the candidate expects to vote for her. The candidate knows that all voters whose preferred policy point is at least $\frac{r+1}{2}$ will vote for her. But, due to the uncertainty, candidate r is not sure about how many voters are included in that set. To this end, she tilts her approximating probability model with the measurable, multiplicative term, $m(x)$, discussed earlier. In effect, candidate r believes that the mass of voters whose preferred policy is in between two points a and b is $\int_a^b m(x)f(x)dx$. If $m(x) > 1(m(x) < 1) \quad \forall x \in [a, b]$, then the candidate believes that there is a larger (smaller) mass of voters in that policy space than is true under the approximating probability model. The second term in this objective function is a penalty term that effectively constrains the size of the endogenous probability tilting by penalizing alternative probability models based on their distance from the approximating probability model using the measure of entropy described earlier.¹³ Finally, the third term is the legitimacy constraint. This constraint guarantees that the probabilities associated with each alternative voter distribution sum to unity and thus can be considered a plausible probability model. The multiplier on this legitimacy constraint is λ .

A critical parameter in this objective function is $\theta_r > 0$. θ_r is a penalty parameter that indexes the degree to which candidate r is uncertain about the distribution of the voters. A small θ_r indicates that the candidate is not penalized too harshly for tilting her probability model away from the approximating probability model. The min operator then yields a set of $m(x)$ that diverges greatly from one. The resulting probabilities $\{m(x)f(x)\}$ are distant from the approximating probability model. Thus, a small θ_r denotes a situation in which the candidate is very unsure about the approximating probability model and so fears a large set of alternative models. A large θ_r means that the candidate faces a sizable penalty for tilting her probability model away from the approximating probability model. As a result, the min operator yields a set of $m(x)$ close to

¹³ We will discuss this term in greater detail in the following paragraph.

unity, implying that the worst-case alternative model is close to the approximating probability model. Thus, a large θ_r denotes a situation in which the candidate has more certainty about the underlying measure and so fears only a small set of alternative models. As $\theta_r \rightarrow \infty$, this model collapses to the rational expectations framework discussed earlier.

To solve for the candidate's optimal platform, we must first solve the minimization problem within this objective function. In this step, candidate r fears that, for a given policy platform to the right of l , the worst-case distribution of voters within the set of possible alternative distributions happens to be correct. Note that for each policy platform considered, the candidate could consider a different worst-case distribution. The solution that results from this minimization is the politician's worst-case voter distribution for a given platform to the right of l .

The first-order conditions (FOCs) from this objective function are

$$\begin{aligned} m(x), \text{ when } x &\in \left[\frac{r+l}{2}, 1 \right] : 1 + \theta_r [1 + \log m^*(x)] - \theta_r \lambda = 0 \\ m(x), \text{ when } x &\in \left[0, \frac{r+l}{2} \right) : \theta_r [1 + \log m^*(x)] - \theta_r \lambda = 0 \\ \lambda : &\int_0^1 m^*(x) f(x) dx - 1 = 0 \end{aligned}$$

where $m^*(x)$ represents the value of m that solves the FOCs. Combining the first two FOCs with the legitimacy constraint, we can derive the following values for $m^*(x)$:

$$\begin{aligned} m^*(x) &= \frac{\exp\left[-\frac{1}{\theta_r}\right]}{\int_0^{\frac{r+l}{2}} f(x) dx + \exp\left[-\frac{1}{\theta_r}\right] \int_{\frac{r+l}{2}}^1 f(x) dx} < 1, \text{ for all } x \in \left[\frac{r+l}{2}, 1 \right] \\ m^*(x) &= \frac{1}{\int_0^{\frac{r+l}{2}} f(x) dx + \exp\left[-\frac{1}{\theta_r}\right] \int_{\frac{r+l}{2}}^1 f(x) dx} > 1, \text{ for all } x \in \left[0, \frac{r+l}{2} \right) \end{aligned}$$

These values represent the candidate's subjective probability tiltings of the voter distribution for each value of x . That is, candidate r fears that the true percentage of voters who have a preferred policy within $x \in [a, b]$ is

$$\int_a^b m^*(x) f(x) dx.$$

^a We can use these probability tiltings to determine the candidate's subjective expected utility for each choice of $r > l$. To do this, plug the optimal values of

$m(x)$ into the candidate's multiplier preferences. With some manipulation, it can be shown that the candidate's objective function simplifies to the following:

$$-\theta_r \log \left[\int_0^{\frac{r+l}{2}} f(x) dx + \exp \left[-\frac{1}{\theta_r} \right] \int_{\frac{r+l}{2}}^1 f(x) dx \right]$$

This robust objective function allows us to determine the candidate's optimal policy platform for given values of l and θ_r while still assuming that $r > l$.

Candidate r , though, might not choose a policy platform to the right of l . Instead, if candidate r chooses a policy platform to the left of l , then the objective of the candidate is to maximize the following function:

$$\min_{m(x)} \left\{ \int_0^{\frac{r+l}{2}} m(x) f(x) dx + \theta_r \int_0^1 m(x) \log m(x) f(x) dx - \theta_r \lambda \left[\int_0^1 m(x) f(x) dx - 1 \right] \right\}$$

Following the same steps as outlined earlier, we would find that the robust objective function is

$$-\theta_r \log \left[\exp \left[-\frac{1}{\theta_r} \right] \int_0^{\frac{r+l}{2}} f(x) dx + \int_{\frac{r+l}{2}}^1 f(x) dx \right]$$

for given values of l and θ_r and assuming that $r < l$.

