Topics

Suren Basov* and Svetlana Danilkina

Bertrand Oligopoly with Boundedly Rational Consumers

Abstract: In this paper we consider a model of Bertrand oligopoly when consumers are boundedly rational and make their purchase decisions probabilistically, according to the Luce model. We consider three different cases: first, we characterize equilibrium when firms face boundedly rational consumers with the fixed irrationality parameter λ; second, we discuss the case of obfuscating oligopoly, when firms can invest in order to confuse consumers, i.e. to increase their λ; and third, we consider educating oligopoly, when firms can choose to invest to decrease λ. We show that while it is worthwhile for the firms to confuse the consumers, it is only optimal to educate them if they are sufficiently rational at default. We also analyze how the social welfare, consumer surplus and the firms' profits depend on the number of firms.

Keywords: oligopolistic competition, Luce choice probabilities, social welfare
JEL Classification Numbers: D43, D60

DOI 10.1515/bejte-2013-0077

1 Introduction

Recent years saw proliferation of economic models that incorporate boundedly rational behavior on the part of economic agents. This explosion followed the seminal paper by Conlisk (1996) and includes Offerman, Schram, Sonnemans (1998) in the area of provision of public goods, and Anderson, Goeree, and Holt (1998) in Tullock contests. For the review of the literature in the mechanism design theory, see chapter by Basov, Bhatti, and Danilkina (2011).

***Corresponding author: Suren Basov,** Department of Finance, LaTrobe University, Bundoora, VIC 3086, Australia, E-mail: s.basov@latrobe.edu.au
Svetlana Danilkina, Department of Economics, The University of Melbourne, Melbourne, VIC 3010, Australia, E-mail: sdanilki@unimelb.edu.au

More recently, there emerged considerable interest in studying the consequences of boundedly rational behavior on the oligopolistic pricing and other activities of the firms – see Ellison (2006) for a detailed review of this literature. Here we will mention only a few papers related to the current topic.

Grossman (1981) and Milgrom (1981) demonstrated that if information disclosure is costless, the firms will reveal all the relevant information, i.e. they will educate rather then obfuscate the consumers. However, Ellison and Ellison (2009) argue that this prediction is not born out by many real world observations. Ellison and Wolitzky (2012) developed a model where obfuscation is theoretically justified in a search-theoretic framework. In their model obfuscation takes a form of an unobservable action that makes it more difficult to inspect the product and learn its price or quality. One way to think about it is to assume that the consumers have to spend time to make a product selection. They go through the list of product's characteristics to determine its value. If the list is too long they may end up making a mistake in their assessment. By devising concise and relevant description of the product, a firm may ease the task of the consumer, thereby decreasing the probability of a mistake. On the contrary, by putting a lot of irrelevant characteristics on the list or by engaging in an uninformative advertising, firms can make it more difficult for the consumers to go through the product description, increasing the probability of an error. One just needs to remember the plethora of different mobile plans to see the point. The complexity of different rates and features leaves most consumers being unable to compare mobile plans, let alone to choose the best one.

Ellison and Wolitzky (2012) describe several scenarios for achieving obfuscation. For example, they suggest that in online shopping application the firm can choose the number of screens the consumer has to go through to find all the relevant information and they may instruct the salespeople how long to talk to consumers during face-to-face shopping. Ellison and Wolitzky assume that consumers go sequentially through description of products supplied by the firms. The consumers' search costs depend on the sum of obfuscation levels chosen by the firms that they are visiting. Consumers have rational expectations about the obfuscation levels and can rationally choose to stop the search.

Our paper is similar in spirit to Ellison and Wolitzky (2012), however, instead of assuming that consumers experience direct costs from reading the products' description and learning relevant information about their quality and price, we assume that this process affects quality of consumers' decision making; the longer and more confusing the product description, the lower the quality of decision making. The crucial difference of this assumption from the one used by Ellison and Wolitzky is that while in the latter model the consumers are fully aware of the search costs and will rationally stop search at some point, in our model consumers unaware of the effect of obfuscation on their quality of the decision making. For example, a consumer who already has spent a couple of hours at a computer screen reading description of different house insurance policies, will still continue going through the exhaustive list, since she does not experience direct costs from search and is not aware that the very act of continuing the search will decrease the quality of her decision making.[1] For another example, think about choosing among different mobile plans and trying to understand how much they will cost per month.[2]

A related but different model will be one where the value of the products has a potentially heterogenous element that differs across firms.[3] In such a model, advertising can be seen as a way for firms to teach consumers to differentiate between the good and the bad versions of the product. In such a model firms will act as experts conveying information to the consumers and the model will have potential links with cheap talk literature (see, for example, Hagenbach and Koessler 2010) where the dominant insight is that if there are multiple experts around, there always exists equilibrium with perfect communication. Krishna and Morgan (2001) reached a similar conclusion[4] in a model of expertise, provided that experts have opposite biases. In our framework, however, the products are homogenous and quality is not an easily communicable piece of information, but can instead be learned only by reading a lengthy description of the product.

