Research Article

Mariko Tanaka*

Currency Exchange in an Open-Economy Random Search Model

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Abstract: This paper studies how endogenous currency exchange arises in a two-country, two-currency monetary search model. Although currency exchange is widely observed in the globalized economy, Zhou (1997) is one of the exceptional studies that adopted a search model to generate currency exchange endogenously. Moreover, in her model, currency exchange occurs only in the case where agents rarely consume foreign goods, which is contrary to the well-known fact that international trade increases the number of currency exchanges. We construct a monetary search model that has a feature that increased international trade increases the volume of currency exchange. We develop a two-country model in which each country has two types of agents: local traders who consume only the goods produced in the home country, and international traders who consume only the goods produced in the foreign country. For international traders, the foreign currency gives more opportunity to obtain the foreign goods than the home currency does. Thus, when an international trader holds the home currency, he has an incentive to exchange the home currency for the foreign currency. Unlike Zhou, the model has a feature that currency exchange is more likely to occur when many agents are engaged in international trade.

Keywords: currency exchange, international trade, search model

JEL Classifications: D83, E44, F41

1 Introduction

In general, domestic trade is conducted through the home currency, international trade is conducted through international currencies, and there is active currency exchange between the home currency and each foreign currency. Why is active currency exchange observed in the foreign exchange market? The mechanism to generate such currency exchange has been an intriguing topic

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*Corresponding author: Mariko Tanaka, Faculty of Economics, Musashino University, 3-3-3 Ariake, Koto-ku, Tokyo 135-8181, Japan, E-mail: matanaka@musashino-u.ac.jp

in international finance. However, constructing a model with a microfoundation of currency exchange is not necessarily a simple task.

Matsuyama et al. (1993) is a seminal work that characterizes equilibria with an international currency in terms of the relative country size and the degree of economic integration by developing a two-country, two-currency monetary search model. However, in their model, currency exchange does not occur in a pure strategy Nash equilibrium since agents accept both the home currency and the foreign currency if both currencies circulate in each country.¹ Zhou (1997) criticizes this point and generates currency exchange in a pure strategy Nash equilibrium by introducing a taste shock under which agents who usually prefer domestic goods come to prefer foreign goods.

Zhou (1997) is a rare exception that succeeds in generating currency exchange by adopting a monetary search model. The paper focuses on the equilibrium in which domestic sellers accept only the home currency under the assumption that the majority of agents prefer domestic goods. The currency choice by each agent depends not only on his present taste, but also on changes in the liquidity of each currency at hand and his expected search cost due to a taste shock. Consequently, agents who prefer domestic goods ensure the circulation of the home currency, while agents who face a taste shock try to obtain the foreign currency; this generates currency exchange. However, such currency exchange occurs only in the case where agents rarely consume foreign goods. This result is contrary to the well-known fact that international trade increases the number of currency exchanges.

For example, Figure 1 depicts the relationship between the currency exchange turnover and the world trade volume from 1998 to 2013. The data of currency exchange turnover is from the BIS Triennial Central Bank Survey 2010-Final results and 2013, and the data of the world trade volume is from CPB World Trade Monitor 2014. This figure shows that the world trade volume temporarily decreases after the Lehman shock, but overall, both the currency exchange turnover and the world trade volume are increasing. According to BIS (2010-Final and 2013), global foreign exchange spot turnover with non-financial customers is increasing from averaged \$99 billion in April 1998 per day to \$217 billion in April 2010 and \$188 billion in April 2013. Although the foreign exchange market consists of the interbank market and the client market, and the growth of trading in the interbank market is remarkable, the figure suggests that currency exchange with non-financial customers and the world trade volume go hand in hand. Hence, this paper focuses on the client market in which currency exchange occurs

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¹ Subsequent studies include Wright and Trejos (2001), Li and Matsui (2009), Kannan (2009), and Zhang (2014). These papers utilize two-country, two-currency search model to study an international currency. However, they do not incorporate currency exchange into their models.

Figure 1: The currency exchange turnover and the world trade volume.

when importers exchange the home currency for the foreign currency and exporters exchange the foreign currency for the home currency.

In previous literature, Rey (2001) points out that an increasing volume of currency exchange decreases the transaction cost on the bilateral foreign exchange market –that is, a “thick market externality”–, and incorporates this property into a three-country, three-currency general equilibrium model, to show that the currency that saves transaction costs most circulates as a vehicle currency. The paper also confirms that the dollar replaces the pound as the vehicle currency around the same time when U.S. merchandise exports exceeded their U.K. counterparts. This finding implies that active transactions with the United States decreases the currency exchange cost between each currency and the dollar and increases currency exchange with the dollar, which leads the dollar to be the vehicle currency. Since the model assumes the “thick market externality” and imposes the CIA constraint, the choice of currency is exogenous. In contrast, this paper develops a two-country, two-currency monetary search model where the choice of currency is endogenous, to show that international trade increases the number of currency exchanges.

Besides Zhou (1997), a closely related literature which analyzes currency exchange in a monetary search model include Waller and Curtis (2003), and Craig and Waller (2001 and 2004). Unlike this model, they do not explain the stylized fact that increased international trade increases the volume of currency exchange. For example, Waller and Curtis (2003) show that currency exchange increases if each government imposes a strong restriction on the internal use of the foreign currency, which decreases international trade. Craig and Waller (2001, 2004) show that the currency risk, which acts as a tax on the home currency, increases currency exchange due to the portfolio rebalance effect at first, but then decreases it because the home currency becomes too risky. Craig

and Waller (2001, 2004) focus on money as a store of value and do not examine the volume of international trade,² whereas this paper uses a microfoundation of currency exchange by focusing on money as a medium of exchange.

In the following analysis, the model firstly sets the environment in which each country has two types of agents: local traders, who consume only the goods produced in the home country, and international traders, who consume only the goods produced in the foreign country, by extending Matsuyama et al. (1993). Next, the paper constructs a new basic model in which an exogenous shock, such as the taste shock in Zhou (1997), does not occur and shows that currency exchange exists in a pure strategy Nash equilibrium. In this newly constructed basic model, the currency that circulates in the home country is the home currency as in Zhou (1997), and local traders accept only the home currency. However, unlike Zhou (1997), this paper analyzes the environment in which local traders always prefer domestic goods, while international traders always prefer foreign goods. Consequently, currency exchange occurs without a trade shock when an international trader has the home currency.

This paper then introduces a taste shock into the basic model to reproduce Zhou's (1997) model. Zhou (1997) assumes that the taste shock follows a Markov process and that the probability of the taste switch from domestic goods to foreign goods is higher than that of the reverse – that is, the preference for domestic goods is more stable than that for foreign goods. As a result, currency exchange occurs when the probability of the taste shock is low in the equilibrium in which agents who prefer domestic goods accept only the home currency and agents who prefer foreign goods accept both currencies but prefer the foreign currency to the home currency. However, in this currency exchange equilibrium, most of the agents accept only the home currency, and the few agents who happen to temporarily prefer foreign goods exchange the home currency for the foreign currency, which is contrary to the well-known fact that international trade increases the number of currency exchanges.

In contrast, in the basic model of this paper, currency exchange occurs between international traders who want to exchange the home currency for the foreign currency to obtain foreign goods, while international traders always prefer foreign goods. Hence, unlike Zhou (1997), as the proportion of international traders increases, the volume of currency exchanges increases.

The rest of the paper is organized as follows. Section 2 describes the structure of the basic model. Section 3 derives the existence conditions of currency exchange based on the basic model. Section 4 introduces the

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² Camera et al. (2004) show that a higher currency risk promotes dollarization by utilizing a simplified model of Craig and Waller (2001, 2004), although they do not analyze currency exchange.

assumption, corresponding to a taste shock in Zhou (1997), that the proportion of local traders and in international trade follows a Markov process and that the majority of agents are engaged in domestic trade to reproduce the model of Zhou (1997), and compares with the basic model. Section 5 considers the optimal quantity of money in the asymmetric equilibrium. Section 6 concludes.