Finally, the third category of possible policy platforms for candidate r is to set $r = l$. With this choice, regardless of the value of θ_r and the position of l , candidate r can guarantee a subjective expected utility equal to $\frac{1}{2}$. This is because the voters cannot distinguish between the two candidates and therefore each candidate expects to win half of the votes.

Collecting these results, we can write down the subjective expected utility for candidate r for any possible platform chosen, for a particular value of θ_r and for a fixed platform of candidate l :

$$\text{Subjective expected utility} = \begin{cases} -\theta_r \log \left[\int_0^{\frac{r+l}{2}} f(x) dx + \exp \left[-\frac{1}{\theta_r} \right] \int_{\frac{r+l}{2}}^1 f(x) dx \right], & \text{if } r > l \\ -\theta_r \log \left[\exp \left[-\frac{1}{\theta_r} \right] \int_0^{\frac{r+l}{2}} f(x) dx + \int_{\frac{r+l}{2}}^1 f(x) dx \right], & \text{if } r < l \\ \frac{1}{2}, & \text{if } r = l \end{cases}$$

[1]

Notice the difference between this objective function under model uncertainty and the analogous one under rational expectations. Equation [1] must be divided into three components because the candidate's worst- case voter distribution depends on the policy platform under consideration. That is, if the candidate considers choosing a platform to the right of her opponent, she fears that the true voter distribution will be weighted toward the left end of the spectrum; if the candidate considers choosing a platform to the left of her opponent, then she fears that the true voter distribution will be weighted toward the right end of the spectrum. These fears are consistent with the type of behavior exhibited in Ellsberg (1961) and the subsequent literature on uncertainty aversion. Only when $r = l$ does uncertainty aversion not imply that the candidate fears a harmful voter distribution. This is because, when $r = l$, the candidate knows that she and her political opponent are identical, meaning that she has guaranteed herself half the vote.

We can use eq. [1] to derive the optimal policy platform of candidate r . These equations suggest that candidate r 's optimal platform depends upon three factors: the approximating distribution of voters, the position of candidate l , and candidate r 's level of uncertainty, θ_r . In the theorem below, we establish r 's optimal choice of policy for given values of l and θ_r .

Theorem 1: Define $\underline{l}$ through the equation $\frac{1}{2} = -\theta_r \log \left[\int_0^{\underline{l}} f(x) dx + \exp \left[-\frac{1}{\theta_r} \right] \int_{\underline{l}}^1 f(x) dx \right]$ and $\bar{l}$ through the equation $\frac{1}{2} = -\theta_r \log \left[\exp \left[-\frac{1}{\theta_r} \right] \int_0^{\bar{l}} f(x) dx + \int_{\bar{l}}^1 f(x) dx \right]$.

Also, assume that f is continuous. Then, if $l \in [\underline{l}, \bar{l}]$, $r = l$. If, instead, $l \notin [\underline{l}, \bar{l}]$, r will choose a policy platform that is $\epsilon > 0$ closer to the bound $[\underline{l}, \bar{l}]$, meaning that r has no best response.

To help with the flow of the paper, we prove this theorem in the Appendix.

This theorem has two parts. First, the theorem introduces the bounds $\underline{l}$ and $\bar{l}$. All values of l within these two bounds are within what we will call candidate r 's "range of uncertainty." This range is important because if candidate l chooses a policy platform within candidate r 's range of uncertainty, then candidate r believes that the median voter's preferred policy could be on either side of l . That is, there are some alternative probability models that suggest the median voter is to the right of l , while others that suggest she/he is to the left of l . Candidate r is therefore unable to definitively say whether the median voter is to one side of candidate l . However, if candidate l chooses a policy platform that is outside of the range of uncertainty, candidate r believes that the median voter is strictly on one side of candidate l . That is, all alternative probability models considered by candidate r suggest the median voter is strictly on one side of l .

The second part of the theorem indicates what policy platform candidate r should choose, conditional on the location of l . If l is within r 's range of uncertainty, then the theorem indicates that candidate r should optimally choose $r = l$. This behavior is intuitive because candidate r believes that the median voter could be on either side of l . As a result, if candidate r would choose $r \neq l$, her feared, worst-case voter distribution would imply that the median voter was on the other side of candidate l , leading to an electoral loss for candidate r . To prevent this outcome, then, candidate r chooses the same policy platform as candidate l .

If, though, candidate l chooses a policy platform outside of candidate r 's range of uncertainty, then policy divergence is optimal. This is because candidate r can reject the idea that the median voter could be on either side of candidate l . Consequently, candidate r finds it optimal to choose a platform that is slightly closer to the median voter under the approximating probability model. This choice leads candidate r to believe that she will receive the majority of votes in the election.

One additional point is worth noting about candidate r 's range of uncertainty: the size of candidate r 's range of uncertainty depends positively on the size of that candidate's uncertainty. That is, as θ_r falls and candidate r 's uncertainty grows, then candidate r 's range of uncertainty grows as well. This means that there is a wider range of policy platforms that candidate l could choose such that candidate r would choose $r = l$.