To formalize the idea of quality of choice we use the Luce model of probabilistic choice (Luce 1959). In the Luce model the degree of bounded rationality is captured by a single parameter, λ, with $\lambda = 0$ corresponding to rational behavior and higher λ corresponding to more irrational behavior. Note that consumers in our model are boundedly rational in two ways: they may fail to choose optimal product and they fail to take into account the effect of going through lengthy product description on λ.

We consider three different cases: first, we characterize equilibrium when firms face boundedly rational consumers with the fixed irrationality parameter λ; second, we discuss the case of obfuscating oligopoly, when firms can invest in

1 Since the consumer will always go through the exhaustive set of the alternatives, which will affect the overall quality of choice, a decision of one firm to increase level of obfuscation will affect the relative probabilities of all choices.

2 We are grateful to anonymous referee for this example.

3 We are grateful to anonymous referee for this observation.

4 The conclusion does not hold, however, if one of the senders can veto messages. See, Newton (2014).

confusing consumers, i.e. in increasing their λ; and third, we consider educating oligopoly, when firms can choose to invest in decreasing λ.

In this paper we arrive at the following results. First, firms will in general engage in obfuscating activities, but will only engage in educating ones if consumers are sufficiently rational at default. It is in contrast to the conclusion of full information revelation reached by Grossman (1981) and Milgrom (1981). Though definition of education and obfuscation is formally different in our framework and that of Grossman (1981) and Milgrom (1981), comparison of the conclusion is still illuminating. Second, in this framework competition has two beneficial effects from the social point of view: it increases the chance that a consumer will buy the product and decreases the amount of obfuscation. From the consumers' point of view, there is another beneficial effect of competition – it leads to decrease in price.

The paper is organized in the following way. In Section 2 we briefly remind the reader the Luce model of boundedly rational behavior. In Section 3 we consider price competition when firms face boundedly rational consumers with given λ.[5] In Section 4 we allow firms to engage in obfuscating activities. In Section 5 we assume that the firms cannot engage in obfuscating activities, but may choose to educate consumers (assume, for example, that the new law requires that all advertising and product description must be informative and simple). Section 6 concludes.

2 The Luce model

In this paper we assume that the consumers are boundedly rational. To capture it we assume that their choice is probabilistic, i.e. the utilities associated with different choices determine the probabilities with which these choices are made. The first probabilistic choice model in economics was proposed by Luce (1959). He showed that if one requires the choice probabilities to be independent of a sequence in which choices are made, then they must be represented by[6]:

$$q_i = \frac{\exp(u_j/\lambda)}{\sum_{j=1}^{n} \exp(u_j/\lambda)}, \qquad [1]$$

5 We restrict our attention to pure strategy Nash equilibria.

6 These probabilities are also known in empirical IO as a logit probabilities, but motivation there is different.

where n is the number of alternatives, q_i is the probability that alternative i is chosen, and u_i is the utility associated with alternative i. Note that according to this model any two alternatives that have the same utility are selected with the same probabilities. Parameter λ, which can take values from zero to infinity, can be usefully thought of as representing the degree of the consumer's irrationality. If $\lambda = 0$ then

$$q_i = \begin{cases} 1/k, & \text{if } u_i = \max\{u_1, \ldots, u_n\} \\ 0, & \text{otherwise} \end{cases}, \qquad [2]$$

where integer k is the cardinality of the set of the utility maximizers, which is consistent with rational behavior. At the other extreme, as $\lambda \to \infty$ the choice probabilities converge to $1/n$, i.e. the choice becomes totally random, independent of the utility level – the case of fully irrational consumer.