2 The Basic Model

Time is discrete and goes from zero to infinity. The economy is populated by a continuum of infinitely lived agents with unit mass. In the economy, there are two countries: Country 1 and Country 2. Let $n \in (0, 1)$ denote the size of the population of Country 1. Then, the size of the population of Country 2 is $1 - n$.

In each country, there are two types of agents. One is local traders who are engaged in domestic trade. The other is international traders who are engaged in international trade. Let π be the proportion of local traders in Country 1. Then, the proportion of international traders in Country 1 is $1 - \pi$. We assume that the two countries have a symmetric structure, and the variables of Country 2 are denoted symmetrically with $*$.

Under the above assumptions, there are four kinds of agents in this economy: local traders in Country 1, international traders in Country 1, local traders in Country 2, and international traders in Country 2. Each kind of agent produces $k \geq 3$ types of indivisible goods in equal proportion, where k reflects the degree of specialization in each country. Under the assumption, a double coincidence of wants never happens.

It is assumed that a local trader derives utility only from his type of domestic goods, while an international trader derives utility only from his type of foreign goods. After a type- i agent consumes a type- i good, he produces one unit of good of type- $i + 1 \pmod k$ costlessly, and then, he either keeps the production good or exchanges it for a currency that other agents hold to consume the good of his type in the future.

Let δ be the discount rate. Then, the expected discounted utility of an agent in Country 1 at time t is given as follows:

$$V(t) = E \left[\sum_{s=0}^{\infty} \frac{u}{(1+\delta)^s} I_{t+s} \middle| \Omega_t \right],$$

where Ω_t is the information set available at period t , $u = u_d$ is the utility of a local trader, $u = u_f$ is the utility of an international trader, and $I_{t+s} = 1$ if the agent consumes his consumption good at period $t + s$, and $I_{t+s} = 0$ otherwise.

In this economy, in addition to the goods described above, there exist two kinds of currencies: Currency 1, issued by the central bank of Country 1, and Currency 2, issued by the central bank of Country 2. Each currency is indivisible, and up to one unit can be stored costlessly. Hence, the inventory that each agent holds contains either one unit of currency or one unit of his production good. Let m_1^d and m_2^d denote the fraction of Currency 1 and Currency 2 held by local traders in Country 1, respectively. Then, the fraction of production goods held by local traders in Country 1 is $1 - m_1^d - m_2^d$. Hence, the inventory distribution of local traders in Country 1, X^d , is summarized as $X^d = (1 - m_1^d - m_2^d, m_1^d, m_2^d)$. Similarly, the fractions of Currency 1, Currency 2, and production goods held by an international trader in Country 1 are given by m_1^l , m_2^l , and $1 - m_1^l - m_2^l$, respectively. Hence, the inventory distribution of international traders in Country 1, X^l , is summarized as $X^l = (1 - m_1^l - m_2^l, m_1^l, m_2^l)$. Since the two countries have a symmetric structure, the variables of Country 2 are denoted symmetrically with $*$.

This paper analyzes the economy in which only the home currency is accepted in the home country.³ In this environment, local traders in Country 1 do not hold Currency 2, and local traders in Country 2 do not hold Currency 1: $m_2^d = 0$ and $m_1^{d*} = 0$.

Let $M \in (0, 1)$ denote the supply of Currency 1 per Country 1 agent. It is assumed to be exogenous. Then, the following relation holds for Country 1:

$$nM = n\pi m_1^d + n(1 - \pi)m_1^l + (1 - n)(1 - \pi^*)m_1^{l*}.$$

Agents are matched randomly in pairs and decide whether to trade or not. When both agree to trade, a transaction is established. The frequency of meeting the other agents is given in Table 1. For example, the probability with which a

Table 1: The matching technology.

	Country 1 Local trader	Country 1 International trader	Country 2 Local trader	Country 2 International trader
Country 1 agent	$n\pi$	$n(1 - \pi)$	$\beta(1 - n)\pi^*$	$\beta(1 - n)(1 - \pi^*)$
Country 2 agent	$\beta n\pi$	$\beta n(1 - \pi)$	$(1 - n)\pi^*$	$(1 - n)(1 - \pi^*)$

³ Although this assumption does not affect the results of this paper, Appendix C examines the conditions under which local traders endogenously accept only the local currency.

Country 1 agent meets Country 1 local traders is $n\pi$. In the table, β represents the relative frequency with which each agent meets foreign agents. It is interpreted as the degree of economic integration.

The main assumptions that are used to examine the possibility of a transaction are summarized as follows. First, local traders gain utility only from domestic goods. Second, international traders gain utility only from international goods. Third, agents never engage in barter trades. Fourth, local traders use only the home currency.

Table 2: Types of opponents.

Agents with production goods	Types of opponents
Local traders in Country 1	Local traders in Country 1 with Currency 1 International traders in Country 2 with Currency 1
International traders in Country 1	Local traders in Country 1 with Currency 1 International traders in Country 2 with any currency

Under these assumptions, Table 2 shows the types of agents who buy goods from agents who hold production goods. For example, those who buy goods from local traders in Country 1 are local traders in Country 1 who hold Currency 1 and international traders in Country 2 who hold Currency 1. This is because international traders in Country 1 and local traders in Country 2 do not consume goods produced by local traders in Country 1 and because local traders in Country 1 only accept Currency 1. Similarly, those who buy goods from international traders in Country 1 are local traders in Country 1 who hold Currency 1 and international traders who hold Currency 1 or Currency 2. This is because international traders in Country 1 and local traders in Country 2 do not consume goods produced by international traders in Country 1 and because international traders accept both Currency 1 and Currency 2.

Figures 2 and 3 depict the possibility of transactions with local traders and international traders in Country 1, respectively, who hold production goods. In

Figure 2: The possibility of transactions with local traders.

Figure 3: The possibility of transactions with international traders.

the figures, $\times$ indicates that a transaction has not been established. Figure 2 shows that international traders in Country 2 who hold Currency 1 buy from local traders in Country 1, while Figure 3 shows that international traders in Country 2 who hold Currency 1 or Currency 2 buy from international traders in Country 1.

2.1 Strategy and Equilibrium

Each agent chooses his strategy to maximize the expected discounted utility, given the others' strategies and inventory distributions. This paper focuses on pure strategies, which only depend on the type – that is, whether an agent and his opponent are a local trader or an international trader –, their nationalities, and their inventory distributions.

Let $\tau_{a,b}^d$ and $\tau_{a,b}^I$ denote, respectively, the strategies of a local trader and an international trader in Country 1 who decide whether to trade his inventory a for b , where g is a production good, 1 is Currency 1, 2 is Currency 2, $a, b = g, 1, 2$, and $a \neq b$. If the agent agrees to trade, $\tau_{ab}^d = 1$; otherwise, 0. For example, $\tau_{g2}^d = 0$ means that the local trader does not trade his production good for Currency 2. Under the assumption that only the home currency is accepted in domestic trade, $\tau_{g2}^{d*} = 1$, $\tau_{12}^{d*} = 0$, $\tau_{g1}^{d*} = 0$, and $\tau_{21}^{d*} = 0$ hold.

