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Topics

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On Run-preventing Contract Design

Abstract: This study considers how to implement an efficient allocation of a financial intermediation model, including liquidation costs. The main result shows that there is a mechanism such that, for any liquidation cost, an efficient allocation is implementable in strictly dominant strategies. There is no need for third-party assistance, such as deposit insurance. In addition, the mechanism is tolerant of a small, unexpected shock caused by premature withdrawals.

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1 Introduction

Diamond and Dybvig (1983) point out that a demand-deposit contract can achieve a socially efficient allocation, but can also bring about an inefficient *bank run* outcome in equilibrium. Following this study, a number of other studies have addressed bank runs.[1] From a theoretical perspective, it is interesting to investigate the unique implementability of efficient allocations.[2]

Cooper and Ross (1998) extend the model of Diamond and Dybvig (1983) by introducing a *liquidation cost*. Their extended model has enriched the analyses

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1 Many useful surveys on this topic are available; for example, see Brunnermeier (2001, chap. 6) and Thakor and Boot (2008).

2 In a model with *finite* consumers, a single consumer can affect a social outcome. In this case, the problem can be solved easily if there is no *sequential service constraint*. Hence, in a finite model, the theoretical concern lies in implementing efficient outcomes while considering a sequential service constraint. For further details, see Green and Lin (2003), Peck and Shell (2003), Andolfatto and Nosal (2008), and Ennis and Keister (2009b). However, note that these finite models assume no liquidation costs.

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of banking systems, making them more realistic.[3] Cooper and Ross (1998) provide a necessary condition for which a direct mechanism relative to an efficient allocation prevents bank runs in all Nash equilibria.

However, their mechanism only offers a *one-time opportunity* to provide a good to consumers in each period. This restriction generally makes it difficult to achieve the optimum outcome. In fact, their run-preventing mechanism fails to implement an efficient allocation if the liquidation cost is sufficiently high.

In this study, I consider a more general mechanism and revisit the problem of implementing an efficient allocation. The mechanism I consider allows, at most, *two opportunities* to provide a good in one period.[4] The proposed mechanism can be defined regardless of liquidation costs. The main result shows that the mechanism uniquely implements an efficient allocation in strictly dominant strategies (see Section 3).

My mechanism enforces a degree of *illiquidity* on consumers in that there is a *delay* before they can complete premature withdrawals. Consumers can withdraw their deposits up to a limit. However, the remainder of their deposits and interest remains frozen for a time.[5] Without this illiquidity, some efficient allocations are not implementable (see Sections 2.2 and 3.1).

My result requires no deposit insurance. Although deposit insurance is useful to prevent bank runs, some researchers consider it to be controversial because it can cause other problems, for example, a *moral hazard*.[6]

Without aggregate risk or estimation error for withdrawals, the obtained outcome from my mechanism coincides with that of the mechanism of *suspension of convertibility* (Diamond and Dybvig 1983). Although the suspension mechanism appears attractive, some studies have shown its limits.[7] My mechanism is superior to the suspension mechanism in terms of the *continuity*

3 For example, see Allen and Gale (1998, 2000) and Ennis and Keister (2009a).

4 This method of expanding a message space is common in existing literature on implementation theory. For instance, see Moore and Repullo (1988) and Abreu and Matsushima (1992).

5 Recently, McCabe et al. (2012) proposed a scheme named *minimum balance at risk (MBR)* for money market funds (MMF). The MBR scheme prevents large redemptions of MMF. The mechanism in my study contains a scheme similar to the MBR.

6 Cooper and Ross (2002) point out that deposit insurance enhances the moral hazard problem. Martin (2006) shows that the liquidity provision of central banks is superior to deposit insurance in preventing moral hazards. Wallace (1988) criticizes the deposit insurance proposed in Diamond and Dybvig (1983) from the point of view of *sequential service*. However, this critique implicitly assumes that sequential service implies “one-provision-in-one-period.”

7 For example, Engineer (1989) shows that the mechanism of suspension of convertibility fails to work when there is uncertainty as to each consumer’s type. Ennis and Keister (2009a) show that a bank cannot ensure the prevention of bank runs if the bank’s suspension mechanism considers *ex post* efficiency.

of outcomes. That is, my mechanism makes moderate changes to a provision level, depending on the number of unexpected premature withdrawals. In contrast, the suspension mechanism makes drastic changes to a provision level if the number of unexpected premature withdrawals is sufficiently large (see Section 3.2).