3 The Bertrand model with boundedly rational consumers: the case of fixed λ

In this section we assume that each of n firms sets a price and consumers respond by choosing whether to buy a good and from which firm. We assume that the consumers never buy more than one unit of a good, i.e. each consumer has $(n+1)$ different choices. The probability a consumer, who values the good at v, will decide to buy the good from firm i is given by:

$$q_i = \frac{\exp((v - p_i)/\lambda)}{1 + \sum_{j=1}^{n} \exp\left((v - p_j)/\lambda\right)}, \qquad [3]$$

while the probability that she will decide not to buy the good at all is:

$$q_0 = \frac{1}{1 + \sum_{j=1}^{n} \exp\left((v - p_j)/\lambda\right)}. \qquad [4]$$

Here p_i is the price charged by oligopolist i.

It is instructive to compare the demand shares implied by eq. [3] with the ones implied by the Hotelling model.[7] Though parameter λ plays role somewhat similar to the transportation costs, there are important differences. For example,

7 We are grateful to an anonymous referee for suggesting this comparison.

in our model consumers are not partitioned into groups that purchase from a particular firm. Also, in the latter model, the demand shares will depend only on price differences as long as the consumers' value is sufficiently high, so that all consumers purchase the good in equilibrium.[8] In our model the result will only obtain asymptotically, as $v \to +\infty$. Also, as we will see below, unlike the case of the Hotelling model, for finite n the equilibrium price depends on v.

Assuming the marginal cost is constant and equal to c for all firms, the profits of firm i are given by:

$$\Pi_i(p_i, p_{-i}) = (p_i - c)q_i(p_i, p_{-i}), \qquad [5]$$

here p_{-i} denotes the vector of prices chosen by its rivals. The first order conditions are:

$$\frac{1}{p_i - c} = -\frac{\partial \ln q_i}{\partial p_i}, \text{ for } i = \overline{1, n}. \qquad [6]$$

In the unique[9] symmetric equilibrium $p_i = p^{(n)}$,[10] where $p^{(n)}$ solves:

$$p^{(n)} = c + \lambda \frac{1 + n \exp\left(\left(v - p^{(n)}\right)/\lambda\right)}{1 + (n-1)\exp((v - p^{(n)})/\lambda)}. \qquad [7]$$

One can write eq. [7] as

$$p^{(n)} = F\left(p^{(n)}\right), \qquad [8]$$

where $F(\cdot)$ is defined as:

$$F\left(p^{(n)}\right) = c + \lambda \frac{1 + n \exp\left(\left(v - p^{(n)}\right)/\lambda\right)}{1 + (n-1)\exp((v - p^{(n)})/\lambda)}. \qquad [9]$$

It is straightforward to establish that

$$F'(p^{(n)}) = -\frac{\exp\left(\left(v - p^{(n)}\right)/\lambda\right)}{\left[1 + (n-1)\exp((v - p^{(n)})/\lambda)\right]^2} < 0. \qquad [10]$$

8 In our model for any profile of prices and any finite v some consumers will choose not to purchase the good.

9 We restrict our attention to the pure strategy equilibria. For a discussion of a mixed strategy equilibrium that allows firms to earn positive profits in the standard Bertrand game, see Klemperer (2004).

10 From here on, a subscript i will refer to the price charged by firm i, while a superscript (n) will refer to the price charged by any firm in a symmetric equilibrium with n firms.

Therefore, the right hand side of eq. [7] is monotonically decreasing in $p^{(n)}$, while the left hand side monotonically increases from zero to infinity, which implies that the solution exists and is unique. Moreover, since an increase in the number of firms will decrease the right hand side of eq. [7] for any $p^{(n)}$, the equilibrium price will uniformly decrease from the monopoly price $p^{(1)}$ to $c+\lambda$, when the number of firms approaches infinity. Therefore, we arrive at the following Lemma:

Lemma 1 *For any n, the equilibrium price is above the marginal cost and is decreasing in the number of firms.*

Let us concentrate on an important special case when the consumers' valuation of the good is large compared with both the marginal cost and the irrationality parameter. Then for $n \geq 2$ eq. [7] implies:

$$p^{(n)} = c + \frac{n\lambda}{n-1} + O\left(\exp\left(-\frac{v-c}{\lambda}\right)\right). \qquad [11]$$

Note that the equilibrium price is always much smaller than the valuation, therefore one can neglect the possibility that the consumers do not buy the good (the probability of it is proportional to $\exp(-(v-p_n)/\lambda)$, which is exponentially small).

3.1 The monopoly pricing

In this Subsection we analyze the behavior of a monopoly, derive a general formula for $p^{(1)}$ and provide simple expressions for two opposite cases, when the consumers are nearly rational or strongly irrational.

