

Step by Step Innovation without Mutually Exclusive Patenting: Implications for the Inverted U

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Abstract

The step by step model of innovation is a benchmark model in research investigating the relationship between competition and innovation. The model assumes an industry can be in one of two states; leveled or unleveled. In an unleveled state the lagging firm is the only innovator. In a leveled state firms compete in a patent race. In this patent race successful innovation probabilities are mutually exclusive. This formulation provides mathematical tractability, but it has no other justification. I relax this assumption and use numerical simulation to demonstrate that allowing for non mutually exclusive success in innovation has important consequences for the inverted U relationship. The inverted U relationship is no longer a prediction of the model. In addition, the model predicts that patent measures will under count innovation from the leveled state, allowing for an inverted U relationship between competition and patenting under a narrow set of parameter restrictions. This theoretical exercise has important implications for understanding the current state of the empirical record on this topic.

1 Introduction

The purpose of this paper is to take a closer look at the theoretical structure of the step by step model of innovation. This popular model of a patent race is used to study the relationship between innovation and other economic variables. The model is thought to predict an inverted U relationship between innovation and the degree of market competition. It is this relationship, with

its important implications for many areas of economics such as Industrial Organization, Macroeconomic Growth, and International Trade, that is the focus of the present paper.

The relationship between product market competition and innovation receives a good deal of attention from IO and growth economists. The interest in this topic dates back many decades (see e.g. Arrow 1962, Gilbert and Newbery 1982), and recently economists have attempted to align empirical evidence with the theory of Schumpeterian growth (Aghion et al. 1997, Aghion et al. 2001, Aghion et al. 2005). The most basic Schumpeterian model of growth (Aghion and Howitt 1992, Grossman and Helpman 1991) suggests that product market competition is a discouragement to innovation and growth. In this early framework monopoly profits, secured by leapfrogging the technological leader, provide the sole source of incentive to undertake R&D. Empirical evidence, however, has long suggested that the relationship is more complicated. Early attempts to clearly identify a relationship between competition and innovation failed to provide sharp conclusions. Simple linear relationships failed to capture consistent and significant results (Scott 1984, Baldwin and Scott 1987, Carlin, Shaffer and Seabright 2004).

The most significant advancement in the theoretical treatment of the role of market competition in determining innovation and patenting is presented, in a number of papers, by Aghion, Howitt and others (see e.g. Aghion et al. 1997, Aghion et al. 2001, Aghion et al. 2005, Aghion et al. 2015). The model, known as the model of step by step innovation, requires firms catch up to a technological leader before moving ahead to a monopoly position. This implies that industries will spend time in two states. In one state a firm is a leader with market power. In the following state the firms are in leveled or neck and neck competition and the incentive to innovate is an incentive to escape this competition. Therefore, the traditional Schumpeterian effect, suggesting an inverse relationship between competition and innovation, is counter balanced by an escape competition effect allowing for a direct relationship between competitive forces and innovation. Aghion et al (2005, 2015) argue that these forces can explain the mixed empirical evidence through the existence of an inverted U relationship between product market competition and innovation. Aghion et al. (2005) motivate their theoretical presentation by demonstrating an empirical inverted U relationship for publicly traded UK firms. Their empirical results are, however, challenged by Correa (2012) who demonstrates that a structural break occurs in the Aghion et al. data set following the establishment of the US Court of Appeals for the Federal Circuit in 1982. He shows that controlling for this structural break eliminates the inverted U found in the econometric analysis of the data. He reports a positive relationship before the break and a lack of any relationship following the structural break. Hashmi (2013) uses US patenting data and finds a weak negative relationship between markups and patenting. On the other hand Autor et al. (2019) find that enhanced foreign competition discourages domestic innovation as measured by domestic patenting activity. It should be pointed out that this paper does not directly test the hypothesis of an inverted U, an exercise that would require investigating the

relationship between competition and total patenting activity (i.e. both foreign and domestic patenting). It is fair to conclude that the econometric evidence is mixed, with a preponderance of evidence suggesting that increased competition is more typically associated with a higher level of innovation (Nickell 1996, Gilbert 2006, Correa 2012; De Bondt and Vandekerckhove 2012) rather than an inverted U.

The mixed evidence on the relationship between competition and innovation leads some researchers to alternative theoretical frameworks that attempt to reconcile the theory and empirical record. Marshall and Parra (2019), using computational techniques similar to those used in this paper, provide a framework allowing for product market innovation (higher profit gaps for industry leaders encourages more innovation) and innovation competition (increasing in the number of existing research labs). Contrary to much of the empirical record already discussed the model emphasizes a Schumpeterian inverse relationship between product market competition and innovation, i.e. the right hand side of the inverted U relationship. Chernyshev (2016) uses a model of Cournot competition to derive a model that preserves the hump shaped relationship between R&D output (i.e. patents) and competition. R&D inputs/intensity can be hump shaped, rising, or falling in competition. The choice of the variable used to proxy innovation becomes an important consideration. In this paper similar issues arise, but the emphasis remains focused on the output from R&D effort. Delbono and Lambertini (2020) also use a Cournot model to demonstrate that this framework is rich enough to predict a direct, inverse, or inverted U relationship between innovation and competition.

The key assumption of the step by step model investigated in this paper concerns the nature of the patent race that defines the probability of firm and industry innovation when firms are in neck and neck or leveled competition. The existing step by step model assumes that firms undertake independent, individual research efforts. It is also specified, as is clearly demonstrated below, that the outcome of the patent race relies upon mutual exclusion in the sense that the probability of firms simultaneously innovating as they pursue independent and individual research agendas is assumed away. This assumption is a result of the model specification in continuous time. This assumption is justified, not theoretically but practically, because it leads to a much more tractable solution.¹ Indeed, this assumption dates back to the early work in IO theory exemplified by Gilbert and Newbery (1982) and reviewed in Triole (1988). I relax this assumption and demonstrate that doing so has important consequences for the step by step model's ability to produce an inverted U relationship between competition and innovation. Allowing for non mutually exclusive innovation in leveled competition leads to a considerably more complex mathematical structure, resulting in the need to find the roots of a

¹Note that the existence of empirical examples where one firm definitively wins a patent race exist, but this is not a theoretical justification for ignoring the possibility of simultaneous innovation. For a recent example of simultaneous we do not need to look further than the COVID-19 crisis. Pharmaceutical companies simultaneously produced 4 vaccines, each with substantial market share.

sixth order polynomial. Therefore, the implications and comparisons are fully explored using computer based numerical evaluation.

