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Informal Insurance Networks

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Abstract: This paper develops a model of risk sharing in which each individual's income shock is locally shared ex-post given an ex-ante strategically formed network. Emphasizing the informational constraint of the network such that transfers can only be contingent on local information, the model provides characterizations of the ex-ante efficient network and the pairwise stable networks. While it is no surprise that the unique efficient network is the complete graph, it is interesting that any pairwise stable network features low average degree and almost 2-regular structures, even under individual risk heterogeneity, and it tends to exhibit positive assortativity in terms of risk variances. If expected incomes are also locally shared in addition to income shocks, the pairwise stable network may become more densely connected, achieving efficiency under certain parameter values.

Keywords: network, informal insurance, risk sharing, pairwise stability

JEL Classification: D85, O12, O17

1 Introduction

In the absence of a formal insurance mechanism, individuals and households are observed to rely on their social networks to make informal insurance arrangements and thus to mitigate the risks associated with their incomes. Informal insurance arrangements may take various forms, such as exchanging gifts and services, lending to each other at zero interest rates and providing direct financial support. These social activities have been documented by a series of empirical findings in the literature. In particular, risk sharing within villages has been widely studied with the available data on village networks (Townsend 1994; Dercon and Krishnan 2000; Fafchamps and Lund 2003; De Weerd and Dercon 2006; Fafchamps and Gubert 2007).

This raises the question of how these informal insurance mechanisms differ from formal structures. This paper provides an analytical account of informal risk-sharing activities by developing a model of local risk-sharing networks that

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incorporates two features of real-world insurance practices, which differentiate network-based insurance mechanism from formal schemes.

First, in social networks, information is better provided and transmitted locally. Frequently we observe that high-quality and low-cost information are only available between “neighbors”¹ because the short social distance between them, which may be a result of geographical proximity or strong social relationship, facilitates the transmission of information. As risk sharing is achieved by transfer payments contingent on income realizations, the sharing rule is more likely to be clearly defined and mutually accepted *ex post* if high-quality information on the income realizations of the individuals concerned is available at low cost. Otherwise, individuals will tend to understate their incomes to avoid payments to others or to solicit transfers from others, creating an environment of limited trust and thus reducing the stability of the mechanism.

Second, the social obligations that motivate and enforce risk sharing also tend to be local, as they are typically generated by personal relationships (social links), such as family, friendship, work relations, etc. As a result, an individual’s incentive to undertake the risk of a non-neighbor is generally very weak, even if the risk is passed to her through a path of social links. For example, an individual may be willing to lend money to a friend but less willing to lend money to a friend of his friend, who is unknown to the individual, especially when the individual can observe that his friend is not experiencing any bad luck himself. Put in another way, helping friends to help their friends is a lower-level consideration than helping friends, so it may demand stronger social norms for enforcement, and it is dubious how strong such norms are in real-world societies. As an informal insurance mechanism depends primarily on such local obligations, it may be hard for risks to be transmitted globally with an informal insurance mechanism, as opposed to the case of a formal insurance mechanism.

In consideration of the two aforementioned aspects, this paper develops a model of local risk-sharing networks that incorporate them. The model proposed in this paper features equal risk sharing² within an individual’s neighborhood, corresponding to the benchmark case of maximal risk diversification subject to the locality constraints. Furthermore, an individual shares the income risks of her neighbors only, which captures the second aspect.

1 Pairs of individuals that are “directly linked” to each other in a social network

2 It is worth mentioning the bilateral partial sharing (Belhaj and Deroïan 2012) which fixes an arbitrary fraction of income for own and for neighbors. They set a pre-fixed proportion for sharing incomes and that proportion is independent of individuals’ number of neighbors. This model is distinguishable from the bilateral partial sharing rule in terms of endogenously decided proportions.

By addressing these two features, this paper seeks to analyze the implication of “informality” on risk-sharing behaviors. It is shown that, with the model specification of this paper, the complete network achieves maximum risk reduction. From the equilibrium point of view, pure risk-sharing networks feature low average degree and almost 2-regular³ structures, even under general individual heterogeneity. Put in another way, people only have a very small number of friends in equilibrium if friendship is purely about risk sharing. Furthermore, pairwise stable networks tend to exhibit assortativity or homophily phenomenon, i.e., individuals tend to link with individuals of similar characteristics, that is, income shock variances in this context. Additionally, it is shown that sharing expected income in addition to income shocks can improve efficiency. In particular, the complete network is pairwise stable when a shock is minor or expected income is high.

This paper is built upon a growing literature on informal risk sharing in networks. Most closely related is Bramoullé and Kranton (2007b) who build the first model of ex ante formed network with ex post risk sharing, and characterize the efficient network and the pairwise stable network. This paper seeks to complement their work by relaxing the assumption of complete income equalization within components, in order to explore the implications of locality.

Another line of literature on this topic focuses more on the enforcement issues than the risk-sharing process itself, as identified in Bramoullé and Kranton (2007a, 2007b). In particular, Bloch, Genicot, and Ray (2008) models the consistency of global and bilateral transfer rules and the self enforceability of these transfer rules with high discount factors in a repeated game setting, while Karlan et al. (2009) and Ambrus, Mobius, and Szeidl (2014) address the enforcement issues by characterizing the capacities and the constrained efficient levels of risk sharing given an exogenous network structure.

Regarding the issue of functional evolution and assortativity, Wang (2013b) and Wang (2013a) show that equilibrium risk-sharing networks feature assortative matching and formal insurance mechanisms crowd out informal insurances. Pelliccia (2013) explores the opposite direction to the aforementioned papers by studying how a given risk-sharing network will affect each individual’s strategic choice of risky investment. (In addition, Jaramillo et al. (2015) develop a cooperative game of risk sharing to show that higher heterogeneity in risk amongst individuals causes inefficiency and social segregation of risk-sharing coalitions. These papers all provide important insights into different aspects of the informal risk-sharing networks, and this paper strives to complement these works from another point of view.

3 A network is 2-regular if every individual in the network has exactly two links.

Finally, a parallel work of this paper by Ambrus and Gao (2016) provides a characterization of the Pareto efficient transfer rules under the local information constraints. In particular, they argue that the local equal sharing rule proposed in this paper is actually highly representative of the set of Pareto efficient transfers in certain settings, providing a formal theoretical justification of the transfer rules to be applied in this paper.

The remainder of the paper is organized as follows. Section 2 specifies the model and discusses the validity of the modeling assumptions. Sections 3 and 4 characterize the efficient network structure and the pairwise stable network structure, respectively. Section 5 considers several extensions of the model by relaxing certain assumptions, and Section 6 concludes the paper.

2 The Model

2.1 Setup

Let N denote a society of n strictly risk-averse individuals. Each individual's income, denoted as y_i , is the sum of the individual's expected income $\bar{y}$ and a random monetary income shock ϵ_i , i. e., $y_i = \bar{y} + \epsilon_i$, where ϵ_i is independently distributed with mean 0 and variance σ_i^2 . Each agent's von Neumann-Morgenstern utility function is given by $v: \mathbb{R} \rightarrow \mathbb{R}$, a function of the agent's final consumption measured in monetary units, or equivalently, her final disposable income. v is assumed to be strictly increasing, strictly concave and twice continuously differentiable.

In the absence of formal insurance mechanisms, the risk-averse individuals rely on the social network to mitigate their income risks. Let G denote the social network structure, which is *undirected* and *unweighted*, i. e., $G_{ij} = G_{ji}$ and $G_{ij} \in \{0, 1\}$ for all i, j . Individual i and j are defined to be *linked* if $G_{ij} = 1$. By convention we define $G_{ij} = 0$. Let $N_i = \{j \in N: G_{ij} = 1\}$ denote the *neighborhood* of i . Ex post information may be only locally available in the sense that individual i 's income y_i is observed only by i himself and i 's neighbors.⁴

The local risk-sharing process is modeled as follows. Ex post (after incomes are realized), only neighbors can engage in (direct) risk-sharing. For each individual i , her income shock ϵ_i is equally shared by i and i 's neighbors. In other words, i and each of i 's neighbors, if any, undertake $\frac{1}{d_i + 1}$ of ϵ_i , where

⁴ j is a *neighbor* of i if j is in the neighborhood of i .

$d_i = \#(N_i)$ is the degree of i in G . Hence, after local sharing of income shocks, individual i 's final disposable income can be written as

$$w_i(\epsilon; G) = \bar{y}_i + \frac{\epsilon_i}{d_i + 1} + \sum_{j \in N_i} \frac{\epsilon_j}{d_j + 1},$$

where ϵ is the n -dimensional vector of income shocks.

Taking the risk sharing process into consideration, individuals decide ex ante on whom they want to link with. A link is successfully formed if and only if both individuals concerned want to form the link. We assume here that the cost of linking is zero to focus on the tradeoffs generated by risk sharing only.