The monopoly price can be obtained from eq. [7] by setting $n = 1$ and it is the unique solution of the equation:

$$p^{(1)} = c + \lambda\left(1 + \exp(v - p^{(1)})/\lambda\right). \qquad [12]$$

One can rewrite eq. [12] as:

$$\frac{v-c}{\lambda} = \Phi\left(\frac{v-p}{\lambda}\right), \qquad [13]$$

where

$$\Phi(x) = 1 + x + \exp(x). \qquad [14]$$

Since function $\Phi(.)$ is strictly increasing, its inverse function, $\Phi^{-1}(\cdot)$, is well defined. Therefore we arrive at the following Lemma:

Lemma 2 *The equilibrium monopoly price is given by*:

$$p^{(1)} = v - \lambda\Phi^{-1}\left(\frac{v-c}{\lambda}\right). \qquad [15]$$

It is interesting to note that $p^{(1)} > v$ provided that $(v-c)/\lambda < 2$, i.e. if either total social surplus, $v-c$, is small or the consumers are sufficiently irrational, i.e. λ is large, the monopolist will charge the price above the valuation. To get some intuition for that result let us consider the case when value is close to the marginal cost. Though charging price below value increases the probability of a purchase, the monopolist will make more money by charging price above value and capitalizing on irrationality of consumers.[11]

Now let us look at the special case when the consumers are strongly irrational. Then the equilibrium price satisfies

$$p^{(1)} = v - \lambda\left[x_0 + O\left(\frac{v-c}{\lambda}\right)\right], \qquad [16]$$

where x_0 solves equation

$$\Phi(x_0) = 0. \qquad [17]$$

Solving it numerically one obtains $x_0 = -1.28$, which implies in the main order of approximation

$$p^{(1)} = v + 1.28\lambda. \qquad [18]$$

In the opposite case, when $(v-c)/\lambda$ is large, i.e. either the consumers are nearly rational or the total social surplus, $v-c$, is large, the optimal monopoly price is approximately equal to:

$$p^{(1)} = v - \lambda\ln\frac{v-c}{\lambda}, \qquad [19]$$

where we used that

$$\Phi^{-1}(y) = \ln y + O\left(\frac{\ln y}{y}\right). \qquad [20]$$

11 Making a mistake that leads to pay above one's value is not qualitatively different from buying from a wrong firm.

3.2 Number of firms and social welfare

Next, we would like to study how social welfare changes depending on the number of firms. Recall, that in the case of fully rational buyers the social welfare under Bertrand competition is independent of the number of firms. However, this conclusion does not hold under bounded rationality.

First, note that since $v > c$, the social efficiency requires that all consumers buy a unit of the good. Therefore, the welfare loss, WL, is proportional to the fraction of the consumers who do not buy the good.

More precisely, welfare loss is given by

$$WL(n) = q_0(v - c) = \frac{v - c}{1 + n\exp((v - p^{(n)})/\lambda)}. \qquad [21]$$

Let us formulate the following Proposition.

Proposition 1 *If the consumers are boundedly rational both the total welfare and the consumer surplus increase when there are more firms in a Bertrand competitive industry. A firm's profits decrease as the number of firms increases, but are always bounded away from zero.*

Proof. It is easy to see that the welfare loss declines in the number of firms. Indeed

$$\frac{\partial WL(n)}{\partial n} = -\frac{(v - c)\exp\left((v - p^{(n)})/\lambda\right)}{\left[1 + n\exp((v - p^{(n)})/\lambda)\right]^2}\left(1 - \frac{n}{\lambda}\frac{dp^{(n)}}{dn}\right) < 0 \qquad [22]$$

where we used that according to Lemma 1

$$\frac{dp^{(n)}}{dn} < 0. \qquad [23]$$

Since increase in n leads to both an increase of consumer participation and a decrease in price, the consumer surplus increases.

A firm's profits are given by:

$$\pi = \frac{1 - q_0}{n}\left(p^{(n)} - c\right) = \frac{p^{(n)} - c}{n + \exp(-(v - p^{(n)})/\lambda)}. \qquad [24]$$

Therefore

$$\frac{d\pi}{dn} = \frac{\partial \pi}{\partial n} + \frac{\partial \pi}{\partial p^{(n)}}\frac{dp^{(n)}}{dn}. \qquad [25]$$

It is straightforward to show that

$$\frac{\partial \pi}{\partial n} < 0. \quad [26]$$

On the other hand

$$\frac{\partial \pi}{\partial p^{(n)}} = \frac{n\lambda - \left(p^{(n)} - c - \lambda\right)\exp\left(-\left(v - p^{(n)}\right)/\lambda\right)}{\lambda[n + \exp(-(v - p^{(n)})/\lambda)]^2}. \quad [27]$$