Section two of the paper is split into three parts. First, a brief presentation of the common aspects of the competing models is provided. Modifications to the framework are highlighted during the presentation and appear near the end of the discussion where the nature of equilibrium research intensities are discussed. The next part of section two demonstrates how the Aghion et al.(2005) model is derived as a special case through the assumption of mutually exclusive innovation. Full numerical estimates of the Aghion et al. (2005) model are developed and reviewed. For the purposes of this paper I will refer to the Aghion et. al. (2005) model as the "standard step by step model." Part three of section two explores the implications of relaxing the assumption of mutually exclusive innovation when in a neck and neck patent race. Again, full numerical estimates are derived, reviewed, and comparisons to the Aghion et al.(2005) model are highlighted. Section three provides some concluding thoughts.

2 Analysis

2.1 Theoretical Model

The basic structure of the model follows Aghion et al. (2005) so the presentation of the model foundations provided here are brief. Households exist along the unit interval. At each point in time each household inelastically supplies a unit of labor toward the production of intermediate goods. Their utility function is logarithmic.

Final goods producers purchase intermediate goods, also indexed on a unit interval. Given these assumptions and denoting the final consumption good as y_t and intermediate goods as x_{jt} the production function is:

$$\ln y_t = \int_0^1 \ln x_{jt} dj \quad (1)$$

Each x_j is produced by an industry pair of duopolists who are simply denoted firm A and firm B. Therefore $x_j = x_{Aj} + x_{Bj}$. At each point in time expenditures on x_j are the numeraire and equilibrium is symmetric. Households supply the only input to intermediate goods production, labor. With the assumption of constant returns to scale across all firms. Unit labor requirements are defined

$$a_i = \frac{1}{\gamma^k}, i = A, B, \gamma > 1 \quad (2)$$

where k is a positive number indexing the level of technology. Each industry at each point in time is characterized by the level of technology at the frontier and the technology gap between firms A and B. It is assumed that the maximum

gap between to firms is equal to 1. This defines two possible states for any industry at any point in time. First the firms can be leveled or neck and neck. In the neck and neck state the technological gap between the firms is zero. Profits for firms and the resources devoted to R&D by each firm independently are defined as π_o and η_o respectively. In this model the credit needed to fund research is not explicitly modeled and credit to fund research is unconstrained. Recently, Aghion et al.(2019) use this type of model to explain the relationship between the degree of credit constraints and innovation. The relationship predicted between the degree of credit constraints and innovation is an inverted U.

The other possibility is that one firm, the leader, is a step ahead in technology. This is the unleveled state where the leader engages in no R&D (due to the spillover) and enjoys a monopoly profit margin of $\pi_1 = 1 - \frac{1}{\gamma}$. The laggard firm is also helped by a spillover of knowledge from the industry leader. This spillover is denoted as h . The lagging firm's profits are equal to zero and they engage in a research effort that benefits from the knowledge spillover, $\eta_{-1} + h$. R&D is costly and defined by the function $\psi(\eta) = \eta^2/2$.

Competition is defined by the ability of firms in the leveled state to collude and share the profits earned by the unleveled monopoly firm. The degree of competition, Δ , varies on interval between one half and unity:

$$1/2 \leq \Delta \leq 1 \quad (3)$$

and

$$\pi_0 = (1 - \Delta)\pi_1 \quad (4)$$

Up to this point in the discussion I have not deviated from the standard step by step model. I deviate at this point in the analysis by formulating a solution to the equilibrium research intensities η_0 and $\eta_{-1} + h$ allowing for non mutually exclusive innovation when the industry is in a leveled or neck and neck state. I follow standard practice and view innovation as a Poisson arrival process. Under this assumption η_0 and $\eta_{-1} + h$ are interpreted as the research intensities over a small time period dt . Thus, I add the requirement innovation probabilities are bounded.

$$\begin{aligned} 0 &\leq \eta_0 dt \leq 1 \\ 0 &\leq (\eta_{-1} + h)dt \leq 1 \\ 0 &\leq \eta_0 dt + \bar{\eta}_0 dt - \eta_0 \tilde{\eta}_0 dt^2 \leq 1 \end{aligned} \quad (5)$$

Only solutions where the constraints outlined in equation 5 hold with a strict inequality are considered. In this case the Lagrange multipliers associated with the constraints are equal to zero and the solutions are interior solutions. More importantly, and in keeping with the existing literature, R&D remains an

uncertain and non deterministic process². Note the following concerning these restrictions. The first two conditions have obvious economic interpretations. $\eta_0 dt > 1$ clearly implies a non profit maximizing strategy. Why would a firm invest more resources into an R&D project than is necessary to secure a 100% chance of success? The same logic clearly holds for the second condition, a lagging firm in an unleveled competition is wasting resources if $(\eta_{-1} + h)dt > 1$. The last restriction in equation 5 is the probability of firm A innovating or firm B innovating. $\tilde{\eta}_0 dt$ is the conditional probability of one firm innovating given that the other firm has innovated. Under a continuous time setting and mutually exclusive innovation with independent research and symmetry the 3rd condition together with $\eta_0 = \bar{\eta}_0$ and $\lim_{dt \rightarrow 0} \eta_0 \tilde{\eta}_0 dt = 0$ further restricts $\eta_0 \leq 1/2$. There is no economic interpretation or justification for this additional restriction on η_0 . It is the arbitrary restriction implicitly imposed through a continuous time framework in the existing literature, and it is an assumption that is needed to simplify the model and provide a closed form solution. In the model without mutually exclusive innovation condition 1 automatically satisfies condition 3. If $0 \leq \eta_0 dt \leq 1$ then it must be the case that $0 \leq \eta_0 dt + \bar{\eta}_0 dt - \eta_0 \tilde{\eta}_0 dt^2 \leq 1$ if $\eta_0 = \bar{\eta}_0 = \tilde{\eta}_0$. Thus the model without mutually exclusive innovation contains two economically meaningful restrictions while the common step by step model adds an arbitrary restriction that has no economic interpretation.

I specify the discrete time Bellman equations as:

$$V_{1,t} = \pi_1 dt + (1 - rdt)[(\eta_{-1} + h)dt(V_{0,t+dt}) + (1 - (\eta_{-1} + h)dt)V_{1,t+dt}] \quad (6)$$

$$V_{-1,t} = (1 - rdt)[(\eta_{-1} + h)dt(V_{0,t+dt}) + (1 - (\eta_{-1} + h)dt)(V_{-1,t+dt})] - (\eta_{-1}^2/2)dt \quad (7)$$

$$V_{0,t} = \pi_0 dt + (1 - rdt)[\eta_0 dt(1 - \bar{\eta}_0 dt)(V_{1,t+dt} - V_{0,t+dt}) + \bar{\eta}_0 dt(1 - \eta_0 dt)(V_{-1,t+dt} - V_{0,t+dt})] - (\eta_0^2/2)dt \quad (8)$$

Equation 8 is the Bellman equation for a firm in leveled or neck and neck competition. This specification is different from the Bellman equation in the standard step by step model of innovation in that it requires the firm to fully assess the probability of becoming a leader, $\eta_0 dt(1 - \bar{\eta}_0 dt)$ or a lagging firm, $\bar{\eta}_0 dt(1 - \eta_0 dt)$ inclusive of the possibility that both firms can produce a successful innovation simultaneously, an occurrence with a probability of $\eta_0 \bar{\eta}_0 dt^2$. If this occurs two innovations occur but the industry remains leveled as it does if neither innovate. Since the leveled firms are identical they will choose the same research effort in equilibrium, $\eta_0 = \bar{\eta}_0$. A fuller discussion is presented in the next two subsections.