Strategies $(\tau^d, \tau^I, \tau^{d*}, \tau^{I*})$, inventory distributions $(X^d, X^I, X^{d*}, X^{I*})$ and the matching of agents generate the Markov process that each agent's inventory distributions follow. The transition matrices of local traders and international traders in Country 1, which are denoted by Π^d and Π^I , respectively, are given as follows:

$$\Pi^d = \begin{bmatrix} 1 - P_{g1}^d - P_{g2}^d & P_{g1}^d & P_{g2}^d \\ P_{1g}^d & 1 - P_{1g}^d - P_{12}^d & P_{12}^d \\ P_{2g}^d & P_{21}^d & 1 - P_{2g}^d - P_{21}^d \end{bmatrix},$$

$$\Pi^I = \begin{bmatrix} 1 - P_{g1}^I - P_{g2}^I & P_{g1}^I & P_{g2}^I \\ P_{1g}^I & 1 - P_{1g}^I - P_{12}^I & P_{12}^I \\ P_{2g}^I & P_{21}^I & 1 - P_{2g}^I - P_{21}^I \end{bmatrix},$$

where P_{ab}^d and P_{ab}^I mean that the inventories held by local traders and international traders, respectively, in Country 1 switch from a to b . For example, P_{g1}^d means that a local trader in Country 1 trades his production good for Currency 1. This transaction is established when the local trader in Country 1 is matched with another local trader in Country 1 with Currency 1 or with an international trader in Country 2 with Currency 1 and if they agree to exchange their inventories, that is, if $\tau_{g1}^d \cdot \tau_{1g}^d = 1$ or $\tau_{g1}^d \cdot \tau_{1g}^{I*} = 1$ holds. Hence, P_{g1}^d is given by $P_{g1}^d = \tau_{g1}^d \{n\pi m_1^d \tau_{1g}^d + \beta n^* (1 - \pi^*) m_1^{I*} \tau_{1g}^{I*}\} / k$.

In the steady state, the inventory distributions and the transition probabilities are constant, that is, $X^d \Pi^d = X^d$, $X^I \Pi^I = X^I$, $X^{d*} \Pi^{d*} = X^{d*}$, and $X^{I*} \Pi^{I*} = X^{I*}$ hold. This paper examines a pure strategy Nash equilibrium in the steady state. The equilibrium consists of strategies $(\tau^d, \tau^I, \tau^{d*}, \tau^{I*})$, inventory distributions $(X^d, X^I, X^{d*}, X^{I*})$, and transition matrices in the steady state. In equilibrium, each agent chooses his strategy to maximize his discounted expected utility given the other agents' strategies and inventory distributions. In addition, the transition matrices and inventory distributions are consistent with strategies under rational expectations.

2.2 General Properties

This section describes agents' optimization problem by adopting dynamic programming. Let V_g^d , V_1^d , and V_2^d be the value functions of a local trader in Country 1 who holds his production, Currency 1, and Currency 2, respectively, and V_g^I , V_1^I , and V_2^I be those of an international trader in Country 1:

$$V_g^d = [(1 - P_{g1}^d - P_{g2}^d)V_g^d + P_{g1}^d V_1^d + P_{g2}^d V_2^d] / (1 + \delta),$$

$$V_1^d = [P_{1g}^d(u_d + V_g^d) + (1 - P_{1g}^d - P_{12}^d)V_1^d + P_{12}^d V_2^d] / (1 + \delta),$$

$$V_2^d = [P_{2g}^d(u_d + V_g^d) + P_{21}^d V_1^d + (1 - P_{2g}^d - P_{21}^d)V_2^d] / (1 + \delta),$$

$$V_g^I = [(1 - P_{g1}^I - P_{g2}^I)V_g^I + P_{g1}^I V_1^I + P_{g2}^I V_2^I] / (1 + \delta),$$

$$V_1^I = [P_{1g}^I(u_I + V_g^I) + (1 - P_{1g}^I - P_{12}^I)V_1^I + P_{12}^I V_2^I] / (1 + \delta),$$

$$V_2^I = [P_{2g}^I(u_I + V_g^I) + P_{21}^I V_1^I + (1 - P_{2g}^I - P_{21}^I)V_2^I]/(1 + \delta). \quad [1]$$

The Bellman equations and equilibrium strategies satisfy the incentive compatibility constraints as follows:

$$\begin{aligned} \tau_{gb}^d = 1 \text{ iff } V_g^d < V_b^d, \tau_{ag}^d = 1 \text{ iff } V_a^d < u_d + V_g^d, \tau_{ab}^d = 1 \text{ iff } V_a^d < V_b^d, \\ \tau_{gb}^I = 1 \text{ iff } V_g^I < V_b^I, \tau_{ag}^I = 1 \text{ iff } V_a^I < u_I + V_g^I, \text{ and } \tau_{ab}^I = 1 \text{ iff } V_a^I < V_b^I, \end{aligned}$$

where $a, b = 1, 2$ and $a \neq b$.

In the steady-state equilibrium, the strategies $(\tau^d, \tau^I, \tau^d, \tau^I)$, inventory distributions (X^d, X^I, X^d, X^I) , and transition matrices (Π^d, Π^I) satisfy the following lemma.

Lemma (See Appendix A)

- (a-d) $0 \leq V_g^d, V_1^d, V_2^d < u_d + V_g^d$, (a-I) $0 \leq V_g^I, V_1^I, V_2^I < u_I + V_g^I$,
- (b-d) $\text{Max}\{V_1^d, V_2^d\} > V_g^d > 0$, (b-I) $\text{Max}\{V_1^I, V_2^I\} > V_g^I > 0$,
- (c-d) $V_1^d \geq V_2^d$ iff $P_{1g}^d \geq P_{2g}^d$ (c-I) $V_1^I \geq V_2^I$ iff $P_{1g}^I \geq P_{2g}^I$,
- (d-d) $V_1^d \geq V_g^d$ iff $P_{1g}^d(\delta + P_{g2}^d + P_{2g}^d + P_{21}^d) + P_{12}^d P_{2g}^d \geq P_{g2}^d P_{2g}^d$,
- (d-I) $V_1^I \geq V_g^I$ iff $P_{1g}^I(\delta + P_{g2}^I + P_{2g}^I + P_{21}^I) + P_{12}^I P_{2g}^I \geq P_{g2}^I P_{2g}^I$,
- (e-d) $V_2^d \geq V_g^d$ iff $P_{2g}^d(\delta + P_{g1}^d + P_{1g}^d + P_{12}^d) + P_{21}^d P_{1g}^d \geq P_{g1}^d P_{1g}^d$,
- (e-I) $V_2^I \geq V_g^I$ iff $P_{2g}^I(\delta + P_{g1}^I + P_{1g}^I + P_{12}^I) + P_{21}^I P_{1g}^I \geq P_{g1}^I P_{1g}^I$,
- (f-d) $\delta[(1 - m_1^d - m_2^d)V_g^d + m_1^d V_1^d + m_2^d V_2^d] = [m_1^d P_{1g}^d + m_2^d P_{2g}^d]u_d$.
- (f-I) $\delta[(1 - m_1^I - m_2^I)V_g^I + m_1^I V_1^I + m_2^I V_2^I] = [m_1^I P_{1g}^I + m_2^I P_{2g}^I]u_I$.

The following sections characterize the currency exchange equilibrium using this lemma.

2.3 Ranking of Value Functions and Equilibrium Strategies

In the equilibrium, a local trader in Country 1 trades his production good for Currency 1 or Currency 1 for his consumption good but does not accept Currency 2, while an international trader in Country 1 trades his production good for Currency 1 or Currency 2, Currency 1 for Currency 2 or his consumption good, or Currency 2 for his consumption good. Then, the ranking of the value functions of local traders and international traders in Country 1 is given as follows:

$$u_d + V_g^d > V_1^d > V_g^d \geq V_2^d \text{ and } u_I + V_g^I > V_2^I > V_1^I > V_g^I.$$

2.4 Inventory Distributions in the Steady State

In the equilibrium, a local trader in Country 1 holds his production good or Currency 1, while an international trader in Country 1 holds his production good, Currency 1, or Currency 2, that is, $m_1^d > 0$, $m_2^d = 0$, $m_1^I > 0$, $m_2^I > 0$. Hence, Country 1 agents' inventory distributions are described as $X^d = (1 - m_1^d, m_1^d, 0)$ and $X^I = (1 - m_1^I - m_2^I, m_1^I, m_2^I)$.