2 Banking model

2.1 Preliminaries

This section describes the setting for Cooper and Ross's (1998) extended model. Consider an economy with a single consumption good, homogeneous competitive banks, and a continuum of consumers. The economy has three periods: $t_0, t_1,$ and t_2.

Short-term and long-term investment opportunities are available for the consumption good. For a unit of input in t_0, a short-term investment yields one unit of the consumption good in t_1. For the same input, a long-term investment yields $R > 1$ units of the good in t_2. The long-term investment can be liquidated, but this yields $1 - \kappa$ in t_1 per unit of input, where $\kappa \in [0, 1]$ denotes a *liquidation cost*.

Each consumer, i, is an element of $I = [0, 1]$ and has one unit of the consumption good as an endowment. At the commencement of t_1, a fraction of the consumers, $\theta \in (0, 1)$, face a liquidity shock and can only obtain utility from consumption in t_1. The remaining consumers can obtain utility in both t_1 and t_2. I refer to the former type as *early* consumers and the latter as *late* consumers. Each consumer's type is known only to that consumer (i.e. it is private information).

Let u denote the consumers' utility function over consumption, where u is a von Neumann–Morgenstern utility function, $u : \mathbf{R}_+ \to \mathbf{R}$. This function is strictly increasing, strictly concave, twice differentiable, and satisfies $\lim_{c \to 0} u'(c) = \infty$.[8] Here, $u(c_1 + c_2)$ applies to late consumers, where c_k is consumption in t_k, and $u(c_1)$ applies to early consumers.

8 Diamond and Dybvig (1983) further assume that $-cu''(c)/u'(c) > 1$, for all $c > 0$; that is, the relative risk aversion coefficient is greater than one, which makes the optimal consumption level greater than one. Following Cooper and Ross (1998), I do not make this assumption.

An efficient allocation, say (c_e^*, c_l^*), is defined as the solution to the following optimization problem:

$$\begin{aligned} &\max_{c_e, c_l, \psi} \theta u(c_e) + (1-\theta)u(c_l) \\ \text{s.t.} \quad &\theta c_e \leq 1-\psi, \ (1-\theta)c_l \leq R\psi, \end{aligned} \qquad [1]$$

where $\psi \in [0,1]$ is the ratio of long-term investment to bank deposits. In the optimum solution, $\psi^* = 1 - \theta c_e^*$ and $c_l^* = R(1-\theta c_e^*)(1-\theta)^{-1}$. In addition, the first-order condition implies that $c_e^* < c_l^*$.

2.2 Demand-deposit contracts and the bank-run problem

In this section, I consider the problem of uniquely implementing an efficient allocation (c_e^*, c_l^*) by constructing a *mechanism*. Let $\mathcal{M} = ((M_i), (g_i))$ denote a mechanism, where M_i is the message space of consumer i, and g_i denotes a provision for consumer i. For convenience, I denote a consumer's message by $m_i \in M_i$ and a profile of all consumers' messages by m.

For example, the Cooper–Ross mechanism assumes that $M_i = \{0,1\}$, for all $i \in I$, where $m_i = 0$ shows the willingness to withdraw. It further assumes that g_i is described as $g_i = (g_i^1, g_i^2)$, where g_i^k represents a provision in t_k. The mechanism is subject to the following *sequential service constraint*: (1) $g_i^k = g_j^k$, for all $k \in \{1,2\}$ and $i, j \in I$; (2) $g_i^1 = g_i^1(m_i)$ and $g_i^2 = g_i^2(m)$, for all $i \in I$.[9] If $M_i = \{0,1\}$, a contract is said to be a *demand-deposit* if $g_i^1(0) \geq 1$, for all $i \in I$. Under demand-deposit contracts, if $m_i = 0$ for all $i \in I$, the bank has no asset in t_2, so it is plausible to assume that $g_i^2(0) = 0$.[10]

Then, as Diamond and Dybvig (1983) show, any demand-deposit contract may cause a bank run under the sequential service constraint. Cooper and Ross (1998) consider a *run-preventing* contract, which prevents bank runs in all Nash equilibria, although their mechanism no longer embodies a demand-deposit contract if consumers are less risk-averse.

9 Rule (2) ignores the feasibility of provisions. However, I only consider the case in which g_i is feasible for all $i \in I$.