²There may be interesting things to learn about innovation from investigating the corner solutions and deterministic R&D in the model without mutually exclusive innovation. This is a separate issue from those studied in this paper and I leave it to future research.

Before moving to a look at how this modification influences the predictions of the model the steady state is defined, incorporating the modified Bellman equation. The equilibrium for an industry is a Markov steady state where μ_0 and μ_1 denote the fraction of time that an industry is in the leveled or unleveled state respectively. Therefore I have:

$$\mu_0 + \mu_1 = 1 \quad (9)$$

$$2\mu_0\eta_0dt(1 - \eta_0dt) = \mu_1(\eta_{-1} + h)dt. \quad (10)$$

Note that, unlike in Aghion et al. (2005), here the probability of leaving the leveled state, $2\eta_0dt(1 - \eta_0dt)$, follows a binomial probability function.³

2.2 Standard Step by Step Model

The standard step by step model is expressed in continuous time, thereby eliminating the possibility of mutual innovation. In this case it is straight forward to show that the Bellman Equations become:

$$rV_1 = \pi_1 + (\eta_{-1} + h)(V_0 - V_1) \quad (11)$$

$$rV_{-1} = (\eta_{-1} + h)(V_0 - V_{-1}) - \eta_{-1}^2/2 \quad (12)$$

$$rV_0 = \pi_0 + \eta_0(V_1 - V_0) + \overline{\eta_0}(V_{-1} - V_0) - \eta_0^2/2 \quad (13)$$

The standard step by step model assumes that, in the leveled state, firms independent research efforts lead to mutually exclusive innovation. Equation 10 becomes $2\mu_0\eta_0 = \mu_1(\eta_{-1} + h)$. Without the possibility of both firms simultaneously innovating Aghion et. al. (2005) demonstrate a number of key results. First, using equations 11-13 and first order conditions from differentiating equations 12 and 13 it is shown that the model is a mathematically simple recursive system:

$$\eta_0 = \sqrt{h^2 + 2\Delta\pi_1} - h \quad (14)$$

$$\eta_{-1} = \sqrt{h^2 + \eta_0^2 + 2\pi_1} - h - \eta_0. \quad (15)$$

equation 9 and the modified equation 10 are used to define the flow of innovation:

³For example setting $dt = 1$ the binomial probability that exactly one firm is successful and the industry moves to unleveled is $\binom{2}{1} \eta_0^1(1 - \eta_0)^{2-1} = 2\eta_0(1 - \eta_0)$. The more general formula is $\binom{n}{x} p^x(1 - p)^{n-x}$ where x is the number of successes and p is the probability of a success in a single Bernoulli trial. The experiment consists of two independent Bernoulli trials, one for each firm's research effort.

$$I = \frac{4\eta_0(\eta_{-1} + h)}{2\eta_0 + \eta_{-1} + h}. \quad (16)$$

Equation 16 makes an important assumption regarding the innovations developed by firms when they lag in an unleveled state. When moving from unleveled to leveled competition the contribution of the innovation by the lagging firm to the measure of innovation in equation 16 is $2\mu_0\eta_0$. Since empirical studies of the inverted U relationship rely on patents as a measure of innovation it is assumed that the laggard firm receives intellectual property protection when catching up to the leading firm. Thus, the innovation that closes the gap is distinct, patentable and appears in a measure of innovation that is patent based. This issue will play an important role when we allow for non mutually exclusive innovation in the next section.

Finally, the model produces the celebrated inverted U pattern whenever $\tilde{x} = \sqrt{(h^2 + 2\pi_1)/3}$ is on the interval $[\sqrt{h^2 + \pi_1} - h, \sqrt{h^2 + 2\pi_1} - h]$. When \tilde{x} is below this interval the relationship between competition, Δ , and I is direct only. When \tilde{x} is above this interval the relationship is inverse only. Table 1 shows numerical estimates of innovation calculated using equations 14-16. I focus on values of \tilde{x} that produce the inverted U pattern. The parameters chosen to solve the model numerically include the degree of competition, Δ , the degree of spillover, h , and the profit margin, π_1 . Δ can vary between .5 and 1 and changes in increments of .05 as seen in the columns of the table. The spillover parameter can take a value on the interval $0 < h < 1$. The numerical summary begins with $h = .01$, moves to $h = .99$ and varies in tenths from .1 to .9. Each sub table is constructed for a given profit margin. The profit margins reported are 10%, 25%, 50%, 75%, and 100%. The underlying probabilities η_0 and η_{-1} are reported in Appendix 1 of the paper.

The numerical results for the standard step by step model are consistent with expectations given the theoretical treatment of Aghion et. al. (2005) summarized above. In general an inverted U pattern emerges for each specification of the profit rate under the restrictions outlined in the previous paragraph. Figure 1 graphs the predicted relationship between Δ and I for various profit rates using a value of $h = .2$, the most common spillover value consistent with the inverted U pattern. In general the range of values of Δ and h that produce an inverted U increases with the profit rate. The degree of spillovers consistent with an inverted U also increases with the rate of profit. As the profit rate grows, however, the restrictions outlined in equation 5 become important. The table cells blanked are calculations violating the first two bounded probability restrictions in equation 5. The parameter combinations that violate restriction 3, $\eta_0 \leq 1/2$, are greyed out in Table 1. This further limits the range of parameter values that produce the inverted U pattern.