In addition, since the money supply of Currency 1 is given by M , and the ratios of production goods to each currency in agents' inventories are equalized in the steady state, $m_1^d = M$, $m_1^I = m_1^{I*}$ and $m_2^I = m_2^{I*}$ hold. Hence, the amounts of the currencies in inventories in the steady state are given as follows:

$$\begin{aligned} m_1^d = M, m_1^I = m_1^{I*} &= \frac{n(1-\pi)M}{n(1-\pi) + (1-n)(1-\pi^*)}, \\ m_2^{d*} = M^*, \text{ and } m_2^I = m_2^{I*} &= \frac{(1-n)(1-\pi^*)M^*}{n(1-\pi) + (1-n)(1-\pi^*)}. \end{aligned} \quad [2]$$

The next section derives the existence conditions of currency exchange based on this basic model.

3 Currency Exchange Equilibrium in the Basic Model

3.1 Transition Probabilities

Utilizing the agents' strategies, inventory distributions, and the matching of agents in the basic model, the transition probabilities of Country 1 local traders are given as follows:

$$\begin{aligned} P_{g1}^d &= \frac{n\pi m_1^d + \beta(1-n)(1-\pi^*)m_1^{I*}}{k}, \quad P_{g2}^d = P_{12}^d = P_{21}^d = P_{2g}^d = 0, \\ P_{1g}^d &= \frac{n\pi(1-m_1^d) + n(1-\pi)(1-m_1^I - m_2^I)}{k}. \end{aligned} \quad [3]$$

The transition probabilities of the other kinds of agents are similarly given as follows:

$$P_{g1}^I = \frac{n\pi m_1^d + \beta(1-n)(1-\pi^*)m_1^{I*}}{k}, \quad P_{g2}^I = \frac{\beta(1-n)(1-\pi^*)m_2^{I*}}{k},$$

$$P_{1g}^I = \frac{\beta(1-n)(1-\pi^*)(1-m_1^{I^*}-m_2^{I^*})}{k}, P_{12}^I = \beta(1-n)(1-\pi^*)m_2^{I^*},$$

$$P_{2g}^I = \frac{\beta(1-n)\pi^*(1-m_2^{d^*}) + \beta(1-n)(1-\pi^*)(1-m_1^{I^*}-m_2^{I^*})}{k}, P_{21}^I = 0. \quad [4]$$

$$P_{g2}^{d^*} = \frac{(1-n)\pi^*m_2^{d^*} + \beta n(1-\pi)m_2^I}{k}, P_{g1}^{d^*} = P_{12}^{d^*} = P_{21}^{d^*} = P_{1g}^{d^*} = 0,$$

$$P_{2g}^{d^*} = \frac{(1-n)\pi^*(1-m_2^{d^*}) + (1-n)(1-\pi^*)(1-m_1^{I^*}-m_2^{I^*})}{k}. \quad [5]$$

$$P_{g2}^{I^*} = \frac{\beta n(1-\pi)m_2^I + (1-n)\pi^*m_2^{d^*}}{k}, P_{g1}^{I^*} = \frac{\beta n(1-\pi)m_1^I}{k},$$

$$P_{2g}^{I^*} = \frac{\beta n(1-\pi)(1-m_1^I-m_2^I)}{k}, P_{21}^{I^*} = \beta n(1-\pi)m_1^I,$$

$$P_{1g}^{I^*} = \frac{\beta n\pi(1-m_1^d) + \beta n(1-\pi)(1-m_1^I-m_2^I)}{k}. \quad [6]$$

3.2 Equilibrium Conditions

The conditions under which the ranking of value functions and the transition probabilities satisfy the lemma stated above give the following proposition.

Proposition 1 (See Appendix B).

In the basic model, currency exchange equilibrium always exists.⁴

This proposition means that the home currency increases the possibility of consuming goods much more than does the foreign currency. Thus, when international traders hold the domestic currency, they would like to trade the home currency for the foreign currency. Hence, currency exchange occurs between international traders in two countries who hold their respective home currencies.

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⁴ Appendix C examines the conditions under which local traders endogenously accept only the local currency. This exercise does not affect Proposition 1.

The frequencies of currency exchange by international traders in Country 1 and Country 2, P_{12}^I and $P_{21}^{I^*}$, respectively, are calculated by using eqs [5] and [6] as $P_{12}^I = \beta(1-n)(1-\pi^*)m_2^{I^*}$ and $P_{21}^{I^*} = \beta n(1-\pi)m_1^I$, respectively. This implies that as the population of international traders increases, the frequency of currency exchange increases.

A proposition regarding the ratio of currency exchange to international transactions, which we call the ratio of currency exchange in the following passages, is given as follows.

Proposition 2 (See Appendix D).

As the population of international traders increases, the ratio of currency exchange increases.

Note that an increase in the degree of economic integration, β , increases the ratio of currency exchange since it increases the frequency of trade with agents who hold foreign goods. An increase in the money stock also increases the ratio of currency exchange since it increases the frequency of trade with agents who hold Currency 1 or Currency 2.

For example, in the symmetric equilibrium with $n = 1/2$, $\pi = \pi^*$, and $M = M^*$, the ratio of currency exchange is $\frac{\beta(1-\pi)^2 Mk^2}{4(1-M)\{\beta(1-\pi)(3-\pi)+4\pi\}}$. Figure 4(a) depicts the relation between the population of international traders and the ratio of currency exchange in the case of $k = 15$ and $M = 0.5$. The graphs show that as the population of international traders, $1 - \pi$, increases, the ratio of currency exchange increases. They also show that as the degree of economic integration, β , increases, the ratio of currency exchange increases.

According to BIS (2013), the average daily spot trading with non-financial customers amounts to \$188,109 million in April 2013. On the other hand, IMF International Financial Statistics shows that the monthly value of world exports of merchandise amounts to \$1,541.62 billion in April 2013. Assuming that there are 22 trading days per month, the average monthly spot trading in April 2013 amounts to \$4,138.398 billion. These field data shows that the ratio of currency exchange to international trade per month is about 2.68, i.e. 268%. In Figure 4(a), the ratio of currency exchange is about 2.6, i.e. 260% when $\beta = 0.5$ and $1 - \pi = 0.5$. Hence, the basic model can reproduce the currency exchange ratio that is consistent with the field data.

Next, Figure 4(b) depicts the relation between the population of international traders and the ratio of currency exchange in the case of $\beta = 0.5$. The graphs show that as the population of international traders, $1 - \pi$, increases, the ratio of currency exchange increases. They also show that as M increases, the ratio of currency exchange increases.

Figure 4: The ratio of currency exchange in the case of (a) $M = 0.5$ and (b) $\beta = 0.5$.

4 Comparison with the Reproduction Model of Zhou (1997)

This section reproduces the model of Zhou (1997), which we call the reproduction model in the following passages, by introducing a taste shock into the basic

model, and compares the two models. Since we assume that the two countries have a symmetric structure, the variables of Country 2 are denoted symmetrically with $*$.

4.1 Assumptions on a Taste Shock

We assume that the ratios of local traders and international traders follow Markov processes. The present local traders in Country 1 become local traders with probability A and international traders with probability $1 - A$ in the next period, while the present international traders in Country 1 become local traders with probability B and international traders with probability $1 - B$ in the next period.

In the basic model, the fraction π is local traders, while the fraction $1 - \pi$ is international traders in Country 1. Reflecting this setting, this section assumes that $(1 - A)\pi = B(1 - \pi)$ in the steady state in Country 1. Moreover, Zhou (1997) assumes that the probability of the taste switch from domestic goods to foreign goods is higher than that of the reverse. Taking this assumption into consideration, this section assumes that the probability of switching from local traders to international traders is smaller than that from international traders to local traders, that is, $B > 1 - A$.