Let $u_i(G)$ denote the ex ante expected utility of individual i given a network structure G , i. e.,

$$u_i(G) := E[v(w_i(\epsilon; G))].$$

For analytical tractability, we assume that the expected utility admits a mean-variance utility representation, or that risks are relatively small so that the Arrow-Pratt approximation of risk premium is applicable. In either case, there exists a $U: \mathbb{R} \times \mathbb{R}_+ \rightarrow \mathbb{R}$ s.t

$$E[v(Y)] = U(E[Y], \text{Var}[Y])$$

(at least approximately) for any random variable Y , and strict risk aversion is then translated to the requirement that $\partial U(\mu, \sigma^2) / \partial (\sigma^2) < 0$.

$$u_i(G) = U\left(\bar{y}_i, \frac{\sigma_i^2}{(d_i + 1)^2} + \sum_{j \in N_i} \frac{\sigma_j^2}{(d_j + 1)^2}\right)$$

2.2 Discussion of Modeling Assumptions

2.2.1 Local Equal Sharing

A distinguishing feature of this paper lies in the local risk-sharing, which is characterized by the assumption that an individual's income shock is shared only among her and her direct neighbors. Formally, let $t_{ij}(\epsilon; G)$ be the net transfer payment from i to j . Then the local risk-sharing process described above can be represented by the following bilateral transfer rules,⁵ which we may call as the *local equal sharing rule*: $\forall i, j$,

⁵ Note that $w_i(\epsilon; G) = \bar{y}_i + \epsilon_i - \sum_{j \in N_i} t_{ij}(\epsilon; G)$.

$$t_{ij}(\epsilon; G) = G_{ij} \left(\frac{\epsilon_i}{d_i + 1} - \frac{\epsilon_j}{d_j + 1} \right)$$

It should be emphasized that the local equal sharing rule defined above can incorporate a more general risk-sharing arrangement where only a portion of individual's income shock is shared. Suppose each individual “has full, unqualified claim access” (Bloch, Genicot, and Ray 2008) to a proportion α_i of the income shock, keeping $\alpha_i \epsilon_i$ as “private” and sharing only the rest $1 - \alpha_i$ of his income shock in his neighborhood (including himself). Then the corresponding risk sharing rule will be given by

$$t_{ij}(\epsilon; G) = G_{ij} \left(\frac{(1 - \alpha_i)\epsilon_i}{d_i + 1} - \frac{(1 - \alpha_j)\epsilon_j}{d_j + 1} \right).$$

To some extent this sharing rule is more appealing in consideration of the reality, as individuals' income shocks may not be fully observed even by neighbors. Yet by redefining $\tilde{\epsilon}_i := (1 - \alpha_i) \epsilon_i$, we reduce it to the original local equal sharing rule by translating the proportions of private (non-shareable) incomes into the heterogeneity in income shocks. Hence, for the convenience of modeling and without loss of generality, we will focus on the simple local equal sharing rule thereafter, incorporating the possibility of private income.

In Bramoullé and Kranton (2007b), risk-sharing behaviors are modeled as a process of ex post bilateral transfers in which pairs of linked individuals randomly meet each other and equally share their monetary holdings upon meeting. As the rounds of such meetings tend to infinity, the monetary holdings gradually become equalized across the individuals within components, and hence the risks become maximally shared within components. With homogeneous preferences and income distributions, the payoffs can be written as a function of component sizes, and this simplification significantly improves the analytical tractability.

This paper follows the insight of Bramoullé and Kranton (2007b), but considers an opposite benchmark: the risk-sharing process is modeled as one-shot net transfers of a proportion of each individual's observable income shock. First, the requirement for information is minimal: $t_{ij}(\epsilon; G)$ is only contingent on the local information about neighbors' income realizations. Furthermore, only income shocks are insured. Specifically, non-local obligations are not “insured”, and any differences in expected incomes are accepted as a reasonable outcome of other individual characteristics, such as ability and occupation, which will not be eliminated in the risk sharing process. In Section 5, we extend the model to the case where expected incomes are shared among neighbors.

This highly stylized transfer structure accurately captures the main features of Pareto efficient transfer rules under the local information constraints, where agents have homogeneous CARA preferences and income shocks are independently and normally distributed (Ambrus and Gao 2016). In that paper, Ambrus and Gao provide a formal characterization of the set of Pareto efficient transfer rules under the local information constraints in a general setting, and then argue that with homogeneous CARA preferences and independently normal endowments, the local equal sharing rule proposed here must be Pareto efficient, and all other Pareto efficient local transfer rules only differ from the local equal sharing rule above by a constant.

2.2.2 Mean-Variance Expected Utility

Introduced by Markowitz (1952), the mean-variance utility representation is widely used in modern portfolio theory. For example, if ϵ_i 's are normally distributed, or if the von Neumann-Morgenstern utility function $v(\cdot)$ is quadratic, then the expected utility can be written in such way that risk is completely captured by variance. For more general configurations, another approach to achieve the same goal is to obtain a second-order Taylor approximation for risk premium. Let X be a random variable with finite variance, the associated risk premium $RP(X)$ is a solution to the equation

$$E[v(X)] = v(E[X] - RP(X)).$$

When risks are relatively small, by expanding the two sides of the equation above using Taylor's expansion, Pratt (1964) and Arrow (1971) provide an approximation of the risk premium as

$$RP(X) \approx \frac{1}{2} \cdot \text{Var}[X] \cdot AR(E[X]),$$

where $AR(x)$ is the Arrow-Pratt measure of absolute risk aversion at x , defined as $AR(x) := -v''(x)/v'(x)$.

The assumption of the mean-variance utility representation allows great tractability in our analysis of the strategic trade-offs the individuals are faced with in the network formation game.

Furthermore, the model is actually more flexible than specified above. In particular, individuals' expected income levels, $\bar{y}_i$ and their utility functions, $v_i(\cdot)$, may be allowed to vary across individuals, and the results on equilibrium networks will remain intact regardless of the more general heterogeneity. Hence, for simplicity of notation, it has been assumed $\bar{y}_i = \bar{y}$ and $v_i = v$. A more detailed discussion of such robustness is available in Section 4.4.

3 Efficient Networks

We are now able to characterize the efficient network for the model. Let the social welfare of a given network be denoted by an utilitarian welfare function, $W(G)$, which sums the net utility of all individuals:

$$W(G) := \sum_i u_i(G)$$

A network structure G is *efficient* if it maximizes the social welfare $W(G)$ over the set of all possible network structures. Mathematically, G is efficient if and only if for any network structure G' , $W(G) \geq W(G')$. As the set of all possible networks is finite, which, to be precise, equals $2^{n(n-1)/2}$, the efficient network must exist.

Proposition 1: *The unique efficient network structure is the complete graph C .*

In Proposition 1, it is not surprising that the efficient risk-sharing network structure is the complete graph C , which is unique and does not depend on the idiosyncratic levels of risk, σ_i^2 . Intuitively, by expanding every individual's neighborhood as much as possible, each income shock ϵ_i is maximally shared by the society; in other words, each individual's portfolio is maximally diversified.

4 Equilibrium Networks

In this section, a network formation game is considered, and characterizations of the equilibrium networks are provided. In Section 4.1, the notion of pairwise stability is introduced, followed by a discussion of the trade-off that each individual is faced with and the restriction on equilibrium network structures imposed by this trade-off. In Section 4.2, conditions (in terms of individual risk heterogeneity) for the existence of equilibrium are given along with a discussion about the features of the equilibrium networks. Section 4.3 then considers the relationship between risk sharing and other social activities and its implication on the equilibrium networks, and Section 4.4 discusses the robustness of our findings.

4.1 Pairwise Stability

In this paper, equilibrium is characterized by the notion of pairwise stability, proposed by Jackson and Wolinsky (1996). Pairwise stability requires that, at equilibrium, if two individuals are neighbors, they must both prefer to link with

each other, and if two individuals are not neighbors, one of them must prefer not to be linked with the other. Formally, a network structure G is *pairwise stable* if and only if

- (PS1) $\forall ij$ s.t. $G_{ij} = 1$, $u_i(G) \geq u_i(G - ij) \wedge u_j(G) \geq u_j(G - ij)$.
- (PS2) $\forall ij$ s.t. $G_{ij} = 0$, $u_i(G + ij) < u_i(G) \vee u_j(G + ij) < u_j(G)$.