2.3 Step by Step Innovation without Mutually Exclusive Innovation.

The solutions summarized by equations 14, 15 and 16 assume that firms in the leveled state cannot simultaneously innovate. This leads to the recursive system expressed in equations 14 and 15. If the assumption of mutually exclusive innovation is relaxed the more complex Bellman equation 8, and its derivative must be accounted for in a solution for η_0 and η_{-1} .⁴ Following the literature I set $r = 0^5$ and $dt = 1$. With some basic algebraic manipulation the model can be reduced to two equations in η_0 and η_{-1} .

$$0 = -(1 - \Delta\pi_1) - \eta_0^2/2 + \eta_{-1}^2/2 + (\eta_0 + h)\eta_{-1} \quad (17)$$

$$0 = \eta_0^2/2 - \Delta\pi_1 + h \frac{\eta_0(1 - \eta_{-1})}{1 - \eta_0} + \eta_0\eta_{-1} \frac{\eta_0 - \eta_{-1}}{1 - \eta_0} \quad (18)$$

These two equations replace equations 14 and 15 in the standard step by step model. In principle these two equations solve for η_0 and η_{-1} , however, the system is not recursive. The solution can be expressed as:

$$\left\{ \left[\eta_0 = -\frac{1}{-8h+2h^2-4h^3+4} \left(\begin{aligned} &14h + 14\eta_{-1} - 39h\eta_{-1}^2 + 6h^2\eta_{-1} \\ &+ 9h\eta_{-1}^3 - 48h^3\eta_{-1} + 27h\eta_{-1}^4 - 16h^4\eta_{-1} \\ &+ 9h\eta_{-1}^5 - 16\Delta\pi_1 - 50h^2\eta_{-1}^2 + 8h^2\eta_{-1}^3 \\ &- 28h^3\eta_{-1}^2 + 42h^2\eta_{-1}^4 + 64h^3\eta_{-1}^3 + 32h^4\eta_{-1}^2 \\ &- 26h\eta_{-1} + 16h^2 + 16h^3 - 34\eta_{-1}^2 \\ &- 25\eta_{-1}^3 + 12\eta_{-1}^4 + 9\eta_{-1}^5 - 12h\Delta\pi_1 \\ &- 28\eta_{-1}\Delta\pi_1 - 12h^2\Delta\pi_1 - 16h^3\Delta\pi_1 + 12\eta_{-1}^2\Delta\pi_1 \\ &+ 12h^2\Delta\pi_1 - 16h^3\Delta\pi_1 + 12\eta_{-1}^2\Delta\pi_1 + 18\eta_{-1}^3\Delta\pi_1 \\ &+ 48h^2\eta_{-1}^2\Delta\pi_1 + 12h\eta_{-1}\Delta\pi_1 - 6h\eta_{-1}^2\Delta\pi_1 + 36h^2\eta_{-1}\Delta\pi_1 \\ &+ 18h\eta_{-1}^3\Delta\pi_1 + 32h^3\eta_{-1}\Delta\pi_1 + 20 \end{aligned} \right) \right], \eta_{-1} \in \rho_1 \right\}$$

where ρ_1 is a root of

$$\begin{aligned} &-\frac{16}{9}h + \hat{Z}^4 \left(-\frac{32}{9}h + 2\Delta\pi_1 + \frac{64}{9}h^2 - \frac{31}{9} \right) + \hat{Z}^5 \left(\frac{14}{3}h - \frac{2}{3} \right) - \\ &\hat{Z}^2 \left(-\frac{56}{9}h + \frac{16}{9}\Delta\pi_1 + \frac{56}{9}h^2 + \frac{32}{9}h^3 + \frac{56}{9}h\Delta\pi_1 - \frac{32}{9}h^2\Delta\pi_1 - \frac{32}{9} \right) + \\ &\frac{8}{9}\Delta\pi_1 + \hat{Z} \left(\frac{40}{9}h + \frac{16}{9}\Delta\pi_1 + \frac{16}{3}h^2 + \frac{8}{9}h^3 - \frac{16}{9}h\Delta\pi_1 - \frac{32}{9}h^2\Delta\pi_1 - \frac{8}{9} \right) + \\ &\hat{Z}^6 - \frac{8}{9}h^2 + \hat{Z}^3 \left(-\frac{88}{9}h - \frac{8}{3}\Delta\pi_1 - \frac{56}{9}h^2 + \frac{32}{9}h^3 + \frac{16}{3}h\Delta\pi_1 + \frac{16}{9} \right) + \\ &\frac{16}{9}h\Delta\pi_1 + \frac{8}{9}h^2\Delta\pi_1 - \frac{4}{3} \end{aligned}$$

It is not possible to produce a simple closed form solution from a system that involves a 6th order polynomial. It is possible, however, to evaluate the model

⁴The first order conditions are with respect to η_{-1} for equation 7 and η_0 for equation 8 are $V_{0,t+dt} - V_{-1,t+dt} - \eta_{-1} = 0$ and $(1 - \eta_0 dt)(V_{1,t+dt} - V_{0,t+dt}) - \eta_0 dt(V_{-1,t+dt} - V_{0,t+dt}) - \eta_0 = 0$.

⁵As with the standard model, the results are robust to values of the discount rate that are greater than zero. The level of the discount rate has no bearing on the forces that create an inverted U in the standard model. A positive discount rate has no bearing on re-establishing an inverted U in the model without mutually exclusive innovation.

numerically. Each solution provides one set of positive real numbers. Tables 2, 3 and 4 as well as Appendix 2 contain solutions summarizing the results of the model. Following the format of the presentation of the standard model, results were calculated for values of h between .01 and .99 with increments of .1 between .1 and .9. Values of h are seen in the rows of the tables and are reported for profit margins of 10%, 25%, 50%, 75%, and 100%. Competition is measured from a low value of .5 to a maximum value of 1 in increments of size .05. These values appear in the columns of the tables. If a combination of profit margin, spillover, and competition leads to probabilities that violate equation 5 then results are not reported. For example, with a profit margin of 10% all values of h greater than .01 produce $\eta_{-1} + h > 1$. Therefore, only estimates for $h = .01$ are reported for the profit margin of 10%. Note that the third restriction in equation 5, for the current model, is $0 \leq 2\eta_0 - \eta_0^2 \leq 1$. This is satisfied for all $0 \leq \eta_0 \leq 1$.

The tables in Appendix 2 report the computed values of η_0 and η_{-1} . Comparing innovation in a leveled state in Table 1 to innovation in the leveled industry state for the standard step by step model (see Appendix 1 Table 1) an important feature that stands out. The relationship between profit margin and innovation in the leveled state within the standard model is positive. Higher profit margins, *ceteris paribus*, encourage more innovation in neck and neck competition. This relationship is reversed in the model without mutually exclusive innovation. At first glance this might appear to be at odds with intuition. Should not a larger prize in the unleveled state encourage more innovation in the leveled state? This reasoning is appealing but demonstrates a lack of full understanding of the structure of the model. A leveled firm's incentive to innovate is based on the change in the value of the firm when moving from the leveled state to the unleveled state, $V_{1,t+1} - V_{0,t+1}$. With non mutually exclusive innovation it is more difficult to move out of leveled competition. The model weights the leveled state relatively more than in the case of the standard model. A change in profit margin, π_1 , increases *both* $V_{1,t+1}$ and $V_{0,t+1}$. The increase in profit margin impacts $V_{0,t+1}$ more, proportionally, than it impacts $V_{1,t+1}$. This is the first important result from the model without mutually exclusive innovation. The level of research in the neck and neck industry state is a *decreasing* function of the profit margin if innovation is non mutually exclusive.