Up to this point, we have analyzed candidate r 's optimal policy platform for a given choice of l . Now, we turn to candidate l 's optimal platform for a given choice of candidate r . As both candidates are assumed to have the same approximating probability model, then after switching the terms r and l , eq. [1] and Theorem 1 also apply to candidate l . This implies that candidate l also has a range of uncertainty, which depends on her value of θ_l . Further, if candidate r chooses a policy platform within that range, then candidate l should choose $l = r$. If, though, candidate r chooses a policy outside of that range, then candidate l will choose a policy that is slightly closer to the median voter under the approximating probability model.

Combining each candidate's best response for a given position of the opponent, we can now characterize the Nash equilibria in this candidate game.

Proposition 1: *Any combination $r = l$ where both $r \in [\underline{r}, \bar{r}]$ and $l \in [\underline{l}, \bar{l}]$ is a Nash equilibrium in the candidate game. Further, there is no other pure strategy Nash equilibrium in the candidate game.*

Proof: Suppose $r \in [\underline{r}, \bar{r}]$ and $l \in [\underline{l}, \bar{l}]$. Then, using Theorem 1, we see that candidate r 's best response is to set $r = l$ and candidate l 's best response is to

set $l = r$. As neither candidate has the incentive to deviate from their announced policy platform, we have a Nash equilibrium. Depending on the size of both candidates' uncertainty, θ_r and θ_l , there could be an infinite number of Nash equilibria or the equilibrium could be unique.

To show that there exists no other pure strategy Nash equilibria in the game, consider without loss of generality an equilibrium in which $l \notin [\underline{l}, \bar{l}]$. If this is the case, then candidate r 's best response is to move slightly closer to the bounds $[\underline{l}, \bar{l}]$. Depending on whether r is within candidate l 's range of uncertainty, candidate l might either set $l = r$ or move closer to her bounds $[\underline{r}, \bar{r}]$. Either way, as long as $l \notin [\underline{l}, \bar{l}]$, candidate r has the incentive to diverge from candidate l 's platform. Thus, an equilibrium in which $l \notin [\underline{l}, \bar{l}]$ cannot exist. As this was without loss of generality, the same implication holds for candidate r : there exists no equilibrium in which $r \notin [\underline{r}, \bar{r}]$. Taken together, this implies that the only pure strategy Nash equilibrium involves both $r \in [\underline{r}, \bar{r}]$ and $l \in [\underline{l}, \bar{l}]$ and therefore $r = l$. ■

Corollary 1: *A sufficient condition for the existence of a Nash equilibrium is that the intersection of the intervals $[\underline{r}, \bar{r}]$ and $[\underline{l}, \bar{l}]$ must be nonempty.*

This proposition suggests that, by introducing model uncertainty and uncertainty aversion into the Downsian spatial model, it is optimal for both candidates to announce the same policy platform. This policy convergence is a common finding in the literature across a number of different assumptions; see Aragones and Palfrey (2002) and Enelow and Hinich (1989) for additional examples. Importantly, though, our model shows that model uncertainty and uncertainty aversion are not sufficient departures from the baseline Downsian voting model to obtain policy divergence. Rather, in order to obtain divergence, one must resort to other departures like candidate valence or three or more candidates.

One novel implication of our model is that, if both candidates have a finite θ_i , where $i \in \{r, l\}$, there is a range of possible Nash equilibria upon which the candidates could converge. This multiplicity stems from the fact that neither candidate is confident about the location of the median voter's preferred policy point. As a consequence, both candidates are willing to coordinate on any policy platform within a larger set so that they can both guarantee that they garner no worse than 50 % of the vote, regardless of the true voter distribution. Effectively, the candidates are behaving in a way that insures themselves against detrimental voter distributions. This result finds a parallel in the auction paper of Bose, Ozdenoren, and Pape (2006), which assumes that the bidders' valuations are drawn from an unknown distribution and that the bidders respond as if they are uncertainty averse. In that paper, the optimal auction involves the seller providing full insurance to the

bidders, as long as the bidders face more uncertainty than the seller. If either candidate faces no uncertainty, then the model collapses to the standard Downsian model where both candidates announce the preferred policy of the median voter.

Assuming that each candidate has a finite θ and that therefore there is a continuum of possible equilibria upon which the candidates could settle, one might wonder which equilibrium will actually be chosen by the candidates. While the above analysis remains silent on this topic, we believe that certain focal points will emerge over time. These focal points then would help the candidates coordinate on particular equilibria at the exclusion of the others. One example of a focal point could be the publicized policy stance of the national political party. Another could be the platform chosen by candidates in past elections. A third focal point could be the platform chosen by a particularly vocal or popular candidate in the election. If this last focal point were the relevant one, for example, then particular candidates might have the incentive to declare their policy stance early in order to “shape the debate” of the other candidates.

Another implication of our model is that model uncertainty offers a potential justification as to why candidates’ platforms can change over time, even if the median voter’s preferred policy has remained fixed. To see this, consider the situation in which the median voter’s location does not move from election to election. In this situation, the baseline model of Downs (1957) would predict that all candidates will announce the same policy platform across time. However, faced with the same situation, uncertain candidates might coordinate on a different equilibrium, as described in the proposition. Intuitively, the candidates are not tethered to the median voter’s preferred policy because the candidates are unsure about that location.

It is important to note that the results described in this paper hold for any combination of the candidates’ uncertainty. That is, suppose one believes that one candidate faces a large amount of uncertainty while the other candidate is relatively confident in the approximating probability model. The set of Nash equilibria described in the proposition still holds: the only pure strategy Nash equilibria in this particular game would involve both candidates converging on the same policy platforms within the intersection of both candidates’ ranges of uncertainty. The fact that one candidate is less uncertain than the other merely reduces the size of the set of possible Nash equilibria. In fact, Downs (1957) should be viewed as a special case of this more general model, where at least one candidate is characterized by $\theta_i \rightarrow \infty$ for $i \in \{r, l\}$.