Starting with any network G in which individual i is linked with j ,

$$\begin{aligned}
 & u_i(G) - u_i(G - ij) \geq 0 \\
 \Leftrightarrow & U_i\left(\bar{y}_i, \frac{\sigma_i^2}{(d_i+1)^2} + \frac{\sigma_j^2}{(d_j+1)^2} + \sum_{k \in N_i \setminus \{j\}} \frac{\sigma_k^2}{(d_k+1)^2}\right) \\
 & - U_i\left(\bar{y}_i, \frac{\sigma_i^2}{d_i^2} + \sum_{k \in N_i \setminus \{j\}} \frac{\sigma_k^2}{(d_k+1)^2}\right) \geq 0 \quad [1] \\
 \Leftrightarrow & \frac{\sigma_i^2}{(d_i+1)^2} + \frac{\sigma_j^2}{(d_j+1)^2} \leq \frac{\sigma_i^2}{d_i^2} \\
 \Leftrightarrow & \frac{d_j+1}{\sigma_j} \geq \frac{d_i+1}{\sigma_i} \cdot \frac{d_i}{\sqrt{2d_i+1}}
 \end{aligned}$$

With the variances fixed, the RHS is increasing in d_i . This suggests that, as the number of i 's neighbors increases, the size of j 's neighborhood must increase for i to be willing to maintain the link with j . (The intuition follows that a more “popular” i only wants a more “popular” j as a friend, which supports the idea of positive assortativity. However, as will be shown below, when j 's neighborhood becomes so large that i is willing to link with j , j herself may no longer be willing to link with i at the same time. In fact, as the RHS of eq. [1] grows very fast with d_i (the speed of growth is approximately of the order $d_i^{1.5}$), no stable links can be maintained in equilibrium if d_i is too large.

Formally, for any linked i, j at any pairwise stable equilibrium, mutual consent requires that eq. [1] must still hold with the script i, j interchanged, i.e.,

$$\frac{d_i+1}{\sigma_i} \geq \frac{d_j+1}{\sigma_j} \cdot \frac{d_j}{\sqrt{2d_j+1}} \quad [2]$$

Multiplying eqs [1] and [2], we arrive at a necessary condition for any pairwise stable network:

$$\frac{d_i^2}{2d_i+1} \cdot \frac{d_j^2}{2d_j+1} \leq 1. \quad [3]$$

Noticing that $\frac{d^2}{2d+1}$ is increasing in d , the values of $\frac{d_i^2}{2d_i+1}$ is calculated for $d_i = 0, 1, \dots, 7$ in Table 1, and $\bar{d}_j(d_i)$ refers to the largest possible d_j s.t. eq. [3] is satisfied given d_i , any pairwise stable equilibrium, if $G_{ij} = 1$, we must have

Table 1: Selected values of $\frac{d^2}{2d_i+1}$ on N .

d_i	0	1	2	3	4	5	6	7
$\frac{d_i^2}{2d_i+1}$	0	$\frac{1}{3}$	$\frac{4}{5}$	$\frac{9}{7}$	$\frac{16}{9}$	$\frac{25}{11}$	$\frac{36}{13}$	$\frac{49}{15}$
$\bar{d}_j(d_i)$	–	6	2	1	1	1	1	0

$$d_j \leq \bar{d}_j(d_i)$$

(and $d_i \leq \bar{d}_i(d_i)$ vice versa).

$\bar{d}_j(d_i)$ imposes degree restrictions on all possible equilibrium network structures, and several immediate conclusions are summarized in Proposition 2.

Proposition 2: Any pairwise stable equilibrium must feature the following properties:

1. No individual has more than 6 neighbors, i.e., degrees are bounded from above by 6.
2. Any individual with 3–6 neighbors is the center of a star component;
3. If an individual has exactly 2 neighbors, then all her or his neighbors have at most 2 neighbors. Hence, this individual is either part of a circle component, or a nonperipheral member of a line component;
4. Any individual i with exactly 1 neighbor, is a peripheral member of a star or a line component.

Hence, any component of any pairwise stable network must take one of the four forms: a star with at most 7 individuals, a circle, a line or a singleton.

The results in Proposition 2 are necessary but not sufficient conditions for pairwise stability. In particular, they hold regardless of the heterogeneity in risks, i.e., the dispersion in σ_i^2 . It is striking that this condition on degrees alone can restrict the pairwise stable network to a collection of components in the most well-understood forms: stars, lines and circles.⁶

Proposition 2 also draws important conclusions on the equilibrium degree distribution. Note that the sizes of neighborhoods are bounded from above by 6 in any equilibrium network, regardless of the size of the society. Furthermore, for every individual with more than 2 neighbors, all her neighbors must have

⁶ Note that this result does not fit to a pairwise equilibrium of a game with local spillovers by Goyal and Joshi (2006). Precisely, they show that a pairwise equilibrium network is either empty or complete, based on strong monotonicity. However, in this network formation game, the strong monotonicity condition is violated because the transfer depends not only on individuals' degree but also on their risk σ_i^2 , σ_j^2 .

degrees of 1. Hence, the average degree in her component is $\frac{2d_i}{d_i+1} < 2$. This establishes an upper bound on the average degree.

Proposition 3: *At any pairwise stable equilibrium, the average degree is at most 2, and it equals 2 if and only if the network structure is a 2-regular graph.*

The low average degree suggested by Proposition 3 can be intuitively interpreted as the “equilibrium locality” of risk-sharing insurance. Note that this is not a trivial result from the model specification that risk is locally shared (only by neighbors); rather, this equilibrium result arises from individuals’ trade-offs between their own risks and their (potential) neighbors’ risks. This result is also consistent with the wide observations that social networks tend to be sparse, i. e., the average degree is vanishing relative to the size of the whole network $n \rightarrow \infty$.

An immediate corollary from Proposition 3 is the inefficiency of equilibrium network in risk sharing.

Corollary 1: *If $n > 3$, any pairwise stable network is inefficient.*

Each link ij not only provides risk-sharing benefits to i and j , but also exerts positive externalities to i ’s and j ’s neighbors, as this extra link reduces their exposures to the risks of i or j . Furthermore, the benefit of the link ij to i is comparable to the benefit of ij to each of i ’s neighbors, in the sense that ij reduces k ’s exposure to ϵ_i by the same ratio for each $k \in \{i\} \cup N_i$ from $\frac{1}{d_i} \epsilon_i$ to $\frac{1}{d_i+1} \epsilon_i$. Intuitively, the externalities are relatively large, which explains why the efficient network features the maximum degrees ($d_i = n - 1$) but the equilibrium network features much lower degrees.

4.2 Implication of Risk Heterogeneity

The previous results on pairwise stability are obtained regardless of risk heterogeneity, and they are necessary for pairwise stability under any possible profile (σ_i^2) . In this subsection, the remaining requirements for pairwise stability are inspected, and some restrictions on (σ_i^2) provide sufficient conditions for pairwise stable equilibrium.

As in the derivation of eq. [1], we can express the conditions for pairwise stability in terms of degrees and variances:

– (PS1) If $G_{ij} = 1$,

$$\frac{d_i+1}{d_j+1} \cdot \frac{d_i}{\sqrt{2d_i+1}} \leq \frac{\sigma_i}{\sigma_j} \leq \frac{d_i+1}{d_j+1} \cdot \frac{\sqrt{2d_j+1}}{d_j} \quad [4]$$

so that both i and j prefer keeping the link to cutting the link.

- (PS2) If $G_{ij} = 0$, it must NOT be true that

$$\frac{d_i + 2}{d_j + 2} \cdot \frac{d_i + 1}{\sqrt{2d_i + 3}} \leq \frac{\sigma_i}{\sigma_j} \leq \frac{d_i + 2}{d_j + 2} \cdot \frac{\sqrt{2d_j + 3}}{d_j + 1} \quad [5]$$

i. e., at least one of i and j prefers not to establish a link.

A benchmark to consider is the case of homogeneous income shocks across individuals, i. e., $\sigma_i^2 = \sigma^2$ for all i . In that case, the variances are removed from both inequalities, and pairwise stability can be translated into an exact requirement on degrees.

Proposition 4: *Assume that $\sigma_i^2 = \sigma^2$ for all i . Pairwise stable networks always exist. In particular, any 2-regular graph is pairwise stable. Furthermore, any pairwise stable network structure is a 2-regular graph with at most one isolated individual and at most one pair component as exceptions.*

Note that any 2-regular graph is comprised of components in the form of circles: it can be a large circle that involves all the individuals in the society, or a collection of small circles that are separated with each other. The smallest possible circle component is a triangle.

As variances are eliminated, Proposition 4 can be derived directly from Table 1. Specifically, a 2-regular network is pairwise stable because every individual in the 2-regular graph prefers to keep the links to her two neighbors, and every individual with 2 neighbors prefers not to have a third neighbor who currently has two neighbors. However, if one or two individuals are left out of the 2-regular graph, they cannot establish links to any individual in the 2-regular graph, and neither can they establish a 2-regular graph on their own, which requires at least 3 individuals. Hence, the equilibrium structure is “almost” 2-regular.