Comparing Table 2 in Appendix 1 to Table 2 in Appendix 2 it is clear that R&D resources from the lagging firm are a decreasing function of the level of competition across all profit rates and spillover values for both models. This, of course, is the well known Schumpeterian effect operating in both models. The standard model predicts an inverted U when this Schumpeterian effect dominates at high levels of competition but the escape competition effect dominates at low levels of competition. Appendix 1 Table 1 shows that this requires that the probability of innovating in the leveled state increases with competition. Appendix 2 Table 1 makes it clear that innovation in the neck and neck industry state can follow a number of patterns in the non mutually exclusive innovation model. Top rows in the table, from a profit rate of 10% and a spillover of 1% through a profit rate of 50% and a spillover of 20% lead to falling rates

of neck and neck innovation as the level of competition increases. The middle of the table shows a U shaped pattern and the bottom of the table, below a profit margin of 75% and a spillover of 40%, shows a pattern of rising innovation in the leveled state as competition increases. What are the implications for the relationship between economy wide innovation and market competition? What are the implications for the relationship between observed patent rates and market competition? I now address these two key questions.

To derive the relationship between innovation and competition use equations 9 and 10. These equations solve for Markov steady state probabilities of being in a leveled state, μ_0 , or an unleveled state, μ_1 . The flow of innovations includes the flow when in a leveled state. Previously this was $2\mu_0\eta_0$. Now, however, the flow of innovation must be consistent with the view of innovation implied by the standard model and reviewed in the previous section. When firms simultaneously innovate (with a probability of $\eta_0^2 dt^2$) they produce two separate innovations. In this case the industry remains in a leveled state and the innovations provide no economic advantage to the firm. Nonetheless, they are innovations and are as important for the advance in technology as any patent produced by a lagging firm in the unleveled state. In this case the flow of innovation in the leveled state of the industry is $2\mu_0\eta_0 dt(1 - \eta_0 dt) + 2\mu_0\eta_0^2 dt^2$. Again, setting $dt = 1$ the equilibrium industry flow of innovation is:

$$I = 2\mu_0\eta_0(1 - \eta_0) + 2\mu_0\eta_0^2 + \mu_1(\eta_{-1} + h). \quad (19)$$

where η_0^2 is the probability of both firms simultaneously innovating. Given equation 10 $I = \mu_0(4\eta_0(1 - \eta_0) + 2\eta_0^2)$. Using equations 9 and 10 to solve for μ_0 and substituting into this equation for I gives:

$$I = \frac{(\eta_{-1} + h)(4\eta_0(1 - \eta_0) + 2\eta_0^2)}{2\eta_0(1 - \eta_0) + \eta_{-1} + h} \quad (20)$$

This equation can be compared directly to equation 16. If $2\eta_0^2$ is removed from the numerator and $(1 - \eta_0)$ is removed from the numerator and denominator then mutual exclusion is imposed and this equation is identical to equation 16.

The question of patenting differs only slightly from the discussion presented above. If both firms simultaneously innovate there is no profit advantage from the point of view of individual firms. Though these innovations lower costs, costs are lowered simultaneously for both firms and the industry remains in a leveled state. Under these circumstances firms have no incentive to patent their innovations.⁶ The flow of patents, P , therefore is:

$$P = \frac{4\eta_0(\eta_{-1} + h)(1 - \eta_0)}{2\eta_0(1 - \eta_0) + \eta_{-1} + h}. \quad (21)$$

⁶Patenting is a costly legal and administrative activity. The model does not explicitly include a cost of patent filing, but it is clear that such a cost strengthens the argument that firms will not pursue intellectual property (IP) protection for innovations unless the economic benefits are strictly positive and greater than the legal and administrative costs of achieving IP protection.

The second key theoretical finding is presented in Table 2. Table 2 provides numerical estimates of the flow of innovations, I , calculated from η_0 and η_{-1} and associated values of π_1 , Δ , and h . At all parameter values an increase in Δ implies more time in the unleveled state. Even at low profit margins and spillovers where resources dedicated to R&D fall with competition in the neck and neck state, the fall in resources in the unleveled state fall by a greater degree on the margin. More time in the unleveled state lowers economy wide innovation because a single lagging firm cannot replace the research efforts of two firms in neck and neck competition. This leads to the second major conclusion from the step by step model without mutually exclusive innovation. Across all parameter values the relationship between competition and innovation is inverse. The model does not produce an inverted U relationship. This result is illustrated graphically in Figure 2 Panel A. A spillover rate of 20% is chosen for the graphic. The increase in competition along the horizontal axis is always associated with falling innovation. For reasons discussed previously higher profit rates are associated with lower schedules.

Table 3 summarizes the relationship between patenting and competition. Parameter values that lead to an inverted U relationship between patenting and competition are highlighted in yellow. Clearly the range of values that lead to this pattern is narrow compared to the standard model. More importantly, however, the table shows that direct, inverse, and inverted U patterns can all be found despite the fact the true relationship between innovation and competition is dominated by the Schumpeterian effect and is an inverse relationship across all parameter values. This is demonstrated in Figure 2 panel B where the relationship between observed patents and competition is graphed for each of the cases presented in panel A. Patents can show a direct relationship as when profits are 25%, an inverted U relationship as when profits are 50%, or an inverse relationship as when profit margins are 75% and 100%. This leads to the third important conclusion from the model. Despite an inverse relationship between innovation and competition across all parameter values the observed pattern between patents and competition can follow a direct, inverse, or inverted U pattern. The relationship between patent activity and competition cannot uncover the relationship between innovation and competition.

These patterns in Table 3 are determined by the complex interactions between research efforts, the incentive to patent in the leveled and unleveled states, and the probability of being in each state in equilibrium. As a final exercise I present Figure 3. First note that more than 75% of corporate patents are awarded to manufacturing industries (Autor, 2019). Focusing on manufacturing, profit margins are not expected to exceed 25% and, empirically, are likely to be between 10% and 25%. While profit margins can be significantly higher in service industries, manufacturing receives more attention due to its large share of patents.⁷ Figure 3 shows that for these "realistic" parameter values the relationship between patents (measured on the left axis) and competition and

⁷E.g. See net profit margins in <https://financialrhythm.com/wp-content/uploads/2018/06/profit-by-industry.png>

the relationship between innovation (measured on the right axis) and competition are remarkably different. More competition leads to less innovation, but the observed relationship between patent activity and competition is upward sloping. The increase in competition causes industries to spend more time in the unleveled state. This lowers innovation because there is only one firm innovating in this state. However, this causes the relationship between patents and competition to be direct because in the unleveled state the primary consideration at the selected parameter values is the fact that all innovations in the unleveled state receive a patent.