4.2 Comparison of the Equilibrium Conditions

This section compares the equilibrium conditions of the two models.⁵ As Section 3 shows, currency exchange equilibrium always exists in the basic model. In contrast, currency exchange equilibrium in the reproduction model does not necessarily exist. In particular, under the following condition, currency exchange does not occur in the symmetric equilibrium with $n = 1/2$, $\pi = \pi^*$, $M = M^*$, $A = A^*$, and $B = B^*$:

$$B \geq \beta(1 - B). \quad [7]$$

This condition means that when the probability of the taste shock to switch from an international trader to a local trader is high, the international trader has no incentive to trade the home currency at hand for the foreign currency since he may become a local trader in the near future.

For example, in the case of $\beta = 0.5$ and $B = 0.5$, currency exchange does not occur. Under the assumption of $B > 1 - A$, suppose that $B = (1 - A) + 0.1$. Then, the fraction of local traders, π , is 0.56 since $(1 - A)\pi = B(1 - \pi)$ holds in the

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⁵ Appendix E derives the equilibrium conditions of the reproduction model.

steady state. Hence, currency exchange never occurs in a realistic case in which the fraction of international traders $1 - \pi$ is 0.44 and the degree of economic integration β is 0.5.

As in the basic model, an increase in the population of international traders increases the frequencies of currency exchange and the ratio of currency exchange for a small population of international traders in the symmetric equilibrium in the reproduction model.⁶

Figure 5: Comparison between the two models.

For example, the solid line in Figure 5 depicts the ratio of currency exchange in the symmetric equilibrium of the reproduction model with $\beta = 0.5$, $k = 15$, $M = 0.5$, $\delta = 0.05$, and $B = (1 - A) + 0.1$. The figure shows that when the population of international traders is small, a decrease in A or B increases the population of international traders and the ratio of currency exchange. On the other hand, when A or B is sufficiently low, international traders are less likely to obtain the home currency because of the small population of local traders, which excludes the possibility of currency exchange.

Figure 6(a) and 6(b) shows that increases in β and M increase the ratio of currency exchange for a small population of international traders. In both

⁶ The derivation of this relationship is given in Appendix F.

Figure 6: The ratio of currency exchange in the case of (a) $M = 0.5$ and (b) $\beta = 0.5$.

figures, once the population of international traders exceeds the critical level, currency exchange never occurs.

The equilibrium conditions in the two models are equivalent only in the case of $A = 1$ and $B = 0$, which implies that all agents are international traders; that is, $\pi = 0$ because $\pi = B/(1 - A + B)$ in the steady state. However, in this case,

currency exchange never occurs in either model since the trade shock does not occur in the reproduction model and local traders do not exist in the basic model. Hence, the structure of the basic model is not a special case of the reproduction model.

4.3 The Amount of Money in the Equilibrium

The original model of Zhou (1997) shows that the amount of money must be sufficiently large for currency exchange. This is because agents who prefer domestic goods may prefer foreign goods in the future, and if the amount of money is small, all agents accept both currencies to alleviate the inconvenience of a double coincidence of wants.

On the other hand, in the basic model and the reproduction model, local traders always prefer domestic goods and accept only the home currency. In this setting, the amount of money does not affect the existence conditions of currency exchange. Hence, the condition on the amount of money in Zhou (1997) is the requirement for the circulation of the foreign currency rather than currency exchange.

4.4 Comparison of the Effects on the Ratio of Currency Exchange and Welfare Levels

The above analysis shows that the degree of economic integration, the fraction of international traders, and the amount of money affect the ratio of currency exchange. This section compares the effects of these factors on the ratio of currency exchange and welfare levels between the two models.

Firstly, in the basic model, the welfare levels of local traders and international traders are given by $W^d = (m_1^d P_{1g}^d + m_2^d P_{2g}^d)u_d/\delta$ and $W^I = (m_1^I P_{1g}^I + m_2^I P_{2g}^I)u_I/\delta$ from Lemma (f-d) and Lemma (f-I), respectively. Utilizing eqs [3] and [4], they are rewritten as follows:

$$W^d = \frac{M(1-M)u_d}{2k\delta}, \quad W^I = \frac{\beta(2-\pi)M(1-M)u_I}{4k\delta}.$$

Increases in β and $1-\pi$ do not affect the welfare level of local traders, while they increase the welfare level of international traders. An increase in M increases the welfare levels of both types of agents if $M \leq 1/2$, but decreases them if $M > 1/2$. Hence, increases in β and $1-\pi$ increase not only the ratio of currency exchange but also welfare levels, while an increase in M increases

them only in the case of $M \leq 1/2$. This means that the degree of economic integration and the ratio of international traders are neutral to international traders, while the amount of money is not neutral in the basic model.

Next, in the reproduction model, the welfare levels of local traders and international traders in Country 1 are calculated as $W^d = \{A(m_1^d P_{1g}^d + m_2^d P_{2g}^d)u_d + (1-A)(m_1^l P_{1g}^l + m_2^l P_{2g}^l)u_l\}/\delta$ and $W^l = \{B(m_1^d P_{1g}^d + m_2^d P_{2g}^d)u_d + (1-B)(m_1^l P_{1g}^l + m_2^l P_{2g}^l)u_l\}/\delta$ from Lemma (f-d) and Lemma (f-l), respectively. Substituting the transition probabilities and utilizing $(1-A)\pi = B(1-\pi)$, they are rewritten as follows:

$$W^d = \frac{M(1-M)[2A\{A + \beta(1-A)(1-\pi)\}u_d + (1-A)\{B + \beta(1-B)\}(2-\pi)u_l]}{4k\delta},$$

$$W^l = \frac{M(1-M)[2B\{A + \beta(1-A)(1-\pi)\}u_d + (1-B)\{B + \beta(1-B)\}(2-\pi)u_l]}{4k\delta}.$$

When the fraction of international traders is small, an increase in β and an increase in M in the case of $M \leq 1/2$, as well as decreases in A and B for sufficiently large A and B , increase not only the ratio of currency exchange but also welfare levels, while when the fraction of international traders is sufficiently large, currency exchange never occurs. This means that the degree of economic integration, the ratio of international traders and the amount of money are not neutral to international traders in the reproduction model.

5 Optimal Quantity of Money in the Asymmetric Equilibrium

This section examines the optimal quantity of money in Country 1 in the asymmetric equilibrium.⁷ The optimal quantity of money is defined as the amount of money which maximizes the welfare levels of traders.

Substituting the transition probabilities and the amount of money into W^d , the welfare level of local traders is written as follows:

$$W^d = Mu_d \left[\frac{n\pi(1-M)\{n(1-\pi) + (1-n)(1-\pi^*)\}}{k\delta\{n(1-\pi) + (1-n)(1-\pi^*)\}} + \frac{n(1-\pi)\{n(1-\pi)(1-M) + (1-n)(1-\pi^*)(1-M^*)\}}{k\delta\{n(1-\pi) + (1-n)(1-\pi^*)\}} \right]. \quad [8]$$

⁷ The derivatives of welfare levels are given in Appendix G.

Differentiating W^d by M , the optimal quantity of the local currency to maximize the welfare level of local traders, $\bar{M}^d$, is given as follows:

$$\bar{M}^d = \frac{\{n(1-\pi) + (1-n)(1-\pi^*)\} - (1-n)(1-\pi)(1-\pi^*)(1-M^*)}{2\{n(1-\pi) + (1-n)(1-\pi^*)\}}, \quad [9]$$

which is positive.

Figure 7: The relation between the amount of the local currency and the welfare level of domestic traders.