The framework becomes more interesting once we allow σ_i^2 to vary across individuals. To begin with, assume that $\sigma_i^2 \in \{\sigma_L^2, \sigma_H^2\}$ where $\sigma_H > \sigma_L > 0$, i. e., the individuals can be categorized into two types: the low-risk type and the high-risk type. Based on Proposition 4, if there are more than three individuals in each type, then any 2-regular graph such that individuals of different types belong to different components is pairwise stable. Hence, existence of pairwise stable networks is established. However, it is more interesting to examine how individuals of different types can mix with each other in a pairwise stable network. It turns out that the ratio σ_H^2/σ_L^2 determines whether different types can mix with each other at equilibrium: if the ratio is low, then the different types are effectively “ignored” so that the conclusion in Proposition 4 still holds; if the ratio is high, the pairwise stable network features greater segregation. This

result provides a strategic explanation for the widely observed assortativity or homophily phenomenon from the risk-sharing perspective of view. Similar results have been provided in the literature with completely different settings. These results are formalized in Proposition 5.

Proposition 5: Assume that $\sigma_i \in \{\sigma_L, \sigma_H\}$ where $\sigma_H > \sigma_L > 0$. If $n > 3$, pairwise stable networks always exist. Furthermore, any pairwise stable network structure is a 2-regular graph with at most 2 isolated individuals and at most 2 pair components as exceptions. Furthermore,

- If $\sigma_H^2/\sigma_L^2 \leq 1.25$, individuals of different types may belong to the same circle component, and there are at most 1 isolated individual and at most 1 pair component;
- If $\sigma_H^2/\sigma_L^2 > 1.25$, no two individuals of different types belong to the same circle component.

In the general case where σ_i^2 may be distinct for every i , a circle component is pairwise stable whenever $\max_{ij} \left\{ \sigma_i^2/\sigma_j^2, \sigma_j^2/\sigma_i^2 \right\} \leq 1.25$ for each pair of adjacent individuals i, j in the component. Hence, two individuals with a very large variance ratio may lie in the same component at equilibrium if they are connected by two paths of individuals such that the variance ratio at each step is no larger than 1.25. Formally,

Lemma 1: A 2-regular network is pairwise stable if and only if each linked pair has a variance ratio no larger than 1.25.

This observation allows the study of the equilibrium networks in large societies. Consider a growing society where (σ_i^2) are uniformly bounded in $[\underline{\sigma}^2, \bar{\sigma}^2]$. When n becomes larger and larger, the gap between variance ratios will get closer and closer to 1 in probability.

Lemma 2: Consider a growing society where (σ_i^2) are independently drawn from any distribution with support $[\underline{\sigma}^2, \bar{\sigma}^2]$. As $n \rightarrow \infty$, the maximum step variance ratio approaches 1 in probability. Formally, $\forall \epsilon > 0$,

$$\Pr \left(\max_{i,j} \left\{ \sigma_i^2/\sigma_j^2 : i, j \leq n, \sigma_i^2 > \sigma_j^2 \wedge \nexists k \leq n \text{ s.t. } \sigma_i^2 > \sigma_k^2 > \sigma_j^2 \right\} \leq 1 + \epsilon \right) \rightarrow 1.$$

When there are sufficiently many individuals with sufficiently similar levels of risks, the existence of 2-regular pairwise stable network is guaranteed.

Proposition 6: Consider a growing society where (σ_i^2) are independently drawn from any distribution with support $[\underline{\sigma}^2, \bar{\sigma}^2]$. As $n \rightarrow \infty$, pairwise stable networks in the form of 2-regular graphs exist with probability approaching 1.

Proposition 6 suggests that, when a society is large, 2-regular graphs can be supported as a pairwise stable equilibrium with high probability. With 2-regular equilibrium networks, the phenomenon of positive assortativity is predicted, as linked individuals must have similar risk levels in the sense that the variances of income shocks cannot differ by a ratio larger than 1.25.

4.3 Implication of Bundled Social Activities

Until this point the equilibrium characterization has been derived under the assumption that risk sharing is the only social interaction individuals are concerned with and there are no other costs or benefits associated with a social link. Hence, the results obtained above can only be understood as the pairwise stable structures of pure risksharing networks. Therefore, the fact that most observed social networks are neither almost 2-regular nor of low average degree does not necessarily contradict the results presented in this section, because the observed social networks are never pure risk-sharing networks. Instead, informal risk-sharing networks tend to overlap with other social networks, such geographical network, friendship network, family network, etc. Individuals may take advantage of an existing network and modify it to accommodate their needs for informal insurance, resulting in an evolved social network with the risk-sharing function. This motivates an enquiry into risk-sharing activities in the context of an existing social network, and an inspection of the effect of risk-sharing behaviors on the network structure. Therefore, the results obtained in this section should be better interpreted as the stand-alone effect of risk sharing activities on equilibrium network structures.

4.4 Robustness of Pairwise Stable Network Structures

In this section, we reconsider the model with the introduction of correlated income shocks and heterogeneity in expected incomes.

First, we consider simple patterns of correlated income shocks (ϵ_i) of the identical and independent distribution assumption in the previous section. Now suppose that an income shock ϵ_i can be decomposed into two components: a global risk and an idiosyncratic one, i. e., $\epsilon_i = \eta + \xi_i$, where $E[\eta] = E[\xi_i] = 0$, $E[\eta^2] = \sigma_\eta^2$, $E[\xi_i] = \sigma_i^2$ and η , (ξ_i) are independently distributed. If individuals can perfectly distinguish between the two components, the model remains unchanged, assuming that only ξ_i is shared. However, if the two components cannot be distinguished, then the transfer rule can be written as

$$\begin{aligned}
 t_{ij}(\varepsilon; G) &= G_{ij} \cdot \left(\frac{\eta + \xi_i}{d_i + 1} - \frac{\eta + \xi_j}{d_j + 1} \right) \\
 &= G_{ij} \cdot \left(\frac{\xi_i}{d_i + 1} - \frac{\xi_j}{d_j + 1} + \frac{d_j - d_i}{(d_i + 1)(d_j + 1)} \cdot \eta \right)
 \end{aligned}$$

Therefore an individual i prefers a neighbor j with larger degree, i. e., $d_j \geq d_i$, so that i does not have additional exposure to the global risk. In 2-regular networks, $d_i = d_j$ implies that no global risk is transferred, and it can still be supported in equilibrium if σ_η^2 is not too large relative to σ_i^2 's. Specifically, starting with a 2-regular network, as an individual i can reduce her exposure to global risk if she severs the link with one of her neighbors, she will keep the link only if the gain from cutting the link is not too large, or equivalently, σ_η^2 is not too large. Hence, global risk imposes extra downward pressure on degrees in addition to the pressure arising from idiosyncratic risks. The intuition lies in that the transfer of global risk is not diversifying or reducing the risk, but is a zero-sum game that worsens the bilateral relationship for linkage. Consider the limit case where $\sigma_i^2 = 0$, i. e., only global risk η is present. Then at equilibrium no individual will have more than one neighbor, because reducing one's degree below the degrees of all neighbors is a profitable move. Furthermore, individuals are indifferent between having one neighbor and having no neighbors, because both guarantee unitary exposure to the global risk. In summary, a global risk impedes risk sharing as well as other social activities, as it triggers more competitive considerations in the network formation game.

Consider the dichotomous case where individuals can be categorized into two types: A and B , where $\{A, B\}$ is a partition of N . Let $T(i)$ denote the type of i . Suppose that there are type-specific risks, η_A and η_B so that $\varepsilon_i = \eta_{T(i)} + \xi_i$, where η_A , η_B and $\{\xi_i\}$ are all independently distributed with mean 0. One particular pairwise stable network in this case takes the form of a bipartite⁷ 2-regular network.⁸ This equilibrium exhibits negative assortativity in the sense that individuals do not link with individuals of the same type, because individuals of the same type have the same type-specific risks η_A or η_B : as previously discussed, such “within-group global risk” impedes social linkage. However, for individuals of different types, type-specific risks are effectively idiosyncratic. Therefore social linkages are bilaterally acceptable, providing the variance ratio is not too large. Hence, assortativity is predicted in two dimensions and two directions: negative assortativity in the origin-of-risk dimension, and positive assortativity in the magnitude-of-risk dimension.

⁷ A network is *bipartite* if the set of individuals can be partitioned into two subsets such that no two individuals in the same subset are linked to each other.

⁸ Strictly speaking, this happens only if $\#(A) = \#(B)$. If $\#(A) > \#(B)$ WLOG, then a bipartite 2-regular component with $2\#(B)$ individuals, along with some singleton or pair component of type A , may constitute a pairwise stable equilibrium.