We would also like to highlight that our results are robust to the timing of the political game. That is, the set of possible Nash equilibria is the same as described in the proposition regardless of whether the game is played

simultaneously or sequentially. To see this, understand that the model we described above assumed that the game was played simultaneously and that each candidate, in constructing her best response function, merely considered the possibility that the other chose a particular policy platform. Well, instead of assuming that the candidates consider the other's possible action, we could have assumed that the candidate views the actual action of the opponent. This change would then lead to the same theorem and the same proposition as we have described. Thus, our results are independent of the timing of the game.

Putting this in the context of a sequential game, we imagine a scenario in which, say, candidate l enters the race and announces a platform that she believes comes close to the median voter's preferred point. When candidate r enters the race, she considers whether the first candidate's choice is within r 's range of uncertainty. If it is, candidate r announces the same platform that l announced. In this case, both candidates are happy with their choice. If it is not, then candidate r chooses a platform that is slightly closer to what she believes the median voter prefers. In this case, assuming that r 's belief that the median voter is strictly on one side of l is accurate, candidate r should be confident of winning the election.

One might wonder whether our model assumes that each candidate knows the other's uncertainty level, θ , in order to construct her best response function. The answer is no: each candidate need only know her own level of uncertainty and the approximating probability model. Given these pieces of information, each candidate can construct her range of uncertainty, which then determines her best response function.

Finally, nowhere in the preceding analysis did we use the fact that the candidates' approximating probability models are the same. This implies that our theorem, proposition, and corollary would still hold even if the candidates had different approximating probability models. Thus, as long as the candidates' ranges of uncertainty overlap, all points within the intersection are Nash equilibria in this political game.

5 Numerical Examples

In this section, we present three numerical examples to better illustrate the results described above. In all three examples, we assume that the approximating probability distribution of candidate r is uniform on $[0, 1]$. This assumption of uniformity allows us to rewrite eq. [1] as

$$\text{Subjective expected utility} = \begin{cases} -\theta_r \log \left[\left(\frac{r+l}{2} \right) + \exp \left(-\frac{1}{\theta_r} \right) \left(1 - \frac{r+l}{2} \right) \right], & \text{if } r > l \\ -\theta_r \log \left[\exp \left(-\frac{1}{\theta_r} \right) \left(\frac{r+l}{2} \right) + \left(1 - \frac{r+l}{2} \right) \right], & \text{if } r < l \\ \frac{1}{2}, & \text{if } r = l \end{cases}$$

In these examples, we will assume particular values of θ_r and l . Given these values, we will then determine candidate r 's optimal policy platform.

In the first example, we will assume that candidate r has a large degree of uncertainty and l is relatively close to the median voter. We will show that it is optimal for candidate r to set $r = l$. In Example 2, we will keep the same value for θ_r , but assume that l is further away from the median voter. This will lead candidate r to choose a policy such that $r \neq l$. Finally, in Example 3, we will return to our original assumption about the location of l , but now assume that candidate r faces a smaller degree of uncertainty. In this case, just like in Example 2, we will show that candidate r will choose a policy in which $r \neq l$.

Example 1: High Uncertainty, Moderate l

Suppose that, in addition to the uniformity assumption, $\theta_r = 1$ and $l = \frac{2}{5}$. Given that $\theta_r = 1$, we can show that candidate r 's range of uncertainty has the following bounds: $\underline{l} \approx 0.3775$ and $\bar{l} \approx 0.6225$.¹⁴ These bounds imply that candidate l has chosen a policy platform that is within candidate r 's range of uncertainty. Consequently, we should expect to see that it is optimal for candidate r to set $r = l$.

To see whether this is indeed true, let us examine candidate r 's subjective expected utility for different choices of r . If $r = l$, then candidate r 's subjective expected utility is $\frac{1}{2}$. If $r = l + \epsilon$ where $\epsilon > 0$ is small, then her subjective expected utility approaches 0.4769. This is the highest expected utility that candidate r can hope to achieve by choosing a policy platform to the right of l . If $r = l - \epsilon$, then her subjective expected utility approaches 0.2915. Again, this is the highest expected utility that candidate r can hope to achieve by choosing a policy platform to the left of l . As $\frac{1}{2}$ is the highest subjective expected utility that candidate r can achieve across all of her possible policy choices, she would choose to set $r = l$.

Example 2: High Uncertainty, Extreme l

In this example, we still assume that $\theta_r = 1$, but now we assume that $l = \frac{1}{3}$. This policy chosen by candidate l is outside of candidate r 's range of uncertainty,

¹⁴ To calculate $\underline{l}$, for example, find the value of l at which $\frac{1}{2} = -1 \log[l + \exp[-1](1-l)]$. The other calculations for the bounds are analogous.

and so we should expect to see that candidate r 's optimal policy announcement is to set $r = l + \epsilon$.

As before, candidate r has three options. If she chooses $r = l$, then her subjective expected utility is $\frac{1}{2}$. If she sets $r = l + \epsilon$, then her utility approaches 0.5471. This utility falls if candidate r chooses any value of r such that $r > l + \epsilon$. Finally, if she chooses $r = l - \epsilon$, her utility approaches 0.2366. Again, her utility falls if she chooses an $r < l - \epsilon$. Thus, to maximize her subjective expected utility, candidate r chooses $r = l + \epsilon$, for some small value of $\epsilon > 0$.