Figure 7 depicts the relation between the amount of the local currency, M , and the welfare level of local traders, W^d , in the case of $\beta = 0.5$, $\delta = 1/0.9 - 1$, $n = 1/2$, $\pi = \pi^* = 0.5$, $u_d = 1$, $u_f = 1$ and $k = 15$. As the figure shows, W^d is increasing for a small M , but decreasing for a large M . Since local traders accept only the local currency, an increase in M facilitates transactions for a small M , which increases W^d . On the other hand, an increase in M reduces the possibilities of obtaining consumption goods for a large M , which decreases W^d . The figure also shows that as the amount of the foreign currency, M^* , increases, the optimal amount of the local currency, $\bar{M}^d$, and W^d decreases.

As eq. [9] suggests, an increase in M^* decreases $\bar{M}^d$, i.e. the local currency and the foreign currency are substitutes. Furthermore, the derivative of W^d with respect to M^* given by eq. [29] means that an increase in M^* decreases W^d .

Figure 8 depicts the relation between the amount of the foreign currency, M^* , and the welfare level of local traders, W^d , in the same case as in Figure 7. The figure shows that W^d is monotonically decreasing in M^* . Since local traders

Figure 8: The relation between the amount of the foreign currency and the welfare level of domestic traders.

accept only the local currency, an increase in traders who hold the foreign currency makes it difficult for local traders to exchange their production goods for the local currency, and then decreases the possibilities of obtaining their consumption goods. Hence, the optimal amount of the foreign currency for local traders, $\bar{M}^d$ is zero. The figure also shows that as M increases, W^d is more rapidly decreasing in M^* . This is because more local traders who hold the local currency cannot obtain their consumption goods from international traders in Country 2.

Substituting the transition probabilities and the amount of money into W^I , the welfare level of international traders is written as follows:

$$W^I = \frac{\beta(1-n)(1-\pi^*)u_I[\{n(1-\pi)M + (1-n)(1-\pi^*)M^* + (1-n)\pi^*M^*(1-M^*)\}]}{k\delta\{n(1-\pi) + (1-n)(1-\pi^*)\}} - \frac{\beta(1-n)(1-\pi^*)u_I\{n(1-\pi)M + (1-n)(1-\pi^*)M^*\}^2}{k\delta\{n(1-\pi) + (1-n)(1-\pi^*)\}^2}. \quad [10]$$

Differentiating W^I by M , the optimal quantity of the local currency to maximize the welfare level of international traders, $\bar{M}^I$, is given as follows:

$$\bar{M}^I = \frac{n(1-\pi) + (1-n)(1-\pi^*)(1-2M^*)}{2n(1-\pi)}. \quad [11]$$

Figure 9: The relation between the amount of the local currency and the welfare level of international traders.

Figure 9 depicts the relation between the amount of the local currency, M , and the welfare level of international traders W^I in the same case as in Figure 7. The figure shows that W^I is increasing for a small M , but decreasing for a large M . Since local traders accept only the local currency, an increase in M increases the possibilities that international traders sell their production goods to local traders, which increases W^I for a small M . On the other hand, an increase in M reduces the possibilities of obtaining consumption goods for a large M , which decreases W^I . The figure also shows that as the amount of the foreign currency, M^* , increases, the optimal amount of the local currency, $\bar{M}^I$ decreases, while W^I increases for a small M^* and then decreases for a large M^* .

Equation [11] suggests that as M^* increases, $\bar{M}^I$ decreases, i.e. the local currency and the foreign currency are substitutes. The derivative of W^I with respect to M^* given by eq. [31], however, means that an increase in M^* does not necessarily decrease W^I . The optimal quantity of the foreign currency to maximize the welfare level of international traders, $\bar{M}^{*I}$, is given as follows:

$$\bar{M}^{*I} = \frac{n(1-\pi) + (1-n)(1-\pi^*) - 2nM(1-\pi)(1-\pi^*)}{2[\pi^*\{n(1-\pi) + (1-n)(1-\pi^*)\} + (1-n)(1-\pi^*)^2]}. \quad [12]$$

Figure 10 depicts the relation between the amount of the foreign currency, M^* , and the welfare level of international traders, W^I , in the same case as in Figure 7. The figure shows that W^I is increasing for a small amount of M^* , but decreasing for a large M^* . An increase in M^* increases the possibilities of obtaining their consumption goods especially from local traders in Country 2 for a small amount

Figure 10: The relation between the amount of the foreign currency and the welfare level of international traders.

of the local currency, M . On the other hand, international traders rarely have an opportunity to meet traders who hold their consumption goods for a large M .

The optimal amount of the foreign currency, $\bar{M}^{*I}$ is zero in the case of $n(1 - \pi) + (1 - n)(1 - \pi^*) \leq 2n(1 - \pi)(1 - \pi^*)M$; otherwise it is positive. This suggests that when M is sufficiently large, an increase in M^* decreases W^I . In contrast, when M is small, if M^* is smaller than $\bar{M}^{*I}$, an increase in M^* increases W^I ; otherwise, it decreases W^I .

The figure also shows that as the amount of the local currency M increases, the welfare level of international traders W^I increases for a small M and decreases for a large M . When M is small, international traders can sell their production goods to more local traders who hold the local currency. On the other hand, when M is large, they are unlikely to buy their consumption goods.

The analysis shows that the optimal quantity of the foreign currency as well as the local currency is different between the local traders and international traders. Since local traders accept only the local currency, the optimal amount of the foreign currency is zero. For international traders, the optimal amount of the foreign currency can be positive, because the foreign currency gives them more opportunities to obtain their consumption goods. Moreover, since the local currency and the foreign currency are substitutes, when the amount of the local currency is sufficiently large, an increase in the amount of the foreign currency decreases the welfare level of international traders, though they need the foreign currency to obtain their consumption goods.

6 Conclusion

We develop a two-country model in which each country has two types of agents: local traders who consume only the goods produced in the home country, and international traders who consume only the goods produced in the foreign country. For international traders, the foreign currency gives more opportunity to obtain the foreign goods than the home currency does. This paper shows that there always exists currency exchange equilibrium in which local traders use only the home currency, while international traders use both the home currency and the foreign currency, and international traders who hold the home currency have an incentive to exchange the home currency for the foreign currency. Hence, unlike Zhou (1997), currency exchange is more likely to occur when many agents are engaged in international trade.

Since this paper focuses on the fundamental mechanism of currency exchange in which a different type of agents swaps his holding for his opponent's holding, we adopt the so-called first generation of monetary search model for tractability. The model assumes that goods and money are indivisible as in Matsuyama et al. (1993), and Zhou (1997). This assumption enables us to simplify the derivation of the existence conditions of equilibria. However, since agents swap their holdings one-to-one basis, the model cannot address fluctuations in prices and exchange rates. Recent studies in monetary search models have developed to analyze much broader topics in macroeconomics by easing the restrictions on holdings of goods and money.⁸ The extension of our model is our future research agenda.

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⁸ The models which assume indivisibility of both goods and money are called the first generation of monetary search model. The second generation including Shi (1995), and Trejos and Wright (1995) makes goods divisible by incorporating bargaining, which enables them to determine prices and exchange rates endogenously. The third generation including Shi (1997), Molico (2006), and Lagos and Wright (2005) makes both goods and money divisible to address the distribution of money holdings, so that they can examine inflation, monetary policy, and welfare issues.

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Appendix A: Proof of the Lemma

This section proves the lemma for local traders in Country 1 as in Matsuyama et al. (1993). The lemma for international traders in Country 1 is proved by replacing d with I for the relevant variables. The lemma for Country 2 is similarly proved with $*$.

The Bellman equations in [1] are summarized as the following matrix:

$$[(1 + \delta)I - \Pi^d]V^d = u_d Q^d,$$

where I is the 3×3 identity matrix, $V^d = [V_g^d, V_1^d, V_2^d]$, $Q^d = [0, P_{1g}^d, P_{2g}^d]$, and $[(1 + \delta)I - \Pi^d]$ has the nonnegative inverse matrix for $\delta > 0$ since Π^d is a stochastic matrix and its Frobenius root is 1. Then, the Bellman equation is rewritten as follows:

$$V^d = u_d[(1 + \delta)I - \Pi^d]^{-1}Q^d. \quad [13]$$

Hence, we get the first inequality of Lemma (a–d).