Secondly, individuals may vary in their expected income, which was overlooked in the preceding analysis for tractability. However, the introduction of heterogeneity in expected income across the population does not alter the results significantly. This is because any individual's choice does not depend on the expected income levels of her own or her neighbors' under the assumption of mean-variance representation or Arrow-Pratt risk premium approximation.

However, from an efficiency point of view, the analysis becomes more complicated. First, heterogeneity in expected incomes may result in different extents of local absolute risk aversion across individuals. Second, even with CARA utility functions, the gains in utilities arising from a given reduction in risk may differ among individuals with different levels of expected income, i. e., $v(\bar{y}_i - RP_1) - v(\bar{y}_i - RP_2)$ is decreasing in $\bar{y}_i$ given any RP_1 and RP_2 . Therefore, introducing heterogeneity in expected incomes demonstrates that an efficient network structure may not necessarily be complete.

Similarly, we could allow v_i to vary across individuals. In this case, it is straight forward to show that the equilibrium results are preserved, providing the variance is accepted as an accurate measure of risk.

5 Extension: Sharing of Expected Incomes

In the previous section, risk-sharing is limited to income shocks, i. e., the random component of individual incomes. When individuals share income shocks only, the expected income does not affect their choice of neighbors and pairwise stable networks are always inefficient because of the trade-offs between the own risk for an individual and the risk arising from a neighbor. In this section, we examine the improvement of efficiency by sharing expected incomes as well as income shocks.

To identify the role of shared incomes, we assume homogeneity in risk preferences across individuals such that $AR(y) = a$ (CARA utility) and $\sigma_i^2 = \sigma^2$ for all i (homogeneous income shock). Since identical risk preferences lead to even connections in a pairwise stable network as shown in the previous section, we confine our attention to individual agents in a d -regular network who share their expected incomes as well as random shocks.

When incomes are shared, the disposable income for individual i is given by

$$w_i(\epsilon; G) = \frac{y_i}{d_i + 1} + \sum_{j \in N_i} \frac{y_j}{d_j + 1}.$$

For $G_{ij} = 1$ in a d -regular network,

$$v\left(\bar{y} - \frac{a\sigma^2}{2} \frac{1}{d+1}\right) - v\left(\bar{y}\left(\frac{1}{d} + \frac{d-1}{d+1}\right) - \frac{a\sigma^2}{2} \left(\frac{1}{d^2} + \frac{d-1}{(d+1)^2}\right)\right) > 0. \quad [6]$$

The inequality [6] is reduced as

$$\begin{aligned} \bar{y} - \frac{a\sigma^2}{2} \frac{1}{d+1} &> \bar{y} \frac{d^2+1}{d(d+1)} - \frac{a\sigma^2}{2} \frac{d^3+2d+1}{d^2(d+1)^2} \\ \Leftrightarrow \bar{y} \frac{d-1}{d(d+1)} &> \frac{a\sigma^2}{2} \frac{d^2-2d-1}{d^2(d+1)^2} \end{aligned} \quad [7]$$

In the inequality [7], the LHS represents the income aspect of an additional link and the RHS is the risk aspect of it. If i removes the link to j , i 's income fraction becomes relatively larger than her neighbors. Since i contributes a larger fraction to the exchange, i 's net income and risk are negative. In other words, i reduces both risks and incomes by removing a link. Similar to the trade-off between own risk and others' risk in the previous section, now we can interpret the inequality [7] as trade-offs between incomes and risks: i keeps the link to j if and only if the benefit from the reduced net risk (the RHS of eq. [7]) does not exceed the de-benefit from the reduced net income (the LHS of eq. [7]).

Pairwise stability also requires no link between i and j s.t. $G_{ij} = 0$,

$$v\left(\bar{y} - \frac{a\sigma^2}{2} \frac{1}{d+1}\right) > v\left(\bar{y}\left(\frac{2}{d+2} + \frac{d}{d+1}\right) - \frac{a\sigma^2}{2} \left(\frac{2}{(d+2)^2} + \frac{d}{(d+1)^2}\right)\right).$$

Then the conditions for pairwise stability simplify to

$$\text{for } G_{ij} = 1, \quad \frac{d^2 - 2d - 1}{(d-1)d(d+1)} < \frac{2\bar{y}}{a\sigma^2} \quad \text{for any } d > 0 \quad [8]$$

$$\text{for } G_{ij} = 0, \quad \frac{d^2 - 2}{d(d+1)(d+2)} > \frac{2\bar{y}}{a\sigma^2} \quad \text{for any } d > 0. \quad [9]$$

With a quick look, the pairwise stability conditions may not be satisfied simultaneously for certain parameter values. For example, there could be a sufficiently high $\bar{y}$ (or low σ^2) in which eq. [8] is satisfied and eq. [9] is not satisfied. In other words, individuals would not sever existing links but add new links under the high $\bar{y}$ (or low σ^2), implying that the complete network is only pairwise stable for a particular range of parameter values.

Denoting $\psi := \frac{2\bar{y}}{a\sigma^2}$ for ease of notation,⁹ the conditions for a non-empty pairwise stable network are

⁹ Note that $\psi := \frac{2\bar{y}}{a\sigma^2} = \frac{\bar{y}}{R\bar{p}}$. It measures the relative value of the expected income to the risk: higher σ^2 reduces the relative value of $\bar{y}$, and more risk-averse agent (higher a) subjectively evaluates $\bar{y}$ lower.

$$\frac{d^2 - 2d - 1}{(d-1)d(d+1)} < \psi < \frac{d^2 - 2}{d(d+1)(d+2)}. \quad [10]$$

Unlike the case where individuals only share income shocks, a pairwise stable network can be efficient by sharing the expected income, and the degree in an income-sharing network is determined by the level of the relative value of income ψ as follows:

Proposition 7: *For d -regular networks, pairwise stable network structures are*

$$d = \begin{cases} 2 & \text{if } 0 \leq \psi \leq \frac{1}{12} \\ 3 & \text{if } \frac{1}{12} \leq \psi \leq \frac{7}{60} \\ 4 & \text{if } \psi = \frac{7}{60} \\ n-1 & \text{if } \psi > \frac{n^2 - 4n + 2}{n(n-1)(n-2)}. \end{cases}$$

Proposition 7 indicates that the higher the relative value of income, the denser the pairwise stable network is. Under a high ψ ($\psi > \frac{7}{60}$), the complete network is uniquely pairwise stable. At the medium level of ψ ($\frac{n^2 - 4n + 2}{n(n-1)(n-2)} \leq \psi \leq \frac{7}{60}$), there are multiple pairwise stable network structures such as the complete network or sparse networks, and under a low ψ and ($\psi < \frac{1}{12}$ and $\psi < \frac{n^2 - 4n + 2}{n(n-1)(n-2)}$), only degree 2 regular networks are pairwise stable. Figure 1 illustrates the result that either sparse networks or the complete network is pairwise stable and there is no such pairwise stable network which is moderately dense.

The model demonstrates that the sharing of expected incomes leads to polarized pairwise stable network structures and the polarization comes from strategic complementarities in linking: if the number of links reaches critical density, the existing links provoke new links and the new links accelerate more links until all agents are fully connected.

This polarized risk-sharing network reflects intrinsic inefficiency of a risk that pairwise stable network becomes sparser under the high risk. Intuitively, when both the expected income and shocks are shared, people make a commitment of sharing highly risky concerns only with a small limited number of social groups like family or close friends, whereas people are willing to share minor risks at the whole community level. This is supported by empirical studies that filed sufficient evidence of polarization in risk-sharing behaviors. Fafchamps and Lund (2003) discover that the average degree in ex ante insurance networks is 4.6 for overall risks. In particular, most consumption, investments, and credits

Figure 1: The vertical axis represents the relative value of income ψ and the horizontal axis shows the network size. The marked line denotes $\frac{n^2 - 4n + 2}{n(n-1)(n-2)}$, the lower bound of ψ of which the complete network is pairwise stable. The dotted line and the broken line represent $\frac{1}{12}$ and $\frac{7}{60}$ respectively. Above the marked line, the complete network is pairwise stable. In the area between the broken line and the dotted line, 3-regular networks are pairwise stable, and under the dotted line 2-regular networks are pairwise stable. The complete network is the unique pairwise stable structure above the broken line and 2-regular networks are uniquely pairwise stable under the marked line. There are overlapped areas in which both the complete network and 3-(or 2-) regular networks are pairwise stable.

appear not to be completely insured except a funeral which is a full risk-sharing case. Another empirical study of risksharing networks by De Weerd and Dercon (2006) examines partial and full insurance. When consumptions are classified into basic food and non-food, full insurance occurs for basic food consumption, on the other hand, average degree 3.5 risk-sharing network is observed for non-food consumption.