Example 3: Low Uncertainty, Moderate l

In our final example, we will assume that $\theta_r = 10$ and $l = \frac{2}{5}$. Given this level of uncertainty, we can show that candidate r 's range of uncertainty has the following bounds: $\underline{l} \approx 0.4875$ and $\bar{l} \approx 0.5125$. These bounds imply that candidate l has chosen a policy platform that is outside of candidate r 's range of uncertainty. As such, we should expect to see that candidate r will choose $r = l + \epsilon$.

If candidate r chooses $r = l$, then her subjective expected utility is $\frac{1}{2}$. If she chooses $r = l + \epsilon$, then her utility is 0.5879. This utility falls as candidate r moves further to the right. Finally, if candidate r chooses $r = l - \epsilon$, her utility is 0.3881. This utility falls as candidate r moves further to the left. Thus, candidate r will choose to set $r = l + \epsilon$ for some small $\epsilon > 0$.

Generalizing across these numerical examples, we see that whenever candidate l announces a policy platform that is within r 's range of uncertainty, r chooses to match l 's platform. A flip of the coin then decides the winner of this election. If, though, l announces a policy that candidate r knows is to one side of the median voter, then r chooses a policy platform slightly closer to the median voter than l 's choice and is guaranteed the victory. Knowing this, candidate l 's strategy when choosing an initial platform will be to try and choose one that is within r 's range of uncertainty.¹⁵ If she succeeds, then she has some hope of winning the election; if not, then she does not.

6 Conclusion

In this paper, political candidates face uncertainty about the true distribution of voters along a unidimensional policy spectrum. The candidates are each endowed with a set of alternative voter distributions, each element of which

¹⁵ This is, admittedly, challenging because each candidate's theta is not assumed to be public information.

could potentially characterize the true voter distribution. We assume that the candidates respond to this uncertainty as if they are uncertainty averse. This response, which has considerable support in the behavioral literature, causes both candidates to choose policies that best protect themselves against the worst-case scenario of all possible distributions of voter policy preferences.

With this assumption, we derive two main results. First, all pure strategy Nash equilibria involve policy convergence. Second, there is a multiplicity of possible equilibria that surround the median voter under the approximating probability model. Put another way, the candidates can coordinate on any policy that is close enough to the median voter, where “close enough” depends upon the size of the candidates’ uncertainty. If their uncertainty is small, then the range of possible equilibria is small; if their uncertainty is large, then there is a large range of possible equilibria. This multiplicity of equilibria implies that changes in the policy position of a candidate over time can occur without a movement in the underlying distribution of voter policy preferences; instead, uncertainty regarding the true voter distribution is a sufficient condition to allow a candidate’s position to vary across elections.

A key assumption in our analysis is that both candidates’ ranges of uncertainty overlap, at least to some degree. This has the advantage that it is possible for both candidates to be correct about the location of the median voter. But it would be interesting to explore the consequences of the candidates having nonoverlapping ranges of uncertainty, a situation that requires that the candidates have different approximating probability models. In this case, the candidates fundamentally disagree about the location of the median voter and (at least) one candidate’s belief will be proven false in the election.

Appendix

In this appendix, we prove Theorem 1. This proof involves three main steps. In the first step, we sign the derivative of candidate r ’s subjective expected utility. This derivative will allow us to say that, conditional on choosing a platform to the right of l , candidate r is best off choosing a platform as close as possible to l and, conditional on choosing a platform to the left of l , candidate r is best off choosing a platform as close as possible to l . This will allow us to narrow our field of possible best responses to platforms close to candidate l ’s platform. The second step then shows that if either $l = \underline{l}$ or $l = \bar{l}$, then $r = l$ is optimal. Step 3 compares the subjective expected utility of candidate r at particular values of

r to the expected utility obtained at the bounds of candidate r 's range of uncertainty. In doing so, we will be able to say whether candidate r 's utility is greater or less than $\frac{1}{2}$, which then allows us to know whether that candidate is better off by setting $r = l$. We will repeat this second step multiple times to complete the proof.

For ease, we rewrite candidate r 's subjective expected utility below:

$$\text{Subjective expected utility} = \begin{cases} -\theta_r \log \left[\int_0^{\frac{r+l}{2}} f(x) dx + \exp \left[-\frac{1}{\theta_r} \right] \int_{\frac{r+l}{2}}^1 f(x) dx \right], & \text{if } r > l \\ -\theta_r \log \left[\exp \left[-\frac{1}{\theta_r} \right] \int_0^{\frac{r+l}{2}} f(x) dx + \int_{\frac{r+l}{2}}^1 f(x) dx \right], & \text{if } r < l \\ \frac{1}{2}, & \text{if } r = l \end{cases}; \quad [2]$$

Step 1: Determine the sign of $\frac{\partial U}{\partial r}$ conditional on the location of r relative to l . If we assume that $r > l$, then candidate r 's subjective expected utility is

$$-\theta_r \log \left[\int_0^{\frac{r+l}{2}} f(x) dx + \exp \left[-\frac{1}{\theta_r} \right] \int_{\frac{r+l}{2}}^1 f(x) dx \right]$$