Can the possibility of simultaneous innovation in the leveled state be ignored? One way to approach this question is to use the numerical estimates to calculate the implied probability of simultaneous innovation. The correct probabilities to use in this exercise are developed from the model without mutually exclusive innovation. These probabilities are reported in Table 4. Across all parameter values the probability of joint innovation in the neck and neck industry state is large and cannot be ignored.

3 Conclusion

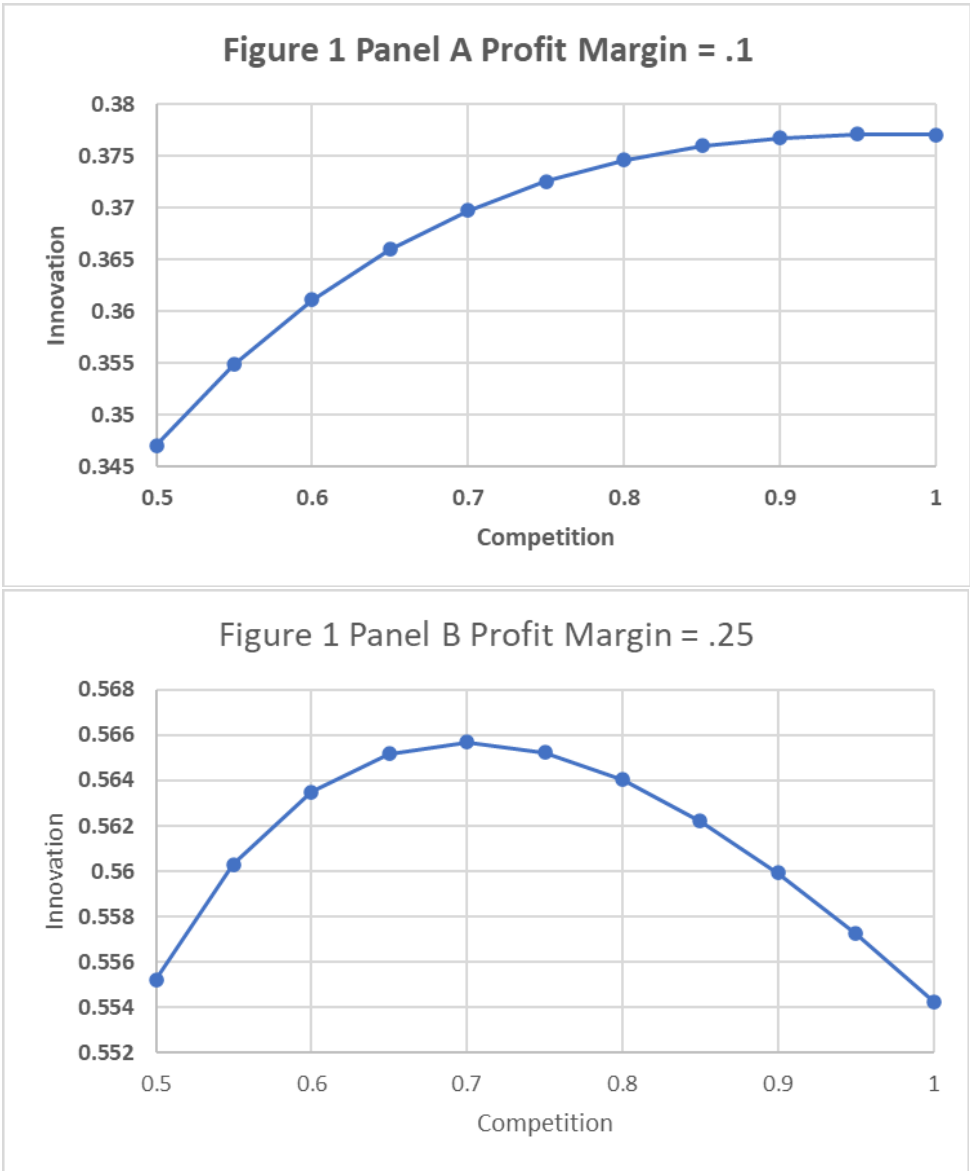
The purpose of this paper is to modify the step by step model of innovation by allowing firms in the leveled state to conduct independent research efforts without mutually exclusive research success. I argue that the only justification for assuming mutually exclusive innovation in the leveled state is one of mathematical convenience. Relaxing this assumption is shown to lead to a mathematical model that is considerably more complex, involving a root of a 6th order polynomial. I demonstrate that the model can be explored numerically and that the modified model has important implications for the relationship between competition, innovation, and patent activity.

First, the level of research in the neck and neck industry state is a *decreasing* function of the profit margin if innovation is non mutually exclusive. This is opposite the pattern observed in the model with mutually exclusive innovation. Empirically, it is factual that manufacturing industries earn lower profit margins but account for a lions share of corporate patents. The most innovation intensive industries in the economy have relatively low profit margins between 10% and 15%. This is an empirical implication of the model that deserves additional attention and testing.

The second major conclusion from the step by step model without mutually exclusive innovation is an elimination of the inverted U relationship between innovation and competition. Across all parameter values the relationship between competition and innovation is strictly inverse. The model does not produce an inverted U relationship. Related to this implication is a third important conclusion from the model. Despite an inverse relationship between innovation and competition across all parameter values the observed pattern between patents and competition can follow a direct, inverse, or inverted U pattern. The relationship between patent activity and competition cannot uncover

the relationship between innovation and competition. This model suggests that the current empirical literature on the topic of competition and innovation is misguided. Aghion et al.(2005), Correa (2012), Hashmi (2013) and Autor et al. (2019) all use patenting data. These studies find no consistent evidence of an inverted U relationship. Some studies find a direct relationship or no relationship (Correa, 2012), while some authors find a weak inverse relationship (Hashmi, 2013). The model presented in this paper suggests that alternative measures of innovation such as R&D spending are the only way forward in uncovering the true relationship between competition and innovation.