Define

$$H(z) = n\lambda - (z - c - \lambda)\exp(-(v - z)/\lambda). \quad [28]$$

Straightforward differentiation shows that

$$H'(z) = -\frac{(z - c)\exp(-(v - z)/\lambda)}{\lambda}. \quad [29]$$

The numerator of [27] is equal to $H(p^{(n)})$ and according to [29] is decreasing in $p^{(n)}$ as long as $p^{(n)} > c$, which is always true according to Lemma 1. Therefore, since $p^{(n)}$ is decreasing in n,

$$H\left(p^{(n)}\right) > H\left(p^{(1)}\right) = (n - 1)\lambda \geq 0, \quad [30]$$

where we made use of eq. [12]. Thus,

$$\frac{\partial \pi}{\partial p^{(n)}} > 0 \quad [31]$$

together with [23]this implies that

$$\frac{d\pi}{dn} < 0. \quad [32]$$

Since, according to Lemma 1, the price remains above marginal cost a firm's profits are bounded away from zero. ■

Note that the derivative in eq. [21] can be represented as a sum of two terms. The first term captures the reduction of the social loss due to the increase in the number of efficient options, while the second captures the decrease of probability of staying out of the market due to the reduction of price. The main result of this section is that if the consumers are boundedly rational both the total social surplus and the consumer surplus increase when there are more firms in a Bertrand competitive industry. It is also worth mentioning that

$$\lim_{n \to +\infty} q_0(n) = 0, \quad [33]$$

therefore taking into account eq. [11] a firm's profits behave as

$$\pi = \frac{\lambda}{n} + o\left(\frac{1}{n}\right) \quad [34]$$

and the total profits of the industry converge to λ.

4 An obfuscating oligopoly

In this Section we assume that the consumers are rational at default ($\lambda = 0$); however, the firms can spend resources to confuse the consumers. This will lead to an increase in the value of λ. In Introduction, we discussed why resources spent by one firm affect probabilities of choice between any alternatives. For simplicity, we also make, admittedly strong, assumption that the single parameter λ captures all these effects. One way to justify it, is to assume that a consumer goes through the list of the alternatives in a random order.

We will assume the following timing. At date 1 all firms simultaneously decide on the amount of resources they spend confusing the consumers. If firm i devotes resources z_i to the obfuscating activities, then

$$\lambda = \varphi\left(\sum_{i=1}^{n} z_i\right), \quad [35]$$

where $\phi(\cdot)$ is a twice differentiable, strictly increasing, concave function, such that $\phi(0) = 0$, and satisfying the Inada conditions. We will refer to it as an "obfuscating technology." At the beginning of date 2 the firms observe λ and play the simultaneous Bertrand game. Introduce

$$\Pi_i\left(\lambda, n, p^{(n)}\right) = \frac{\left(p^{(n)} - c\right)\exp\left(\left(v - p^{(n)}\right)/\lambda\right)}{1 + n\exp\left(\left(v - p^{(n)}\right)/\lambda\right)}, \quad [36]$$

and define $G(\lambda, n)$ to be the profits firm i earns at date 2, assuming the firms at that date play the symmetric Nash equilibrium obtained in the previous section. Then

$$G(\lambda, n) = \Pi_i\left(\lambda, n, p^{(n)}(\lambda)\right), \quad [37]$$

where $p^{(n)}(\lambda)$ is determined by eq. [7]. At date 1 firm i chooses the amount it invests in obfuscation to solve:

$$\max_{z_i}\left(G\left(\varphi\left(\sum_{j=1}^{n} z_j\right), n\right) - z_i\right). \quad [38]$$

The first order conditions are:

$$G_\lambda(\lambda, n)\varphi'\left(\sum_{j=1}^{n} z_j\right) = 1. \qquad [39]$$

In a symmetric Nash equilibrium with n firms $z_i = z^{(n)}$ and

$$G_\lambda\left(\varphi\left(nz^{(n)}\right), n\right)\varphi'\left(nz^{(n)}\right) = 1. \qquad [40]$$

Eq. [40] allows us to find equilibrium amount of obfuscation for any number of firms. Inada conditions guarantee that if at default consumers are rational, the equilibrium amount of obfuscation is positive.