Given this, we can show that

$$\frac{\partial U}{\partial r} = \frac{-\frac{\theta_r}{2} f\left(\frac{r+l}{2}\right) \left[1 - \exp\left(-\frac{1}{\theta_r}\right)\right]}{\int_0^{\frac{r+l}{2}} f(x) dx + \exp\left[-\frac{1}{\theta_r}\right] \int_{\frac{r+l}{2}}^1 f(x) dx} < 0$$

This derivative implies that, conditional on $r > l$, candidate r is always better off by remaining close to l than announcing a platform far away from l . Doing the same thing assuming that $r < l$, we see that

$$\frac{\partial U}{\partial r} = \frac{-\frac{\theta_r}{2} f\left(\frac{r+l}{2}\right) \left[\exp\left(-\frac{1}{\theta_r}\right) - 1\right]}{\int_0^{\frac{r+l}{2}} f(x) dx + \exp\left[-\frac{1}{\theta_r}\right] \int_{\frac{r+l}{2}}^1 f(x) dx} < 0$$

Again, conditional on $r < l$, candidate r is better off by remaining close to l .

Step 2: If either $l = \underline{l}$ or $l = \bar{l}$, then $r = l$ is optimal.

This will be a proof by contradiction. Suppose that, without loss of generality, $l = \underline{l}$, and it was optimal for candidate r to choose $r = \underline{l} - \epsilon$ for some small $\epsilon > 0$. This must mean that two conditions hold:

$$\begin{aligned}
 -\theta_r \log \left[\exp \left[-\frac{1}{\theta_r} \right] \int_0^l f(x) dx + \int_l^1 f(x) dx \right] &\geq \frac{1}{2} \\
 -\theta_r \log \left[\int_0^1 f(x) dx + \exp \left[-\frac{1}{\theta_r} \right] \int_l^1 f(x) dx \right] &= \frac{1}{2}
 \end{aligned}$$

Rewriting these conditions, we get

$$\begin{aligned}
 \exp \left[-\frac{1}{\theta_r} \right] \int_0^l f(x) dx + \int_l^1 f(x) dx &\leq \exp \left(-\frac{1}{2\theta_r} \right) \\
 \int_0^l f(x) dx + \exp \left[-\frac{1}{\theta_r} \right] \int_l^1 f(x) dx &= \exp \left(-\frac{1}{2\theta_r} \right)
 \end{aligned}$$

Adding these two conditions together, we see

$$2 \exp \left(-\frac{1}{2\theta_r} \right) \geq 1 + \exp \left(-\frac{1}{\theta_r} \right)$$

which then implies that

$$\left(\exp \left(-\frac{1}{2\theta_r} \right) - 1 \right)^2 \leq 0$$

which is a contradiction. Thus, if $l = \underline{l}$, then it cannot be optimal for candidate r to choose $r = \underline{l} - \epsilon$. Also, given our definition of $\underline{l}$, we know that if $r = \underline{l} + \epsilon$, then candidate r 's subjective expected utility equals $\frac{1}{2}$.

Finally, using our derivatives from Step 1, we know that candidate r 's subjective expected utility falls as she moves either to the left of $r = \underline{l} - \epsilon$ or to the right of $r = \underline{l} + \epsilon$. Taken together, we see that if $l = \underline{l}$, then candidate r 's optimal response is $r = \underline{l}$. The same pattern of the proof applies to $\bar{l}$, so we suppress that proof for brevity.

Step 3: Utility comparison.

1. Show that if $l \in [\underline{l}, \bar{l}]$, $r = l$. This is again a proof by contradiction. Suppose that $l \in [\underline{l}, \bar{l}]$, but that candidate r 's optimal response is $r \neq l$. Assume first that candidate r 's best response is $r = l + \epsilon$. Then, two conditions hold:

$$\begin{aligned}
 -\theta_r \log \left[\int_0^l f(x) dx + \exp \left[-\frac{1}{\theta_r} \right] \int_l^1 f(x) dx \right] &\geq \frac{1}{2} \\
 -\theta_r \log \left[\int_0^l f(x) dx + \exp \left[-\frac{1}{\theta_r} \right] \int_l^1 f(x) dx \right] &= \frac{1}{2}
 \end{aligned}$$

which implies that

$$\int_0^l f(x) dx + \exp \left[-\frac{1}{\theta_r} \right] \int_l^1 f(x) dx \geq \int_0^l f(x) dx + \exp \left[-\frac{1}{\theta_r} \right] \int_l^1 f(x) dx$$

But, with finite θ_r , then $\exp \left[-\frac{1}{\theta_r} \right] \in (0, 1)$. This means that the left-hand side is smaller than the right-hand side because the left-hand side places a smaller weight on a larger mass of the voter distribution. Thus, we have a contradiction. Combining this result with the derivative from Step 1, we know that if $l \in [\underline{l}, \bar{l}]$, $r \leq l$.

Assume that candidate r 's best response is $r = l - \epsilon$. Then, two conditions hold:

$$\begin{aligned}
 -\theta_r \log \left[\exp \left[-\frac{1}{\theta_r} \right] \int_0^1 f(x) dx + \int_l^1 f(x) dx \right] &\geq \frac{1}{2} \\
 -\theta_r \log \left[\exp \left[-\frac{1}{\theta_r} \right] \int_0^{\bar{l}} f(x) dx + \int_l^1 f(x) dx \right] &= \frac{1}{2}
 \end{aligned}$$

which implies that

$$\exp \left[-\frac{1}{\theta_r} \right] \int_0^{\bar{l}} f(x) dx + \int_l^1 f(x) dx \geq \exp \left[-\frac{1}{\theta_r} \right] \int_0^l f(x) dx + \int_l^1 f(x) dx$$

But, this cannot hold because the left-hand side places a smaller weight on more mass than does the right-hand side. Thus, we have a contradiction. Combining this result with the derivative from Step 1 and with the earlier work in Step 3, we know that if $l \in [\underline{l}, \bar{l}]$, $r = l$.