Figure 1



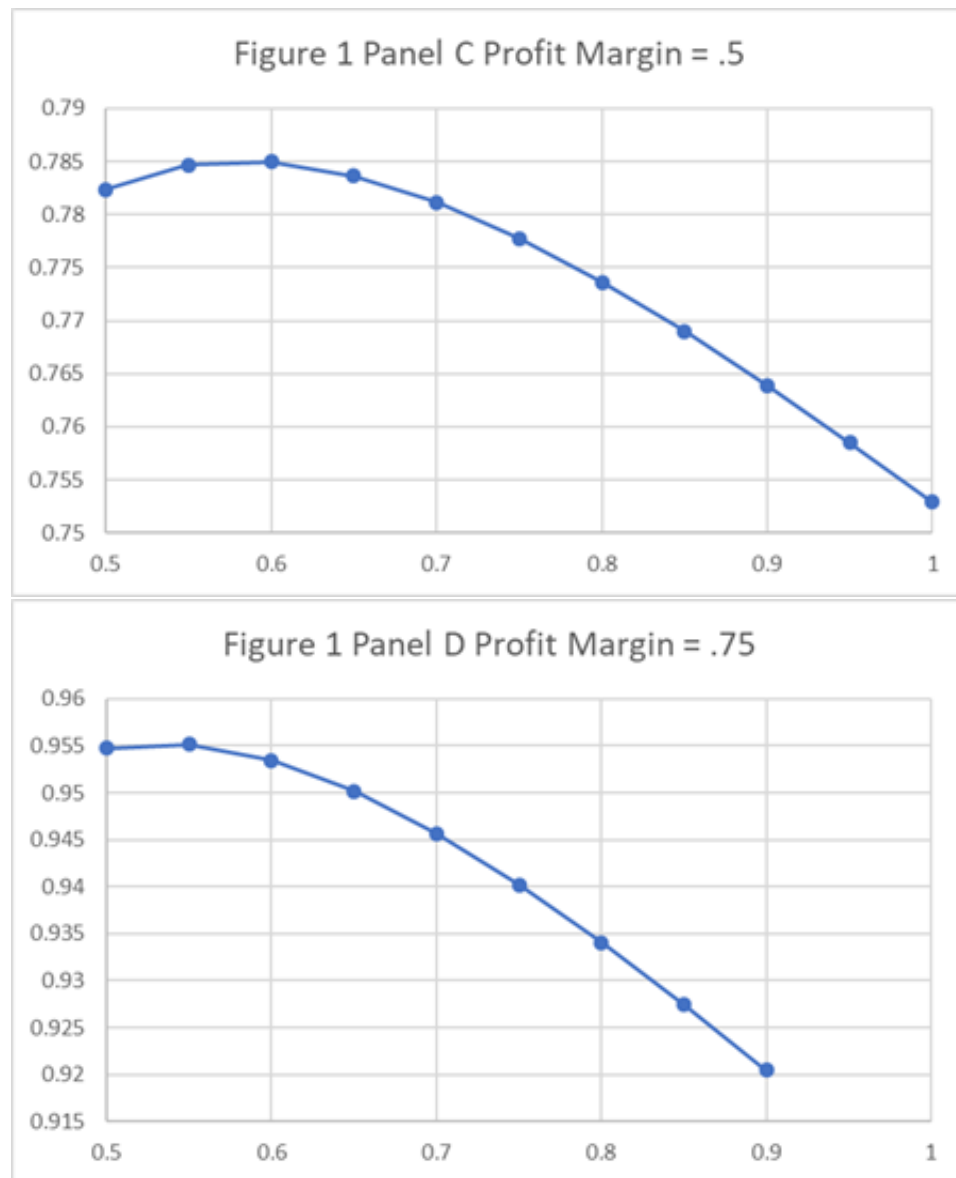


Figure 2

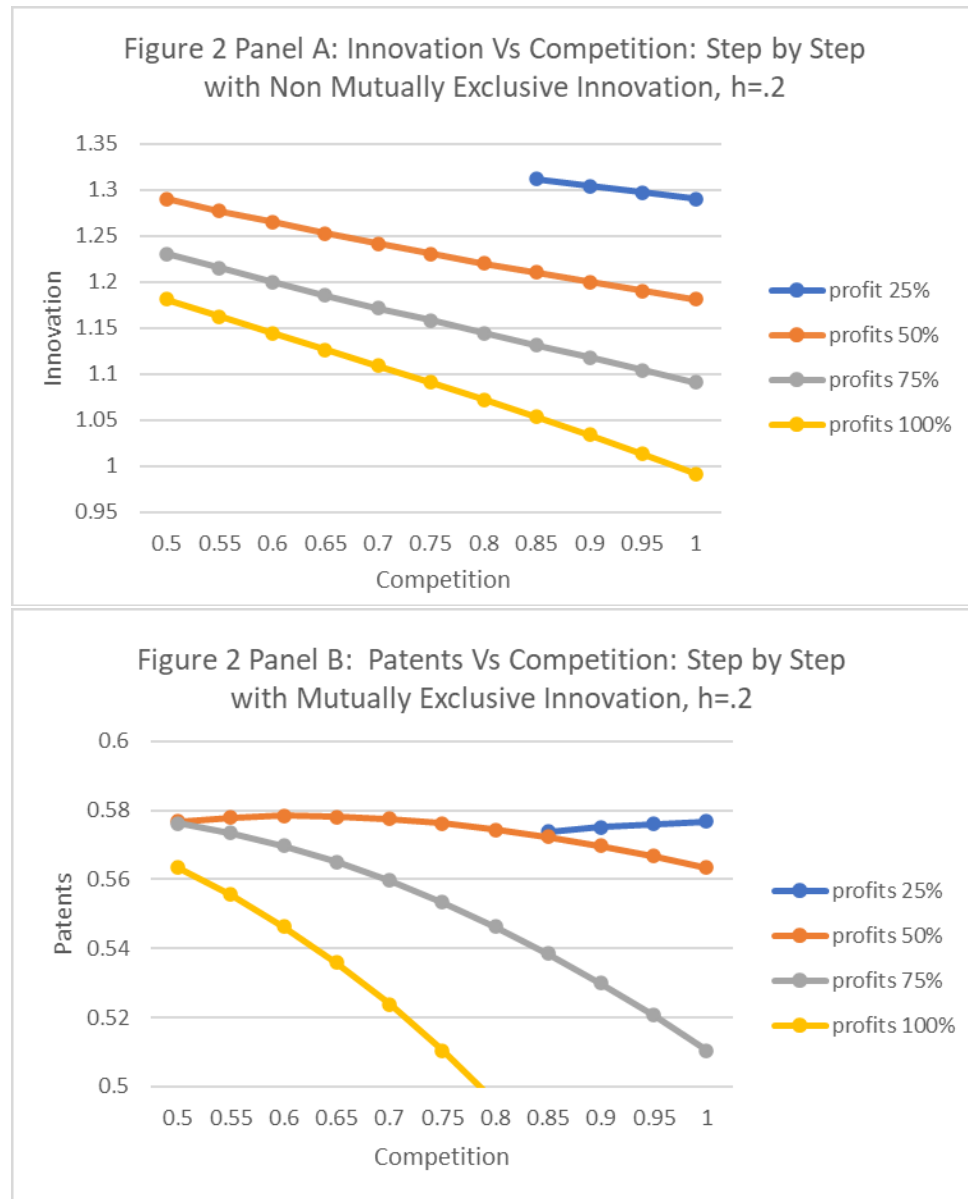


Figure 3

Figure 3 Panel A. Innovation Vs Patents; Profits
10%; Spillovers 1%

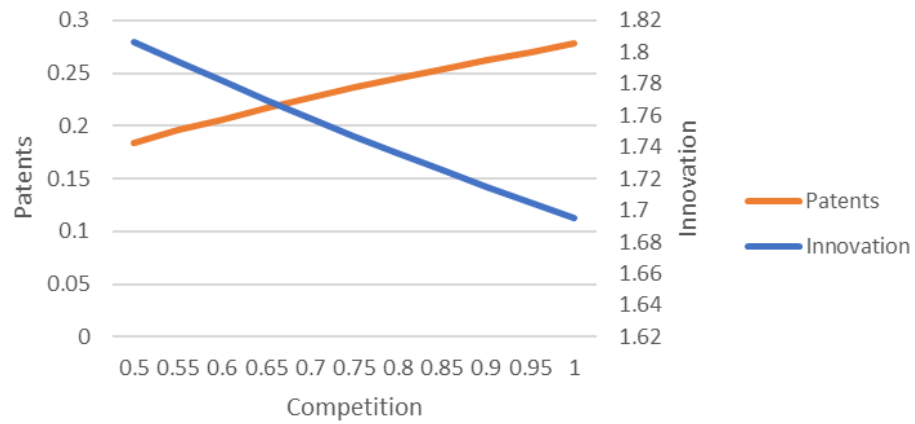


Figure 3 Panel B. Innovation Vs Patents; Profits
25%; Spillovers 1%

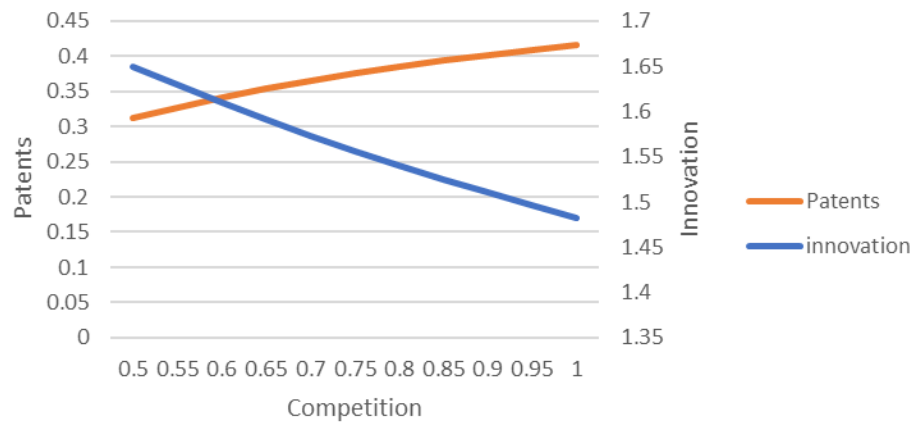


Figure 3 Panel C. Innovation Vs Patents; Profits
25%; Spillovers 10%

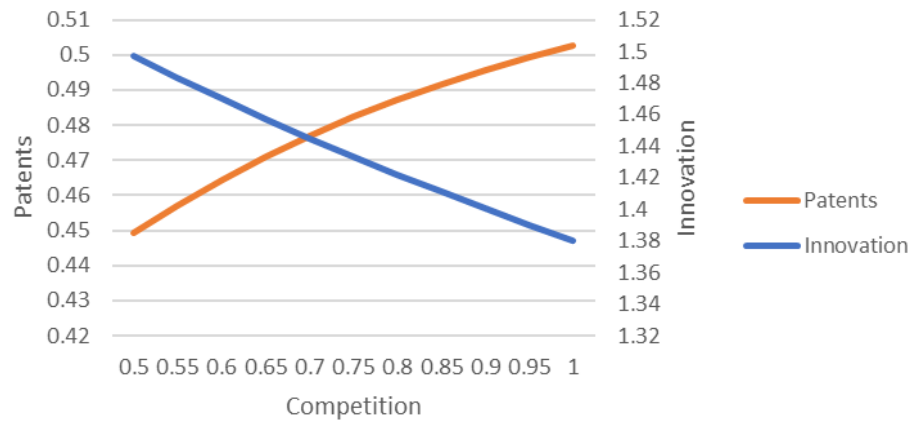


Figure 3 Panel D. Innovation Vs Patents; Profits
25%; Spillovers 20%

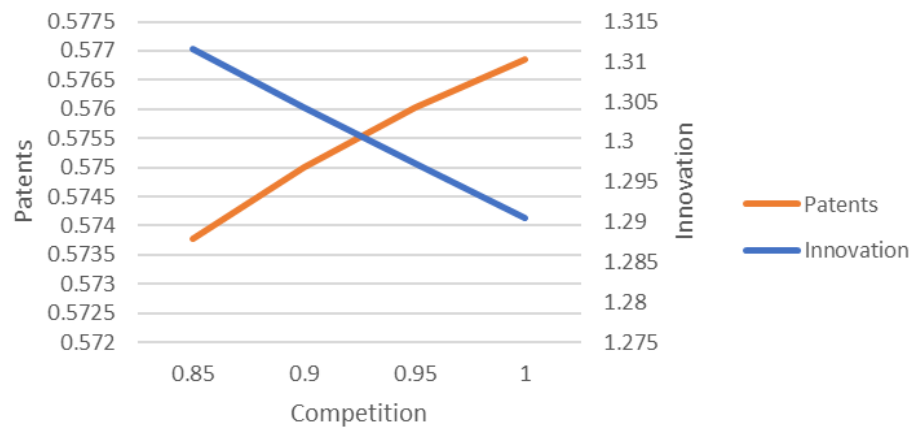


Table 1. Innovation Patterns in the Standard Step by Step Model												
Delta		0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1
h												
Profit Margin 10%												