Calculating $V^d = [V_g^d, V_1^d, V_2^d]$, the following equations are obtained:

$$u + V_g^d - V_1^d = (\delta + P_{12}^d + P_{21}^d + P_{2g}^d)(\delta + P_{g2}^d + P_{g1}^d)\delta u_d / \Delta^d > 0, \quad [14]$$

$$u + V_g^d - V_2^d = (\delta + P_{12}^d + P_{21}^d + P_{1g}^d)(\delta + P_{g2}^d + P_{g1}^d)\delta u_d / \Delta^d > 0, \quad [15]$$

$$V_1^d - V_2^d = (P_{1g}^d - P_{2g}^d)(\delta + P_{g2}^d + P_{g1}^d)\delta u_d / \Delta^d, \quad [16]$$

$$V_1^d - V_g^d = \{P_{1g}^d(\delta + P_{g2}^d + P_{2g}^d) + P_{21}^d P_{1g}^d + P_{12}^d P_{2g}^d - P_{g2}^d P_{2g}^d\}\delta u_d / \Delta^d, \quad [17]$$

$$V_2^d - V_g^d = \{P_{2g}^d(\delta + P_{g1}^d + P_{1g}^d) + P_{21}^d P_{1g}^d + P_{12}^d P_{2g}^d - P_{g1}^d P_{1g}^d\}\delta u_d / \Delta^d, \quad [18]$$

where $\Delta^d = \det[(1 + \delta)I - \Pi^d] > 0$.

The second inequality of Lemma (a–d) is derived from eqs [14] and [15]. Lemmas (c–d), (d–d), and (e–d) are derived from eqs [16], [17], and [18], respectively.

Next, Lemma (b–d) is proved as follows. In the case of $V_1^d > V_g^d$, $P_{g1}^d > 0$ implies $P_{g1}^d V_1^d > 0$, while in the case of $V_2^d > V_g^d$, $P_{g2}^d > 0$ implies $P_{g2}^d V_2^d > 0$. Then, $P_{g1}^d V_1^d + P_{g2}^d V_2^d > 0$ holds. Hence, $V_g^d > 0$ holds from the Bellman equation on the production good in eq. [1].

Finally, Lemma (f-d) is proved as follows. Multiplying X^d from the left on both sides of eq. [13] and utilizing $X^d \Pi^d = X^d$, $\delta X^d V^d = u_d X^d Q^d$ is obtained.

Appendix B: Proof of Proposition 1

This section first analyzes the existence conditions of the currency exchange equilibrium for local traders. From Lemmas (a-d) and (b-d), the currency exchange equilibrium exists if and only if $V_g^d \geq V_2^d$. From Lemmas (e-d) and (d-d), and $P_{2g}^d = P_{21}^d = 0$, $P_{g1}^d > 0$, and $P_{1g}^d > 0$, the existence conditions for local traders always hold.

Next, we examine the existence conditions of the currency exchange equilibrium for international traders. From Lemmas (a-I) and (b-I), the currency exchange equilibrium exists if and only if $V_2^I > V_1^I$ and $V_1^I > V_1^g$. Lemmas (d-I) and (e-I), and $P_{1g}^I(\delta + P_{g2}^I + P_{2g}^I + P_{21}^I) > 0$, $P_{12}^I > P_{g2}^I$, and $P_{2g}^I > 0$, the existence conditions for international traders always hold.

Appendix C: Conditions Where Local Traders Only Accept the Local Currency

The basic model in Section 3 shows that given that local traders only accept the local currency, currency exchange equilibrium always exists. This section derives the conditions under which local traders endogenously accept only the local currency. Note that this extension does not affect Proposition 1 in Section 3 that once only the local currency is accepted, currency exchange equilibrium always exists.

The conditions under which local traders endogenously accept only the local currency consist of $V_g^d \geq V_2^d$. The transition probabilities of local traders are unchanged except for P_{2g}^d and P_{21}^d which are given as follows:

$$P_{2g}^d = \frac{n(1-\pi)(1-m_1^I - m_2^I)}{k}, \quad [19]$$

$$P_{21}^d = n(1-\pi)m_1^I. \quad [20]$$

Utilizing Lemma (e-d), the condition $V_g^d \geq V_2^d$ is rewritten as follows:

$$(1-\pi)(1-m_1^I - m_2^I + km_1^I)\{n\pi(1-m_1^d) + n(1-\pi)(1-m_1^I - m_2^I)\} \\ + k\delta(1-\pi)(1-m_1^I - m_2^I)$$

$$\leq \{n\pi m_1^d + \beta(1-n)(1-\pi^*)m_1^{I^*}\}\pi(1-m_1^d). \quad [21]$$

In the case of the symmetric equilibrium with $n = 1/2$, $\pi = \pi^*$, $M = M^*$ in which $m_1^d = m_2^{d^*} = M$, $m_1^I = m_2^{I^*} = M/2$, and $m_2^I = m_1^{I^*} = M/2$ hold, this condition is rewritten as follows:

$$(1-\pi)(2k\delta+1) \leq M \left\{ \pi^2 + \frac{\beta(1-\pi)\pi}{2} - (1-\pi) \left(\frac{k}{2} - 1 \right) \right\}. \quad [22]$$

This condition suggests that local traders only accept the local currency if the proportion of local traders is high and that the amount of money is large.

Hence, the proportion of local traders should be high and the amount of money should be large for local traders to accept only the local currency.

Appendix D: Proof of Proposition 2

The ratio of currency exchange to total transactions is given by $P_{12}^I P_{21}^{I^*}$, while the ratio of international trade to total transactions is given by $P_{1g}^I P_{g1}^{I^*} + P_{2g}^I (P_{g2}^{d^*} + P_{g2}^{I^*}) + P_{2g}^{I^*} P_{g2}^I + P_{1g}^{I^*} (P_{g1}^d + P_{g1}^I)$. Substituting eqs [3], [4], [5], and [6] into these ratios, the ratio of currency exchange in the basic model is given as follows:

$$\begin{aligned} & \beta n(1-n)(1-\pi)(1-\pi^*)m_1^I m_2^{I^*} k^2 \\ & \times [\beta n(1-n)(1-\pi)(1-\pi^*)\{m_1^I(1-m_1^{I^*}-m_2^{I^*}) + m_2^{I^*}(1-m_1^I-m_2^I)\} \\ & + 2(1-n)\{\pi^*(1-m_2^{d^*}) + (1-\pi^*)(1-m_1^{I^*}-m_2^{I^*})\}\{(1-n)\pi^*m_2^{d^*} + \beta n(1-\pi)m_2^I\} \\ & + 2n\{\pi(1-m_1^d) + (1-\pi)(1-m_1^I-m_2^I)\}\{n\pi m_1^d + \beta(1-n)(1-\pi^*)m_1^{I^*}\}]^{-1}. \end{aligned} \quad [23]$$

This means that as the population of international traders in Country 1 and Country 2, $1-\pi$ and $1-\pi^*$, increases the ratio of currency exchange also increases.

Appendix E: Derivation of the Currency Exchange Equilibrium in Zhou (1997)

This section derives the equilibrium conditions of the currency exchange equilibrium in Zhou (1997). The procedure is the same as in the basic model.