2. Show that if $l \notin [\underline{l}, \bar{l}]$, candidate r will choose a policy platform that is $\epsilon > 0$ closer to the median voter (under the approximating model) than l . This is again a proof by contradiction. Suppose that $l < \underline{l}$, and candidate r chooses $r = l - \epsilon$. Then, we know that the following two conditions hold:

$$\begin{aligned}
 -\theta_r \log \left[\exp \left[-\frac{1}{\theta_r} \right] \int_0^{\bar{l}} f(x) dx + \int_l^1 f(x) dx \right] &\geq \frac{1}{2} \\
 -\theta_r \log \left[\exp \left[-\frac{1}{\theta_r} \right] \int_0^{\bar{l}} f(x) dx + \int_{\bar{l}}^1 f(x) dx \right] &= \frac{1}{2}
 \end{aligned}$$

These conditions imply

$$\exp \left[-\frac{1}{\theta_r} \right] \int_0^{\bar{l}} f(x) dx + \int_l^1 f(x) dx \geq \exp \left[-\frac{1}{\theta_r} \right] \int_0^{\bar{l}} f(x) dx + \int_{\bar{l}}^1 f(x) dx$$

But this cannot hold because the left-hand side places a smaller weight on more of the mass than does the right-hand side. Thus, we have a contradiction. Combining this result with the derivative from Step 1, then we know it is not optimal for candidate r to choose $r < l$ if $l < \bar{l}$.

Now, we know that if $l < \bar{l}$, then candidate r will set $r \geq l$. But, we still must show that it is optimal for candidate r to choose $r = l + \epsilon$. To do this, assume the opposite; namely, $r = l$ is the optimal policy announcement for candidate r . Then, the following conditions must hold:

$$\begin{aligned}
 -\theta_r \log \left[\int_0^l f(x) dx + \exp \left[-\frac{1}{\theta_r} \right] \int_l^1 f(x) dx \right] &\leq \frac{1}{2} \\
 -\theta_r \log \left[\int_0^{\bar{l}} f(x) dx + \exp \left[-\frac{1}{\theta_r} \right] \int_{\bar{l}}^1 f(x) dx \right] &= \frac{1}{2}
 \end{aligned}$$

These conditions imply

$$\int_0^{\bar{l}} f(x) dx + \exp \left[-\frac{1}{\theta_r} \right] \int_l^1 f(x) dx \leq \int_0^1 f(x) dx + \exp \left[-\frac{1}{\theta_r} \right] \int_l^1 f(x) dx$$

But this cannot hold because the left-hand side places a smaller weight on less mass than does the right-hand side, meaning that the left-hand side is greater than the right-hand side. Thus, we have a contradiction. Combining these results with the derivative from Step 1, we can see that it is optimal for candidate r to choose $r = l + \epsilon$ if $l < \bar{l}$.

The proof is similar for $l > \bar{l}$, so we suppress it. Taken together, we have now proven each component of Theorem 1.

References

- Abdellaoui, M., A. Baillon, L. Placido, and P. Wakker. 2011. "The Rich Domain of Uncertainty: Source Functions and Their Experimental Implementation." *American Economic Review* 101:695–723.
- Adams, J., and S. Merrill. 1999. "Modeling Party Strategies and Policy Representation in Multiparty Elections: Why Are Strategies so Extreme?" *American Journal of Political Science* 43:765–91.
- Ahn, D., S. Choi, D. Gale, and S. Kariv. 2008. "Estimating Ambiguity Aversion in a Portfolio Choice Experiment." Working paper.
- Aragones, E., and T. Palfrey. 2002. "Mixed Equilibrium in a Downsian Model with a Favored Candidate." *Journal of Economic Theory* 103:131–61.
- Aragones, E., and T. Palfrey. 2005. "Spatial Competition between Two Candidates of Different Quality: The Effects of Candidate Ideology and Private Information." In *Social Choice and Strategic Behavior: Essays in Honor of Jeffrey Scot Banks*, edited by D. Austen-Smith and J. Duggan, 93–112. Berlin: Springer.
- Bade, S.. 2011. "Electoral Competition with Uncertainty Averse Parties." *Games and Economic Behavior* 72 (1):12–29.
- Banks, J. S.. 1990. "A Model of Electoral Competition with Incomplete Information." *Journal of Economic Theory* 50 (2):309–25.
- Berliant, M., and H. Konishi. 2005. "Salience: Agenda Choices by Competing Candidates." *Public Choice* 125:129–49.
- Black, D.. 1958. *The Theory of Committees and Elections*. New York: Cambridge University Press
- Bose, S., E. Ozdenoren, and A. Pape. 2006. "Optimal Auctions with Ambiguity." *Theoretical Economics* 1:411–38.
- Bossaerts, P., P. Ghirardato, S. Guarnaschelli, and W. Zame. 2010. "Ambiguity in Asset Markets: Theory and Experiment." *Review of Financial Studies* 23:1325–59.
- Calvert, R.. 1985. "Robustness of the Multidimensional Voting Model, Candidate Motivations, Uncertainty and Convergence." *Journal of Political Science* 39:69–95.
- Camerer, C., and R. Weber. 1992. "Recent Developments in Modeling Preferences: Uncertainty and Ambiguity." *Journal of Risk and Uncertainty* V:325–70.
- Congdon-Hohman, J., A. Nathan, and J. Svec. 2014. "Student Uncertainty and Major Choice: A Robust Control Approach." Working paper.
- Chahrour, R., and J. Svec. 2014. "Optimal Capital Taxation and Consumer Uncertainty." *Journal of Macroeconomics* 41:178–198.
- Dennis, R.. 2010. "How Robustness Can Lower the Cost of Discretion." *Journal of Monetary Economics* 57:653–67.
- Dennis, R., K. Leitemo, and U. Soderstrom. 2009. "Methods for Robust Control." *Journal of Economic Dynamics and Control* 33:1604–16.
- Downs, A.. 1957. *An Economic Theory of Democracy*. New York: Harper & Row.
- Ellsberg, D.. 1961. "Risk, Ambiguity and the Savage Axioms." *Quarterly Journal of Economics* LXXV:643–69.
- Enelow, J., and M. Hinich. 1989. "A General Probabilistic Spatial Theory of Elections." *Public Choice* 61:101–13.
- Erikson, R. S., and G. C. Wright Jr. 1985. "Voters, Candidates, and Issues in Congressional Elections." In *Congress Reconsidered*, 3rd ed., edited by L. C. Dodd and B. I. Oppenheimer, 87–108. Washington, DC: Congressional Quarterly.