A related question is when does the system evolve to become an efficient network? To analyze this, consider the marginal effect of a new link for a deviation from a d -regular network:

$$\begin{aligned}
 & v\left(\bar{y}\left(\frac{2}{d+2} + \sum_{k \neq i, j} r_{ik}\right) - \frac{a\sigma^2}{2}\left(\frac{2}{(d+2)^2} + \sum_{k \neq i, j} r_{ik}^2\right)\right) - v\left(\bar{y} - \frac{a\sigma^2}{2} \frac{1}{d+1}\right) \\
 & \Rightarrow \bar{y}\left(\frac{2}{d+2} - \frac{1}{d+1}\right) - \frac{a\sigma^2}{2}\left(\frac{2}{(d+2)^2} - \frac{d}{(d+1)^2}\right) := \Delta_i^+.
 \end{aligned}$$

If $\Delta_i^+ > 0$, i adds a link and this additional link of i triggers others' new link as well, thus the network becomes $d+1$ -regular. In terms of the expected income $\bar{y}$, this condition can be expressed as

$$\Delta_i^+ > 0 \Leftrightarrow \bar{y} > \frac{a\sigma^2}{2} \frac{d^2 - 2}{d(d+1)(d+2)} := \Gamma_{\bar{y}}(d). \quad [11]$$

Observe that $\Gamma_{\bar{y}}(d)$ is decreasing in $d > 1$. It implies that if $\Delta_i^+ > 0$ in a d -regular network, the system converges to the complete network without a change in any other parameters because $\bar{y} > \Gamma_{\bar{y}}(d) \geq \Gamma_{\bar{y}}(d+t)$ for $t=1, \dots, n-d-1$.

For a simple example, consider a 2-regular risk-sharing network in a village with the amount of grain harvest $y_i \sim N(\bar{y}, 50^2)$ and $a = 1$. Suppose that the original level of the expected harvest is $\bar{y} = 100$. Given $\bar{y} = 100$, the current 2-regular network is pairwise stable as $\bar{y} < \Gamma_{\bar{y}}(2) = 104.17$. If villagers work harder and the expected amount of harvest $\bar{y}$ increases to 110, $\bar{y} < \Gamma_{\bar{y}}(2)$ so that everyone would like to have an additional commitment and a 3-regular network becomes pairwise stable because $\Gamma_{\bar{y}}(3) = 145.8$ and the condition [11] is not satisfied. If $\bar{y}$ increases again and becomes 146, then $\bar{y} > \Gamma_{\bar{y}}(3)$ so that one more commitment is added for all individuals and the network becomes a 4-regular network. Even if the expected harvest does not increase further, more links are repeatedly occurred by this additional link until all are fully connected (i. e. $\bar{y} = 146 > \Gamma_{\bar{y}}(2)$ for all $d \geq 4$). Accordingly, the village ends up with the complete network without further increase in $\bar{y}$.

Similarly, it is easy to find a certain level of risk where the complete network arises by lowering the risk σ^2 . Rewriting the inequality [11] in terms of σ^2 ,

$$\Delta_i^+ > 0 \Leftrightarrow \sigma^2 < \frac{2\bar{y}d(d+1)(d+2)}{a(d^2-2)} : \Gamma_{\sigma^2}(d).$$

Continuing the previous example with the income distribution $y_i \sim N(100, \sigma^2)$, suppose that the original variance of y_i is $\sigma^2 = 50^2$ and villagers adapt a new agricultural technology to reduce the fluctuation of the amount of harvest. At the original level of risk 50^2 , degree 2 regular network is pairwise stable (i. e. $50^2 > \Gamma_{\sigma^2}(2) = 2400$). If the technology decreases risk by $\sigma^2 = 48^2 = 2304$, $\sigma^2 < \Gamma_{\sigma^2}(2)$ and individuals create an additional commitment. Now a 3-regular network is pairwise stable as $\Gamma_{\sigma^2}(3) = 1714.29$. If the risk becomes lower than $\sigma^2 = 41^2 = 1681$, the village converges to the complete network without further decrease in σ^2 .

The following result provides specific conditions for the efficient structure.

Proposition 8: Consider a d -regular network. Given a, σ^2 , the network evolves to the full connection if $\bar{y} > \Gamma_{\bar{y}}(d)$.

Given $a, \bar{y}$, the network evolves to the full connection if $\sigma^2 < \Gamma_{\sigma^2}(d)$.

To summarize, when either $\bar{y}$ becomes larger or σ^2 becomes smaller, a pairwise stable network is more likely to be efficient – as either the expected income increases or the risk decreases, the incentive to add a link is strengthened so that the degree reaches the critical density to converge to the complete network.

This result is somewhat counter-intuitive as individuals are less likely to engage in risk sharing under high risk. When an agent deletes an existing link in order to make her own fraction larger, she sacrifices income loss for transferring more risks as exchanging unequal fractions transfers risk and income together for a less connected agent. If the risk is sufficiently high, the benefit of net risk transferring exceeds the loss of income so that people easily remove current links which makes their fraction larger, while they hesitate to add a link which makes their own fraction smaller. Consequently, it seems natural to be reluctant to become involved in a highly risky concern except within a small close social group such as family for minimizing informal insurance.

The expected income $\bar{y}$ works in the opposite direction to σ^2 as shown in Proposition 8. The benefit of adding a link is increasing in $\bar{y}$ and the incentive to deviate by adding a new link is strengthened under the high income so that the pairwise stable network structure is more likely to be the complete network if $\bar{y}$ is high. These findings are supported by Dercon and Krishnan (2000) who demonstrate that poorer households are less likely to engage in complete risk sharing.

6 Conclusion

This paper has developed a theory of local risk-sharing in social networks when formal insurance mechanisms are not available. The model incorporates two aspects of informal mechanisms: locality of information and locality of social obligation. With equal sharing confined within each neighborhood, the model draws conclusions on the efficient and the pairwise stable networks: efficiency in risk reduction promotes complete networks, but pairwise stability requires the network structure be almost 2-regular even if individuals are heterogeneous in their level of expected income and variances of income shocks. Hence, equilibrium networks tend to feature very low degrees of connections. Not only does this conclusion imply that equilibrium network is generally inefficient, but it also implies that risk sharing typically impedes other social activities when other social relationships are introduced. Positive assortativity is also predicted for the equilibrium network structure in terms of individual risk magnitudes (measured by variances): it is more likely for individuals with similar variances to maintain a mutually beneficial link for risk sharing.

It is also shown that sharing the expected incomes as well as the random component in the incomes can improve efficiency. Due to the trade-offs between expected incomes and risks, individuals are more likely to have additional commitment under higher incomes or lower risks. Thus the efficient network structure is more likely to arise with the higher relative value of incomes to risks.

This paper contributes to the theory on informal risk-sharing network by explicitly addressing its major differences from the formal insurance mechanism, i. e. the locality of information and the locality of obligation. In most existing research on this topic, formal insurance mechanisms are not addressed beyond the assumption that they are not available, and it remains unstated how the proposed models characterize the informality of the risk-sharing activities in question. As a first attempt to reveal the implications of the defining features of informal insurance, this paper might introduce a new set of questions about information constraints, multiple-purpose social linkage and functional evolution of social networks.

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Appendix

A.1 Proof of Proposition 1

Proof: By Jensen's inequality,

$$\begin{aligned} W(G) &= \sum_i \mathbb{E}[v(w_i(\epsilon; G))] = n\mathbb{E}\left[\sum_i \frac{1}{n} v(w_i(\epsilon; G))\right] \\ &\leq n\mathbb{E}\left[v\left(\sum_i \frac{1}{n} w_i(\epsilon; G)\right)\right] = n\mathbb{E}\left[v\left(\bar{y} + \frac{1}{n} \sum \epsilon_i\right)\right] \end{aligned}$$

where the equality holds if and only if

$$w_i(\epsilon; G) = \bar{y} + \frac{1}{n} \sum \epsilon_i \quad \forall i$$

i. e.

$$G = C.$$

A.2 Proof of Proposition 2

Proof: The proof is simple with reference to $\bar{d}_j(d_i)$ in Table 1.