Let us determine the equilibrium amount of obfuscation when number of competitors become very large. In that situation, one can write

$$p^{(n)} = c + \lambda + O\left(\frac{1}{n}\right) \qquad [41]$$

and

$$G(\lambda, n) = \frac{\lambda}{n + e * \exp\left(\frac{c-v}{\lambda}\right)} + O\left(\frac{1}{n}\right). \qquad [42]$$

Let $Z^{(n)} = nz^{(n)}$ be the total amount of obfuscation in the equilibrium.

Lemma 3. *As the number of firms increases towards infinity, $Z^{(n)}$converges to zero, i.e.*

$$\lim_{n\to+\infty} Z^{(n)} = 0. \qquad [43]$$

Proof. Eq. [42] implies $G_\lambda \to 0$, therefore eq. [40] implies that $\varphi'\left(nz^{(n)}\right) \to \infty$ and the Inada conditions imply that $Z^{(n)} = nz^{(n)} \to 0$, i.e. the amount of resources the industry devotes to obfuscating activities converges to zero as the number of firms increases to infinity. ■

If instead of assuming that consumers are rational at default one assumes that they are characterized by some positive λ_0, there will exist $n_0 \in \mathbb{N}$ such that whenever number of firms is at least n_0 no resources are spent on obfuscation in equilibrium. Value of n_0 can be one if λ_0 is sufficiently large, in which case no resources will be spent on obfuscation at all.

The reason for the result stated in the Lemma 3 is that the obfuscation is a public good, from the point of view of the firms, and the firms tend to free ride at each other's expenses. It is hard to further analyze eq. [40] in general. Therefore, we will restrict our attention to a special case when the surplus $v - c$, is large compared to the equilibrium level of $\lambda = \lambda^*$ and show that in this case the amount of obfuscation is decreasing in number of firms. Indeed, the price is approximately given by eq. [11] for $n \geq 2$ and [12] in the case of monopoly and with the same precision

$$G(\lambda) = \frac{\lambda^*}{n-1} + O\left(\frac{\lambda^*}{v-c}\right) \quad [44]$$

in the proper oligopoly case, $n \geq 2$. In that case, neglecting O – terms, [40] can be rewritten as:

$$\varphi'\left(Z^{(n)}\right) = \varphi'\left(nz^{(n)}\right) = n-1, \quad [45]$$

where $z^{(n)}$ is the equilibrium value of z, when the number of firms is n. Since $\varphi(.)$ is assumed to be strictly concave, eq. [45] implies that the total amount of resources devoted to obfuscation decreases in the number of firms, i.e. $Z^{(2)} > Z^{(3)} \ldots.$

For example, if

$$\varphi(z) = \sqrt{z}, v - c = 100, \quad [46]$$

$$z^{(n)} = \frac{1}{4n(n-1)^2}, \; Z^{(n)} = \frac{1}{4(n-1)^2}, \; \lambda_n = \varphi\left(Z^{(n)}\right) = \frac{1}{2(n-1)}. \quad [47]$$

for $n \geq 2$. Note that equilibrium value of λ decreases from 0.5 for a duopoly, to zero as n goes to infinity, always remaining much less than $v - c$.

For a monopoly, the special case when $v - c$ is large compared to the equilibrium level of λ never realizes, since the monopoly will always choose the level of obfuscation to bring λ sufficiently close to $v - c$. We will refer to a situation when the total level of obfuscation is decreasing in the number of firms as a regular case. The regular case always realizes when the total surplus is sufficiently greater than the irrationality parameter.

Results of this Section show that from the social point of view increased competition is beneficial for two different reasons. First, as emphasized in the previous section, it decreases the probability that a consumer will choose not to buy the good. Second, it decreases the total amount of resources spent on obfuscation of the consumers.

5 An educating oligopoly

In this Section we show that it is only optimal for the firms to spend resources to educate boundedly rational consumers who are sufficiently rational at default.

Assume that the consumers have $\lambda = \lambda_0 > 0$, i.e. consumers are boundedly rational at default. The firms can spend resources to educate the consumers, for example, they can engage in informative advertising that will lead to a decrease in the value of λ.

The timing is the same as in the previous section. If firm i devotes resources ζ_i to the educating activities, then

$$\lambda = \phi\left(\sum_{i=1}^{n} \zeta_i\right), \qquad [48]$$

where $\varphi(\cdot)$ is a twice differentiable, strictly decreasing, convex function, satisfying the Inada conditions, such that

$$\phi(0) = \lambda_0, \lim_{\zeta \to \infty} \phi(\zeta) = 0, \qquad [49]$$

i.e. spending infinite resources on educating activities will make the consumers fully rational. We will refer to $\phi(.)$ as an "educating technology."