10 One may consider that the bank can offer any positive consumption in t_2 if $m_i = 0$ for all $i \in I$, because there is no consumer who wants to consume in t_2. If so, the bank-run problem itself disappears, including that in Diamond and Dybvig (1983). Suppose that $g_i^1(0) = c > 1$ and that $m_i = 0$ occurs for all $i \in I$. In this case, offering $g_i^2(0) > c$ has to be *possible*. Then, any late consumer, i, is willing to choose $m_i = 1$ unilaterally, which implies that the bank-run problem disappears.

3 Results

This section describes how the efficient allocation $(c_e(\theta), c_l(\theta)) = (c_e^*, c_l^*)$ is uniquely implementable in strictly dominant strategies.

First, I consider the following problem. For an arbitrary $\hat{\theta} \in (0,1)$:

$$\begin{aligned} \max_{c_e, c_l} \quad & \hat{\theta} u(c_e) + (1-\hat{\theta}) u(c_l) \\ \text{s.t.} \quad & \hat{\theta} c_e \le 1 - \psi^* \\ & (1-\hat{\theta}) c_l \le R\psi^* + 1 - \psi^* - \hat{\theta} c_e. \end{aligned} \qquad [2]$$

Let $(a_e(\hat{\theta}), a_l(\hat{\theta}))$ denote the solution to problem [2].

Lemma 1 *For any* $\hat{\theta} \in (\theta, 1)$, $a_e(\hat{\theta}) < c_e^* < c_l^* < a_l(\hat{\theta})$.

Proof. See the Appendix. ■

Next, I define the following functions:

$$f_1(\hat{\theta}) = \begin{cases} c_e^* & \text{if } \hat{\theta} \le \theta \\ a_e(\hat{\theta}) & \text{if } \hat{\theta} > \theta \end{cases}, f_2(\hat{\theta}) = \begin{cases} d_l(\hat{\theta}) & \text{if } \hat{\theta} \le \theta \\ a_l(\hat{\theta}) & \text{if } \hat{\theta} > \theta \end{cases}, \qquad [3]$$

where $d_l(\hat{\theta}) \equiv (1-\hat{\theta})^{-1}[(1-\theta)c_l^* + (\theta - \hat{\theta})c_e^*]$, $a_e(1) \equiv \lim_{\hat{\theta}\to 1} a_e(\hat{\theta})$, and $a_l(1) \equiv M^{-1}[R(1-\theta c_e^*)] > a_e(1)$, for some $M \in (0,1)$.

Lemma 2 *For any* $\hat{\theta} \in [0,1]$, $f_1(\hat{\theta}) < f_2(\hat{\theta})$.

Proof. Lemma 1 and the definitions of $f_1(1)$ and $f_2(1)$ imply that $f_1(\hat{\theta}) < f_2(\hat{\theta})$, for all $\hat{\theta} \in (\theta, 1]$. Thus, we only have to prove the case for each $\hat{\theta} \in [0, \theta]$. The fact that $c_e^* < c_l^*$ implies that:

$$\begin{aligned} (1-\hat{\theta}) d_l(\hat{\theta}) &= (1-\theta)c_l^* + (\theta - \hat{\theta})c_e^* \\ &> (1-\theta)c_e^* + (\theta - \hat{\theta})c_e^* \\ &= (1-\hat{\theta})c_e^*, \end{aligned}$$

which shows the result. ■

I consider a mechanism that comprises two communication phases in t_1 on, say, Day 1 and Day 2, and one phase in t_2 on, say, Day 3. Let $\mathcal{M} = ((M_i^*), (g_i^*))$ denote the mechanism, where $M_i^* = \{0,1\}$ and $g_i^* = (g_i^1, g_i^2, g_i^3)$, where g_i^k represents a provision on Day k. The sequential service constraint is as follows: (1) $g_i^k = g_j^k$, for all $k \in \{1,2,3\}$ and $i,j \in I$; (2) $g_i^1 = g_i^1(m_i)$, $g_i^2 = g_i^2(m)$, and $g_i^3 = g_i^3(m)$, for all $i \in I$.

Here, I consider a mechanism with the following characteristics: (1) the bank does not allow early consumers to withdraw their deposits all at once and (2) consumers can be served all their deposits *with a delay*.