1. Suppose that there is an individual with more than 6 neighbors, i. e., $d_i > 6$. Then $\bar{d}_j(d_i) = 0$ by Table 1, so any neighbor j of i must have 0 neighbors to satisfy the necessary condition [5] for pairwise stability. In particular, j cannot have i as a neighbor, which is a contradiction.
2. If $d_i = 3, 4, 5, 6$, then $\bar{d}_j(d_i) = 1$, i. e., each neighbor i of i can have at most 1 neighbor, who by definition is i . Hence, all neighbors of i are only linked with i , so i is the center of a star component.
3. If $d_i = 2$, then $\bar{d}_j(d_i) = 2$, i. e., each neighbor j of i can have at most 2 neighbors. As one of j 's neighbors must be i , j can either have no other neighbors or just 1 other neighbor. In the former case, j will be a peripheral member of the component that i and j belong to. In the latter case, $d_j = 2$, and thus the same argument extends to the other neighbor of j who is not i . Hence, for all individual k that lies in the same component with j , $d_k \leq 2$. Hence, the component that i lies in can only be of two forms. First, if there are none peripheral members of the component, i. e., $d_k = 2$ for all agents k in the component, then the component must be a circle. Second, if there exists an peripheral member, then the component must be a line.
4. If $d_i = 1$, then $\bar{d}_j(d_i) = 6$. If the only neighbor j of i has 1 neighbor, then i and j form a small line component. If j has two neighbors, case 3 above applies, so that i must lie at one end of a line component. If j has 3, 4, 5 or 6 neighbors, then case 2 above applies, so that i must be a peripheral member of a star component. $\square$

A.3 Proof of Proposition 3

Proof: Note that the average degree of a network is an weighted average of the average degree of each its components, where the weights are given by the sizes of components. By Proposition 3, at any pairwise stable equilibrium, the components of the network must take one of the four form: a star, a line, a circle or a singleton. Notice that every peripheral individual on a line has degree 1 and every non-peripheral individual on a line has degree 2; every individual on a circle has degree 2; every singleton (isolated individual) has degree 0. Consider a star with k peripheral individuals. Then the average degree of the star is

$$\frac{k \cdot 1 + 1 \cdot k}{k+1} = \frac{2k}{k+1} < 2.$$

Note that all four kinds of components have average degree of at most 2. Hence, the average degree of all the individuals in the network must be bounded from above by 2. Furthermore, every component except a circle has an average degree that is strictly less than 2. Hence, the average degree of the network equals 2 if and only if every component is a circle, i. e., the network structure is 2-regular graph. $\square$

A.4 Proof of Proposition 4

Proof: Suppose that $\sigma_i^2 = \sigma^2$ for all i .

Let G be any 2-regular graph with at most one isolated individual and one isolated pair as exceptions, we check that the two conditions for pairwise stability are indeed satisfied.

- (PS1) If $G_{ij} = 1$, any two linked individuals both with 2 neighbors prefer to keep the link, because eq. [4] $\frac{3}{3} \cdot \frac{2}{\sqrt{5}} \leq 1 \leq \frac{3}{3} \cdot \frac{\sqrt{5}}{2}$ holds (this accounts for the 2-regular components of the graph); any two individuals linked only with each other prefer to keep the link, because eq. [4] $\frac{2}{2} \cdot \frac{1}{\sqrt{3}} \leq 1 \leq \frac{2}{2} \cdot \frac{\sqrt{3}}{1}$ holds (this accounts for the isolated linked pair).
- (PS2) If $G_{ij} = 0$, two unlinked individuals both with 2 neighbors will not link with each other, as eq. [5] $\frac{4}{4} \cdot \frac{3}{\sqrt{7}} \leq 1 \leq \frac{4}{4} \cdot \frac{\sqrt{7}}{3}$ does NOT hold; two unlinked individuals with 0 neighbor and 2 neighbors respectively will not link with each other, as eq. [5] $\frac{2}{4} \cdot \frac{1}{\sqrt{3}} \leq 1 \leq \frac{2}{4} \cdot \frac{\sqrt{7}}{3}$ does NOT hold; two unlinked individuals with 1 neighbor and 2 neighbors will not link with each other, as eq. [5] $\frac{3}{4} \cdot \frac{2}{\sqrt{5}} \leq 1 \leq \frac{3}{4} \cdot \frac{\sqrt{7}}{3}$ does NOT hold; any isolated individual and any individual with only 1 neighbor will not link with each other, as eq. [5] $\frac{2}{3} \cdot \frac{1}{\sqrt{3}} \leq 1 \leq \frac{2}{3} \cdot \frac{\sqrt{5}}{2}$ does NOT hold.

Hence, both (PS1) and (PS2) are satisfied, i. e., G is pairwise stable. Therefore pairwise stable networks always exist.

Now let G be any pairwise stable network. By Proposition 3, any component of G must take one of the four forms: a star with at most 7 individuals, a circle, a line or a singleton.

- If G has 2 isolated singletons, then the two isolated individuals would prefer to link with each other as $\frac{2}{2} \cdot \frac{1}{\sqrt{3}} \leq 1 \leq \frac{2}{2} \cdot \frac{\sqrt{3}}{1}$ holds. Hence, PS2 fails, contradicting the pairwise stability of G .

- If G has a line component (with at least 3 individuals) or a star component, then there are at least two peripheral individuals with degree 1. They would prefer to link with each other as $\frac{2}{3} \cdot \frac{2}{\sqrt{5}} \leq 1 \leq \frac{2}{3} \cdot \frac{\sqrt{5}}{2}$ holds. Hence, PS2 fails, contradicting the pairwise stability of G .
- If G has an isolated singleton and an isolated pair, then the isolated singleton and one of the isolated pair will not link with each other as $\frac{2}{3} \cdot \frac{1}{\sqrt{3}} \leq 1 \leq \frac{2}{3} \cdot \frac{\sqrt{5}}{2}$ does NOT hold. Hence, an isolated singleton and an isolated pair can be present in the equilibrium.

Therefore, G is a 2-regular with at most one isolated singleton and one isolated pair as exceptions.

A.5 Proof of Proposition 5

Proof: For $n = 4$, if there are 3 individuals of the same type, then the network where the 3 individuals of the same type form a circle component is pairwise stable; if there are 2 individuals of either type, then any 2-regular network is pairwise stable when $\sigma_H^2/\sigma_L^2 \leq 1.25$, while the network in which individuals of the same types are linked with each other is pairwise stable when $\sigma_H^2/\sigma_L^2 > 1.25$. For $n \geq 5$, there are at least 3 individuals of the same type; then there is a pairwise stable network in which individuals of the same type form a single circle component if possible and forms a pair component if otherwise. Hence, existence is established.

If $\sigma_H^2/\sigma_L^2 \leq 1.25$, then $\frac{4}{5} \leq \sigma_i^2/\sigma_j^2 \leq \frac{5}{4}$ for all i, j . For a 2-regular network, condition [4] for PS1 $\frac{2+1}{2+1} \cdot \frac{2}{\sqrt{5}} = \sqrt{\frac{4}{5}} \leq \frac{\sigma_i}{\sigma_j} \leq \sqrt{\frac{5}{4}} = \frac{2+1}{2+1} \cdot \frac{\sqrt{5}}{2}$ holds. Recall that requirement 2 for pairwise stability is automatically satisfied when two unlinked individuals both have two neighbors. Hence, high-risk individuals and low-risk individuals may link with each other at equilibrium. Let G be any pairwise stable network.

- If G has at least 2 isolated points, then the two isolated individuals of different types would prefer to link with each other as $\frac{2}{2} \cdot \frac{1}{\sqrt{3}} \leq \sqrt{\frac{4}{5}} \leq \frac{\sigma_i}{\sigma_j} \leq \sqrt{\frac{5}{4}} \leq \frac{2}{2} \cdot \frac{\sqrt{3}}{1}$ holds. Recall in the proof of Proposition 5, two isolated individuals of the same type would prefer to link with each other, too. Hence, requirement 2 fail, contradicting with the pairwise stability of G .
- If G has a line component (with at least 3 individuals) or a star component, then there are at two peripheral individuals with degree 1 and at least one non-peripheral individual with degree 2. Note that a high-risk individual

cannot be a peripheral individual at equilibrium, because the non-peripheral individual lined with the high-risk peripheral individual prefers to sever the link, as $\frac{2}{3} \cdot \frac{1}{\sqrt{3}} \leq 1 \leq \frac{\sigma_H}{\sigma_L} \leq \frac{2}{3} \cdot \frac{\sqrt{5}}{2}$ does not hold. Hence, the two peripheral individuals must be both low-risk individuals. However, the two low-risk individuals prefer to link with each other, as $\frac{3}{3} \cdot \frac{1}{\sqrt{5}} \leq 1 \leq \frac{3}{3} \cdot \frac{\sqrt{5}}{2}$ holds. Hence, requirement 2 fail, contradicting with the pairwise stability of G .

Therefore, G is a 2-regular with at most one isolated point and at most one pair as exceptions.