Note that educating activities of one firm affect probability of choices between any pair of products. Intuition for this assumption is that products produced by the firm are homogenous, therefore better understanding characteristics of one of the products helps to understand characteristics of all of the products. Again, we make, an admittedly strong, assumption that the effect is symmetric.

At the beginning of date 2 the firms observe each other's choice of ζ_i and play the simultaneous Bertrand game, assuming that the market is populated by the consumers whose rationality parameter is given by eq. [48]. Denote by $\Pi_i(\lambda, n)$ the profits firm i earns at date 2, assuming the firms at that date play the symmetric Nash equilibrium (obtained in the previous section). Then

$$\Pi_i(\lambda, n) = \frac{\left(p^{(n)} - c\right) \exp\left(\left(v - p^{(n)}\right)/\lambda\right)}{n \exp((v - p^{(n)})/\lambda) + 1}, \qquad [50]$$

where $p^{(n)}$ is determined by eq. [7].

We will argue that firms spend resources on educating the consumers only if consumers are sufficiently rational at default and spend no resources if the consumers are strongly irrational. This happens for two reasons. First, if there are at least two firms on the market, any potential increase in profits due to the higher degree of the consumers' participation will be eroded by price competition. The investment in educating consumers in the case of at least two firms in the market will be decreased further due to the fact that such an investment represents a public good from the point of view of the firms. Second, even in the case of the monopoly, the firm will find it more profitable to cut the price than to spend resources on educating the consumers. The above discussion suggests that if there going to be any investment in educating consumers at all, it should happen in the case of a monopoly.[12] So, we concentrate on this case and argue that even a monopoly will choose not to educate the consumers.

Proposition 2. *If the consumers are sufficiently irrational, i.e.*

$$\lambda_0 > \min\left(\frac{v-c}{2}, \frac{v-c}{\zeta^*}\right), \quad [51]$$

where ζ^* *solves*

$$\frac{\zeta^* \exp(\zeta^*)}{1+\exp(\zeta^*)} = -\frac{1}{\phi'(0)}, \quad [52]$$

the monopolist will spend no resources educating consumers.

Proof. The monopolist solves:

$$\max_{p,\lambda,\zeta}\left(\frac{(p-c)\exp((v-p)/\lambda)}{1+\exp((v-p)/\lambda)} - \zeta\right) \quad [53]$$

$$s.t.\ \lambda = \phi(\zeta), \quad [54]$$

$$\zeta \geq 0. \quad [55]$$

Here we dropped superscript $^{(1)}$ from the equilibrium price for the notational simplicity. The non-negativity constraint was never binding in the obfuscating oligopoly case due to the Inada conditions. The Kuhn-Tucker first order conditions are:

$$\begin{cases} p = c + \lambda\left(1+\exp\left(\frac{v-p}{\lambda}\right)\right) \\ \mu = -\dfrac{(v-p)(p-c)\exp((v-p)/\lambda)}{\lambda^2(1+\exp(v-p)/\lambda))^2} \\ \gamma = 1 - \mu\phi'(\zeta) \\ \lambda = \phi(\zeta) \\ \gamma\zeta = 0, \gamma \geq 0, \zeta \geq 0 \end{cases}, \quad [56]$$

12 At the end of the previous Section we saw that, at least in the case when $v-c$ is large compared to the equilibrium level of λ, the total resources spent on obfuscation decrease in the number of firms and referred to it as the regular case. A similar result can be obtained for the case of educating oligopoly. We will assume that the regular case is realized.

where μ and γ are Lagrange multipliers on constraints [54] and [55] respectively. Eqs. [48] and [49] together with the properties of function $\varphi(\cdot)$ imply that $\lambda > 0$. The first equation in system [56] now implies that $p > c$. Using the first equation in system [56], the second equation can be re-written as

$$\mu = -\frac{(v-p)\exp((v-p)/\lambda)}{\lambda(1+\exp((v-p)/\lambda))}. \quad [57]$$

Note that if

$$\frac{v-c}{\lambda} < 2 \quad [58]$$

then $p > v$ and $\mu > 0$, in which case, the monopolist's problem has the following solution: price p is given by eq. [15] and satisfies $p > v$, $\zeta = 0$, $\lambda = \lambda_0$, $\mu > 0$ is given by eq. [57], and $\gamma = 1 + \mu\phi'(0) > 0$. On the other hand, if

$$\frac{v-c}{\zeta^*} < \lambda_0 < \frac{v-c}{2}, \quad [59]$$

then $\mu < 0$ and

$$\gamma = 1 - \mu\phi'(\zeta) > 1 - \mu\phi'(0) = 1 + \frac{(v-p)\exp((v-p)/\lambda)}{\lambda(1+\exp((v-p)/\lambda))}\phi'(0). \quad [60]$$