Theorem 1 *Suppose that $\theta \in (0,1)$ is public information. There exists a mechanism that uniquely implements the efficient allocation $(c_e(\theta), c_l(\theta)) = (c_e^*, c_l^*)$ in strictly dominant strategies.*

Proof. Let $\underline{c}_e \equiv \inf_{\hat{\theta}\in(\theta,1]} a_e(\hat{\theta})$. Then, $\underline{c}_e > 0$ because $\lim_{c\to 0} u'(c) = \infty$. Consider the following provision functions:

Day 1. For all $i \in I$, $g_i^1(0) = \underline{c}_e$ and $g_i^1(1) = 0$.
Day 2. For all $i \in I$, if $m_i = 0$, then $g_i^2(m) = f_1(\theta_1) - \underline{c}_e$, where θ_1 is the number of consumers whose message m_i is zero; otherwise, $g_i^2 = 0$.
Day 3. For all $i \in I$, if $m_i = 1$, then $g_i^3 = f_2(\theta_1)$; otherwise, $g_i^3 = 0$.

Lemma 1 implies that $g_i^k \geq 0$ for all $k \in \{1,2,3\}$ and $i \in I$. Note that, for all $\theta_1 \in [0,1]$, the bank has $R\psi^*$ on Day 3. Hence, providing $f_2(\theta_1)$ to $1-\theta_1$ consumers is feasible. Lemma 2 implies that all late consumers prefer to be served in t_2, regardless of θ_1. In contrast, all early consumers prefer to be served in t_1, because $f(\theta_1) > 0$ for all $\theta_1 \in [0,1]$. Hence, we obtain $\theta_1 = \theta$ and the mechanism implements (c_e^*, c_l^*) in strictly dominant strategies. ■

This three-day mechanism has three important characteristics. First, it prevents bank runs. Second, the unique equilibrium outcome is efficient. Third, the mechanism is defined independently of the liquidation cost. Note that this mechanism sacrifices some liquidity for the benefit of stability. In particular, it does not embody a demand-deposit contract, as the Cooper–Ross' mechanism does.

3.1 Example

Suppose that consumers have the utility function $u(c) = (1-\gamma)^{-1} c^{1-\gamma}$, with a parameter $\gamma > 0$. Then, we can easily derive that:

$$c_e^* = \frac{1}{\theta + (1-\theta)R^{\frac{1-\gamma}{\gamma}}}, \quad c_l^* = R^{\frac{1}{\gamma}} c_e^*.$$

In the optimum solution to problem [2], we obtain

$$a_e(\hat{\theta}) = \frac{\theta c_e^*}{\hat{\theta}},\ a_l(\hat{\theta}) = \frac{(1-\theta)R^{\frac{1}{\gamma}} c_e^*}{1-\hat{\theta}}.$$

Following our three-day mechanism, the bank allows consumers to withdraw θc_e^* at once. In general, $\theta c_e^* < 1$ holds, hence our three-day mechanism does not represent a demand-deposit contract. However, this type of *illiquidity* is inevitable because a single bank has to avoid bank runs while achieving an *ex ante* Pareto-efficient outcome without any assistance, such as deposit insurance.

Here, we confirm that some efficient allocations fail to be implementable with any "two-day" mechanism, such as that of Cooper and Ross (1998). To achieve both the run-preventing property and efficiency, any efficient consumption, c_e^*, must satisfy

$$c_e^* \le 1 - \psi^* + \psi^*(1-\kappa) = 1 - \kappa\psi^*,$$

where $\psi^* = 1 - \theta c_e^*$, which is equivalent to

$$c_e^* \le \frac{1-\kappa}{1-\kappa\theta}. \qquad [4]$$

Obviously, inequality [4] is violated if κ is sufficiently large. In our example, for any $\theta \in (0,1)$ and $\gamma > 0$, if κ satisfies

$$\frac{\theta + (1-\theta)R^{\frac{1-\gamma}{\gamma}} - 1}{\theta + (1-\theta)R^{\frac{1-\gamma}{\gamma}} - \theta} < \kappa \le 1,$$

then any two-day mechanism must sacrifice efficient allocation $c_e^* = c_e(\theta)$ to prevent bank runs.

3.2 Mechanisms under an unexpected shock

If there is an estimation error for the number of early consumers, our three-day mechanism has better properties than the classic two-day mechanism of suspension of convertibility.[11]

In this section, I consider the following situation. The bank and consumers estimate the probability of a consumer being early as θ in t_0. However, the true

11 Allen and Gale (2000) investigate the robustness of a financial system by introducing a similar unexpected shock.

value is $\theta^* = \theta + \varepsilon$, which remains unknown to the bank and consumers until the end of t_1. Here, $\varepsilon \in [0, 1]$ is an unexpected term, referred to as a *shock*.