- Erikson, R. S., and G. C. Wright Jr. 1993. "Voters, Candidates, and Issues in Congressional Elections." In *Congress Reconsidered*, 5th ed., edited by L. C. Dodd and B. I. Oppenheimer, 91–114. Washington, DC: Congressional Quarterly.
- Erikson, R. S., and G. C. Wright Jr. 2005. "Voters, Candidates, and Issues in Congressional Elections." In *Congress Reconsidered*, 8th ed., edited by L. C. Dodd and B. I. Oppenheimer, 77–106. Washington, DC: Congressional Quarterly.
- Fiorina, M.. 1974. *Representatives, Roll Calls, and Constituencies*. Lexington, MA: Lexington.
- Ghirardato, P., and J. Katz. 2006. "Indecision Theory: Quality of Information and Voting Behavior." *Journal of Public Economic Theory* 8 (3):379–99.
- Gilboa, I., and D. Schmeidler. 1989. "Maxmin Expected Utility with Non-unique Prior." *Journal of Mathematical Economics* 18:141–53.
- Groseclose, T. 2001. "A Model of Candidate Location When One Candidate Has a Valence Advantage." *American Journal of Political Science* 45:862–86.
- Halevy, Y. 2007. "Ellsberg Revisited: An Experimental Study." *Econometrica* 75:503–36.
- Hansen, L., and T. Sargent. 2005. "Robust Estimation and Control under Commitment." *Journal of Economic Theory* 124:258–301.
- Hansen, L., and T. Sargent. 2007. "Recursive Robust Estimation and Control without Commitment." *Journal of Economic Theory* 136:1–27.
- Hansen, L., and T. Sargent. 2008. *Robustness*. Princeton, NJ: Princeton University Press.
- Hansen, L. T., Sargent, G. Turmuhambetova, and N. Williams. 2006. "Robust Control and Model Misspecification." *Journal of Economic Theory* 128 (1):45–90.
- Hanson, L., F. Barillas, and T. Sargent. 2009. "Doubts or Variability?" *Journal of Economic Theory* 144:2388–418.
- Hotelling, H. 1929. "Stability in Competition." *The Economic Journal* 39:41–57.
- Karantounias, A. 2013. "Managing Pessimistic Expectations and Fiscal Policy." *Theoretical Economics* 8:193–231.
- Kartik, N., and R. McAfee. 2007. "Signaling Character in Electoral Competition." *American Economic Review* 97:852–70.
- Klibanoff, P. 1996. "Uncertainty, Decision, and Normal Form Games." Mimeo.
- Leitemo, K., and U. Söderström. 2008. "Robust Monetary Policy in the New Keynesian Framework." *Macroeconomic Dynamics* 12:126–35.
- Levin, A., and J. Williams. 2003. "Robust Monetary Policy with Competing Reference Models." *Journal of Monetary Economics* 50:945–75.
- McDonald, M. 2013. "Presidential Voter Turnout Rates, 1948–2012." United States Elections Project. Accessed 4 January 2013.
- Poole, K., and H. Rosenthal. 1984. "U.S. Presidential Elections 1968–1980: A Spatial Analysis." *American Journal of Political Science* 28:283–312.
- Roemer, J. 2001. *Political Competition: Theory and Applications*. Cambridge, MA: Harvard University Press.
- Schofield, N. 2007. "The Mean Voter Theorem: Necessary and Sufficient Conditions to a Convergent Equilibrium." *Review of Economic Studies* 74:1083–119.
- Stokes, D. 1963. "Spatial Models of Party Competition." *American Political Science Review* 57:368–77.
- Svec, J. 2012. "Optimal Fiscal Policy with Robust Control." *Journal of Economic Dynamics and Control* 36:349–68.
- Wittman, D. 1983. "Candidate Motivation: A Synthesis of Alternatives." *American Political Science Review* 77:142–57.