Firstly, we calculate the transition probabilities by considering not only the change in the preference of each agent but also the change in the preference of his opponent due to the taste shock. For example, the probability with which a local trader in Country 1 trades his production good for Currency 1 is calculated as follows. A local trader in Country 1 accepts Currency 1 regardless of his preference in the next period since Country 1 agents always accept Currency 1. There exist three types of opponents who transact with the agent: the present local trader in Country 1 who is engaged in domestic trade in the next period, the present international trader in Country 1 who is engaged in domestic trade in the next period, and the present international trader in Country 2 who is engaged in international trade in the next period. These types of agents consume goods made in Country 1. Hence, when they hold Currency 1, transactions with Country 1 agents who hold production goods are established. The other transition probabilities are calculated in a similar way. Utilizing the agents' strategies, inventory distributions, and the matching of agents in the basic model, the transition probabilities of local traders in Country 1 are given as follows:

$$\begin{aligned}
 P_{g1}^d &= \frac{nA\pi m_1^d + nB(1-\pi)m_1^I + \beta(1-n)(1-B^*)(1-\pi^*)m_1^{I^*}}{k}, \\
 P_{g2}^d &= \frac{(1-A)\{\beta(1-n)(1-A^*)\pi^*m_2^{d^*} + \beta(1-n)(1-B^*)(1-\pi^*)m_2^{I^*}\}}{k}, \\
 P_{1g}^d &= \frac{A\{n\pi(1-m_1^d) + n(1-\pi)(1-m_1^I - m_2^I)\}}{k} \\
 &\quad + \frac{(1-A)\{\beta(1-n)(1-A^*)\pi^*(1-m_2^{d^*}) + \beta(1-n)(1-B^*)(1-\pi^*)(1-m_1^{I^*} - m_2^{I^*})\}}{k}, \\
 P_{12}^d &= (1-A)\{nB(1-\pi)m_2^I + \beta(1-n)(1-A^*)\pi^*m_2^{d^*} + \beta(1-n)(1-B^*)(1-\pi^*)m_2^{I^*}\}, \\
 P_{21}^d &= P_{2g}^d = 0.
 \end{aligned}
 \tag{24}$$

The transition probabilities of an international trader in Country 1, a local trader in Country 2, and an international trader in Country 2 are calculated in a similar manner.

Secondly, we analyze the equilibrium conditions of the currency exchange equilibrium for local traders. From Lemmas (a-d) and (b-d), the currency exchange equilibrium exists if and only if $V_g^d \geq V_2^d$ for Country 1 agents. From Lemmas (e-d) and (d-d), and $P_{2g}^d = P_{21}^d = 0$, $P_{g1}^d > 0$, and $P_{1g}^d > 0$, the existence condition always holds.

Thirdly, we examine the existence conditions of the currency exchange equilibrium for international traders. For simplicity, the following analysis focuses on the symmetric equilibrium with $n = 1/2$, $\pi = \pi^*$, $M = M^*$, $A = A^*$, and $B = B^*$. In this equilibrium, $m_1^d = m_2^{d^*} = M$, $m_1^I = m_2^{I^*} = M/2$, and $m_2^I = m_1^{I^*} = M/2$ hold. In

addition, $(1 - A)\pi = B(1 - \pi)$ holds in the steady state. From Lemmas (a-I) and (b-I), the currency exchange equilibrium exists if and only if $V_2^I > V_1^I$ and $V_1^I > V_1^g$. From Lemma (c-I), we have the following existence condition:

$$B < \beta(1 - B). \quad [25]$$

Next, Lemma (d-I) gives the following existence condition:

$$\begin{aligned} & \left\{ \left(\frac{\pi}{1 - \pi} - B \right) M + B \frac{M}{2} + \beta(1 - B) \frac{M}{2} \right\} \\ & \times \left[\frac{2k\delta}{1 - \pi} + B\{(1 - B) + k\beta B\} \frac{M}{2} + B\{\beta(1 - B) + kB\}M + (1 - B)\{\beta(1 - B) + kB\} \frac{M}{2} \right. \\ & \left. + \left\{ B^2 + \frac{\beta(1 - B)\pi}{1 - \pi} \right\} (1 - M) + (B + \beta)(1 - B)(1 - M) \right] \\ & > \left[\left\{ B^2 + \frac{\beta(1 - B)\pi}{1 - \pi} \right\} (1 - M) + (B + \beta)(1 - B)(1 - M) \right] \\ & \times (1 - B) \left\{ B \frac{M}{2} + \beta(1 - k)BM + \beta(1 - B)(1 - k) \frac{M}{2} \right\}. \end{aligned} \quad [26]$$

The left-hand side is positive, while the terms including $(1 - k)$ on the right-hand side are negative because $k \geq 3$. In addition, the term including $BM/2$ on the right-hand side is less than that on the left-hand side. Hence, eq. [26] always holds.

These analyses show that the existence condition for the currency exchange equilibrium is given by eq. [25]. Hence, currency exchange never occurs if eq. [7] holds.

Appendix F: The Ratio of Currency Exchange in Zhou (1997)

The ratio of currency exchange to total transactions is given by $P_{12}^I P_{21}^{I*}$, while the ratio of international trade to total transactions is given by $P_{1g}^I P_{g1}^{I*} + P_{2g}^I (P_{g2}^{d*} + P_{g2}^{I*}) + P_{2g}^{I*} P_{g2}^I + P_{1g}^{I*} (P_{g1}^d + P_{g1}^I)$. Hence, the ratio of currency exchange in the symmetric equilibrium of the reproduction model is given as follows:

$$\begin{aligned} & (1 - B)^2 \beta^2 \{2(1 - A)\pi + (1 - B)(1 - \pi)\}^2 M k^2 \times [4(1 - M)(1 - B) \\ & \times [B + \beta(1 - B)\{(1 - A)\pi + (1 - B)(1 - \pi)\}][2\beta(1 - A)\pi + \{\beta(1 - B) + B\}(1 - \pi)] \\ & + [2\beta(1 - B) + 2B\{(1 - A)\pi + (1 - B)(1 - \pi)\}][2A\pi + \{\beta(1 - B) + B\}(1 - \pi)]]^{-1}. \end{aligned} \quad [27]$$

This means that a decrease in taste shocks A and B , which increases the population of international traders $1 - \pi$, increases the ratio of currency exchange for a small population of international traders. However, once the population of international traders exceeds the critical level, currency exchange never occurs.

Appendix G: Derivatives of Welfare Levels

The derivatives of welfare level for local traders and international traders with respect to the amount of the local currency and the foreign currency are given as follows:

$$\frac{dW^d}{dM} = u^d \left[\frac{n\pi(1-M)}{k\delta} + \frac{n(1-\pi)\{n(1-\pi)(1-M) + (1-n)(1-\pi^*)(1-M^*)\}}{k\delta\{n(1-\pi) + (1-n)(1-\pi^*)\}} \right. \\ \left. - \frac{Mn\{n(1-\pi) + (1-n)\pi(1-\pi^*)\}}{k\delta\{n(1-\pi) + (1-n)(1-\pi^*)\}} \right], \quad [28]$$

$$\frac{dW^d}{dM^*} = - \frac{Mn(1-\pi)(1-n)(1-\pi^*)u^d}{k\delta\{n(1-\pi) + (1-n)(1-\pi^*)\}}, \quad [29]$$

$$\frac{dW^I}{dM} = \frac{\beta(1-n)(1-\pi^*)u_I n(1-\pi)\{n(1-\pi)(1-2M) + (1-n)(1-\pi^*)(1-2M^*)\}}{k\delta\{n(1-\pi) + (1-n)(1-\pi^*)\}^2}, \quad [30]$$

$$\frac{dW^I}{dM^*} = \frac{\beta(1-n)^2(1-\pi^*)u^I(1-2M^*)\{n(1-\pi) + (1-n)(1-\pi^*)\}}{k\delta\{n(1-\pi) + (1-n)(1-\pi^*)\}^2} \\ - \frac{2\beta(1-n)^2(1-\pi^*)u^I\{n(1-\pi)M + (1-n)(1-\pi^*)M^*\}}{k\delta\{n(1-\pi) + (1-n)(1-\pi^*)\}^2}. \quad [31]$$

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