Consider the following functions:

$$h_1(\hat{\theta}) = \begin{cases} c_e^* & \text{if } \hat{\theta} \leq \theta \\ 0 & \text{if } \hat{\theta} > \theta \end{cases}, \quad h_2(\hat{\theta}) = \begin{cases} d_l(\hat{\theta}) & \text{if } \hat{\theta} \leq \theta \\ a_l(\hat{\theta}) & \text{if } \hat{\theta} > \theta \end{cases} \qquad [5]$$

Consumer i is served according to $h_k(\hat{\theta})$ in period t_k, where $\hat{\theta}$ is the number of consumers who have been served before i. In contrast to most suspension mechanisms, (h_1, h_2) describes the *suspension of convertibility* independently of $\kappa \in [0, 1]$. Without a shock, that is, $\theta^* = \theta$, (h_1, h_2) implements the efficient allocation (c_e^*, c_l^*) in strictly dominant strategies.

Now, consider the case of a shock. Suppose that $\theta^* = \theta + \varepsilon$ is realized, where $\varepsilon > 0$ is sufficiently small. Then, $h_1(\theta^*) = 0$, while $f_1(\theta^*) \approx c_e^*$ and $f_2(\theta^*) = h_2(\theta^*) \approx c_l^*$, because (f_1, f_2) are continuous. While our three-day mechanism provides goods that are sufficiently close to being efficient, the suspension mechanism (h_1, h_2) ignores the consumption by ε early consumers. Hence, our three-day mechanism is more *prudent* than the suspension mechanism in the case of unexpected shocks caused by premature withdrawals.

4 Conclusions

This paper proposes a provision mechanism in a deposit contract. The mechanism has the following strengths: (1) it is bank run-proof; (2) it uniquely implements an efficient allocation; (3) it is defined independently of liquidation costs; (4) it is defined without any third-party assistance, such as deposit insurance; and (5) it is tolerant of a small unexpected shock, as such a shock does not cause drastic changes in the outcome.

Appendix

Appendix: Proof of Lemma 1

For notational convenience, let $a_j = a_j(\hat{\theta})$, for $j = e, l$. The Lagrangian of the problem described in eq. [2] is

$$L = \hat{\theta}u(c_e) + (1-\hat{\theta})u(c_l) + \lambda(\theta c_e^* - \hat{\theta}c_e)$$
$$+ \mu\Big(R(1-\theta c_e^*) + \theta c_e^* - \hat{\theta}c_e - (1-\hat{\theta})c_l\Big),$$

for some $\lambda \geq 0$ and $\mu \geq 0$. Using the Kuhn–Tucker theorem, we obtain

$$u'(a_e) = u'(a_l) + \lambda, u'(a_l) = \mu, \qquad [6]$$

with complementary slackness conditions:

$$\lambda(\theta c_e^* - \hat{\theta}a_e) = 0 \qquad [7]$$

and

$$\mu\Big(R(1-\theta c_e^*) + \theta c_e^* - \hat{\theta}a_e - (1-\hat{\theta})a_l\Big) = 0. \qquad [8]$$

Eq. [6] implies that $a_e \leq a_l$ and $\mu > 0$.

Suppose that $a_e = a_l$. In this case, eq. [6] implies that $\lambda = 0$. Then, in eq. [8], we have $a_e = R(1-\theta c_e^*) + \theta c_e^*$. As $R(1-\theta c_e^*) = (1-\theta)c_l^*$ and $(1-\theta)c_l^* + \theta c_e^* > c_e^*$, we must have $a_e > c_e^*$. Then, the first inequality of eq. [2] implies that $\hat{\theta} < \theta$, which contradicts the assumption that $\theta < \hat{\theta}$. Hence, we obtain $a_e < a_l$ and $\lambda > 0$ whenever $\theta < \hat{\theta}$. Then, in eq. [7], $\theta c_e^* = \hat{\theta}a_e$ holds and, in eq. [8], as $\mu > 0$, $a_l = (1-\hat{\theta})^{-1}(R(1-\theta c_e^*)) = (1-\hat{\theta})^{-1}(1-\theta)c_l^* > c_l^*$ holds. Thus, we obtain $a_e < c_e^* < c_l^* < a_l$. This result is valid for any $\hat{\theta} \in (\theta, 1)$.

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