Since $p > c$ and $\phi'(0) < 0$ one obtains

$$\gamma > 1 + \frac{(v-c)\exp((v-c)/\lambda)}{\lambda(1+\exp((v-c)/\lambda))}\phi'(0). \quad [61]$$

Define ζ^* to be the solution of eq. [52]. Then for any $\lambda > (v-c)/\zeta^*$ one obtains $\gamma > 0$, which implies $\zeta = 0$, $\lambda = \lambda_0$. ■

Note that though the Proposition holds when

$$\lambda_0 > \min\left(\frac{v-c}{2}, \frac{v-c}{\zeta^*}\right), \quad [62]$$

the case $\lambda_0 > (v-c)/2$ is not very interesting, since when the irrationality parameter is not sufficiently smaller then the total surplus the case may not be regular. However, if $\phi(.)$ is sufficiently flat at the origin, the condition $\lambda_0 > (v-c)/\zeta^*$ can be consistent with the regular case. Numerical simulations tell us that, for example, when $\phi'(0) = -0.2$ then $\zeta^* = 5.03$, when $\phi'(0) = -0.1$ then $\zeta^* = 10$. In general, for $\phi'(0) > -0.1$ one can approximate $\zeta^* = -1/\phi'(0)$. Therefore, in the regular case, the analysis is reduced to that of Section 3 with $\lambda = \lambda_0$.

6 Conclusions

In this paper we developed a model of Bertrand oligopolistic competition with boundedly rational consumers. We used the Luce model of probabilistic choice (Luce 1959) to capture the degree of the consumers' rationality that could be influenced by firms engaging either in educating or obfuscating activities. We have shown that contrary to the results of Grossman (1981) and Milgrom (1981), firms will in general engage in obfuscating activities, but will engage in educating ones only if consumers are sufficiently rational at default.

In this framework competition has two beneficial effects from the social point of view: it increases the chance that a consumer will buy the product and decreases the amount of obfuscation. The second effect arises because obfuscation is a public good from the firms' point of view and, therefore, the firms try to free ride on each other. From the consumers' point of view, there is another beneficial effect of competition – it leads to decrease in price.

References

Anderson, S. P., J. K. Goeree, and C. A. Holt. 1998. "Rent Seeking with Bounded Rationality: An Analysis of All-Pay Auction." *Journal of Political Economy* 106:828–53.

Basov, S., M. I. Bhatti, and S. Danilkina. 2011. "Recent Developments on Mechanism Design Under Bounded Rationality." In *Progress in Economics Research*, Vol. 20, edited by T. L. Wouters, 319–40. New York, NY: Nova Publishers.

Conlisk, J. 1996. "Why Bounded Rationality?" *Journal of Economic Literature* XXXIV:669–700.

Ellison, G. 2006. "Bounded Rationality in Industrial Organization." In *Advances in Economics and Econometrics: Theory and Applications, Ninth World Congress*, edited by R. Blundell, W. Newey, and T. Persson. Cambridge: Cambridge University Press.

Ellison, G., and S. F. Ellison. 2009. "Search, Obfuscation, and Price Elasticities on the Internet." *Econometrica* 77: 427–52.

Ellison, G., and A. Wolitzky. 2012. "A Search Cost Model of Obfuscation." *RAND Journal of Economics* 43:417–41.

Grossman, S. J. 1981. "The Role of Warranties and Private Disclosure about Product Quality." *Journal of Law and Economics* 24:461–83.

Hagenbach, J., and F. Koessler. 2010. "Strategic Communication Networks." *The Review of Economics Studies* 77:1072–99.

Klemperer, P. 2004. *Auctions: Theory and Practice*. Princeton, NJ: Princeton University Press.

Krishna, V., and J. Morgan. 2001. "A Model of Expertise." *The Quarterly Journal of Economics* 116: 747–75.

Luce, R. D. 1959. *Individual Choice Behavior*. New York: Wiley.

Milgrom, P. 1981. "Good News and Bad News: Representation Theorems and Applications." *Bell Journal of Economics* 12: 380–91.

Newton, J. 2014. "Cheap Talk and Editorial Control." *The B.E. Journal of Theoretical Economics* 14(1): 1–25. DOI:10.1515/bejte-2013-0002.