If $\sigma_H^2/\sigma_L^2 > 1.25$, then, for any circle component with both high-risk and low-risk individuals, there must be a high-risk individual i linked with a low-risk individual j . Then their link is not stable as $\frac{2+1}{2+1} \cdot \frac{2}{\sqrt{5}} \leq \sqrt{\frac{5}{4}} \leq \frac{\sigma_i}{\sigma_j} \leq \frac{2+1}{2+1} \cdot \frac{\sqrt{5}}{2}$ does not hold. Hence, at equilibrium, high-risk individuals must be separated from low-risk individuals in circle components. $\square$

A.6 Proof of Lemma 1

Proof: Immediate from in-text argument. $\square$

A.7 Proof of Lemma 2

Proof: For notational simplicity, we only prove the results for uniform distribution on $[\underline{\sigma}^2, \bar{\sigma}^2]$. $\forall \epsilon > 0$, let $K = \min\{k: \underline{\sigma}^2 (1 + \epsilon)^{k/2} \geq \bar{\sigma}^2\}$. Let $p_m = \underline{\sigma}^2 \cdot (1 + \epsilon)^{m/2}$ for $m = 0, 1, 2, \dots, K-1$ and $p_K = \bar{\sigma}^2$. For any $1 \leq m \leq K$, the probability that no σ_i^2 falls in the interval of (p_{m-1}, p_m) is

$$\left(1 - \frac{p_m - p_{m-1}}{\bar{\sigma}^2 - \underline{\sigma}^2}\right)^n \rightarrow 0$$

as $n \rightarrow \infty$. By the sub-additivity of probability measure, the probability that there exists an $m \in \{1, 2, \dots, K\}$ s.t. no σ_i^2 falls in the interval of (p_{m-1}, p_m) is less than or equal to

$$\sum_{m=1}^K \Pr(\text{no } \sigma_i^2 \text{ falls in the interval } (p_{m-1}, p_m)) = \sum_{m=1}^K \left(1 - \frac{p_m - p_{m-1}}{\bar{\sigma}^2 - \underline{\sigma}^2}\right)^n \rightarrow 0.$$

Hence, the probability that there exists an $m \in \{1, 2, \dots, K\}$ s.t. no σ_i^2 falls in the interval of (p_{m-1}, p_m) also approaches 0 as $n \rightarrow \infty$. Hence, the probability that there are at least one σ_i^2 in each (p_{m-1}, p_m) approaches 1.

Note that if there are at least one σ_i^2 in each (p_{m-1}, p_m) , then for any $i, j \leq n$ s.t. $\sigma_i^2 > \sigma_j^2$ and $\exists k \leq n$ s.t. $\sigma_i^2 > \sigma_k^2 > \sigma_j^2$,

$$1 < \frac{\sigma_i^2}{\sigma_j^2} \leq \frac{p_m}{p_{m-2}} = 1 + \epsilon,$$

which implies that

$$\max \left\{ \sigma_i^2 / \sigma_j^2 : i, j \leq n, \sigma_i^2 > \sigma_j^2 \text{ and } \exists k \leq n \text{ s.t. } \sigma_i^2 > \sigma_k^2 > \sigma_j^2 \right\} < 1 + \epsilon.$$

Hence, the probability there are at least one σ_i^2 in each (p_{m-1}, p_m) is less than or equal to

$$\Pr \left(\max \left\{ \sigma_i^2 / \sigma_j^2 : i, j \leq n, \sigma_i^2 > \sigma_j^2 \text{ and } \exists k \leq n \text{ s.t. } \sigma_i^2 > \sigma_k^2 > \sigma_j^2 \right\} < 1 + \epsilon \right).$$

Therefore the fact that the former probability approaches 1 implies that the latter does too, i. e.,

$$\Pr \left(\max \left\{ \sigma_i^2 / \sigma_j^2 : i, j \leq n, \sigma_i^2 > \sigma_j^2 \text{ and } \exists k \leq n \text{ s.t. } \sigma_i^2 > \sigma_k^2 > \sigma_j^2 \right\} < 1 + \epsilon \right) \rightarrow 1.$$

The generalization to any distribution with support $[\underline{\sigma}^2, \bar{\sigma}^2]$ is trivial in observation of the proof above, as the same argument holds as long as

$$1 - \Pr(\sigma_i^2 \in (p_{m-1}, p_m)) < 1,$$

which holds for any distribution with support $[\underline{\sigma}^2, \bar{\sigma}^2]$. □

A.8 Proof of Proposition 6

Proof: Let $\epsilon_0 = \frac{\sqrt{5}}{2} - 1$. Then $(1 + \epsilon_0)^2 = 1.25$. Let $K = \min \{k: \underline{\sigma}^2 \cdot (1 + \epsilon_0)^{k/2} \geq \bar{\sigma}^2\}$. Let $p_m = \underline{\sigma}^2 \cdot (1 + \epsilon_0)^{m/2}$ for $m = 0, 1, 2, \dots, K - 1$ and $p_K = \bar{\sigma}^2$. By the proof of Lemma 2, the probability that there are at least one σ_i^2 in each (p_{m-1}, p_m) approaches 1.

If there are at least one σ_i^2 in each (p_{m-1}, p_m) , we can pick one from each interval $\sigma_{(m)}^2 \in (p_{m-1}, p_m)$. Note that

$$1 < \frac{\sigma_{m+1}^2}{\sigma_m^2} < \frac{p_{m+1}}{p_{m-1}} = (1 + \epsilon_0)^2 = 1.25.$$

Let $\xi = \min \left\{ \frac{\sigma_{m+1}^2}{\sigma_m^2} : m = 1, 2, \dots, K \right\} - 1$. Let $\epsilon = \sqrt{1 + \xi} - 1$. As $\epsilon > 0$, by Lemma 2,

$$\Pr \left(\max \left\{ \sigma_i^2 / \sigma_j^2 : i, j \leq n, \sigma_i^2 > \sigma_j^2 \text{ and } \nexists k \leq n \text{ s.t. } \sigma_i^2 > \sigma_k^2 > \sigma_j^2 \right\} \leq 1 + \epsilon \right) \rightarrow 1.$$

When $\max \left\{ \sigma_i^2 / \sigma_j^2 : i, j \leq n, \sigma_i^2 > \sigma_j^2 \text{ and } \nexists k \leq n \text{ s.t. } \sigma_i^2 > \sigma_k^2 > \sigma_j^2 \right\} \leq 1 + \epsilon$, rank $\left\{ \sigma_j^2 \right\}_{j=1}^n$ from the smallest to the largest. Construct a circle network by the following links:

- Link the individual with variances of $\sigma_{(1)}^2, \sigma_{(2)}^2, \dots, \sigma_{(K)}^2$ into a line according to the order of σ_m^2 .
- Link the individual of $\sigma_{(K)}^2$ with the individual with the highest variance among all n individuals, if they are not the same person, and link the individual of $\sigma_{(1)}^2$ with the individual with the lowest variance, if they are not the same person;
- Complete the circle by connect the individual with the highest variance back to the one with the lowest variance by linking all the remaining individuals in descending order according to their variances.

This large circle is pairwise table, because

- For any linked individuals with variance $\sigma_{(1)}^2, \sigma_{(2)}^2, \dots, \sigma_{(K)}^2$ $\frac{\sigma_{m+1}^2}{\sigma_m^2} < 1.25$;
- For the individual with highest variance and the one with variance $\sigma_{(K)}^2$, $\frac{\max \{ \sigma_i^2 \}}{\sigma_{(K)}^2} \leq \frac{\bar{\sigma}^2}{\bar{\sigma}^2 / (1 + \epsilon_0)} = 1 + \epsilon_0 < 1.25$.

The same holds for the individual with the lowest variance and the one with variance $\sigma_{(1)}^2$

- For any other linked pair of individuals i, j , assume WLOG that $\sigma_i^2 > \sigma_j^2$. Their variance ratio

$$\frac{\sigma_i^2}{\sigma_j^2} \leq (1 + \epsilon)^2 = 1 + \xi < 1.25,$$

as there are at most 1 individual with variance $\sigma_{(1)}^2, \sigma_{(2)}^2, \dots, \sigma_{(K)}^2$ that lies between σ_i^2, σ_j^2 .

Hence, pairwise stable networks exist with probability approaching 1. $\square$

A.9 Proof of Proposition 7

Proof: Firstly, neither $d = 0$ nor $d = 1$ regular network is pairwise stable because eq. [10] is violated for $d = 0$ and $d = 1$. For $d = 2$, the condition [10] is $-\frac{1}{6} < \psi < \frac{1}{12}$.

Since $\bar{y} \geq 0$ and risk-averse agents (i. e. $a > 0$), a 2-regular network is pairwise stable if $0 < \psi < \frac{1}{12}$. Similarly, a 3-regular network is pairwise stable if $\frac{1}{12} < \psi < \frac{7}{60}$. At the border value $\psi = \frac{1}{12}$, either degree 2 or 3 regular network is pairwise stable, and at $\psi = \frac{7}{60}$, either degree 3 or 4 regular network is pairwise stable.

If $d > 4$, d -regular networks cannot be pairwise stable because

$$\frac{d^2 - 2}{d(d+1)(d+2)} < \frac{d^2 - 2d - 1}{(d-1)d(d+1)}.$$

□

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