0.1	0.4254	0.4268	0.4275	0.4278	0.4276	0.4276	0.4272	0.4265	0.4256	0.4246	0.4234	0.4222
0.2	0.3471	0.3549	0.3611	0.3660	0.3698	0.3698	0.3726	0.3746	0.3760	0.3768	0.3771	0.3770
Profit Margin 25%												
0.2	0.5552	0.5603	0.5635	0.5652	0.5657	0.5657	0.5652	0.5640	0.5622	0.5599	0.5572	0.5543
0.3	0.5504	0.5618	0.5708	0.5776	0.5827	0.5827	0.5864	0.5890	0.5905	0.5912	0.5912	0.5906
Profit Margin 50%												
0.2	0.7824	0.7847	0.7849	0.7836	0.7812	0.7812	0.7778	0.7736	0.7690	0.7639	0.7585	0.7529
0.3	0.7852	0.7934	0.7989	0.8021	0.8035	0.8035	0.8035	0.8024	0.8004	0.7976	0.7941	0.7903
0.4	0.7806	0.7952	0.8064	0.8148	0.8208	0.8208	0.8250	0.8276	0.8289	0.8290	0.8283	0.8268
Profit Margin 75%												
0.2	0.9547	0.9552	0.9535	0.9501	0.9456	0.9456	0.9402	0.9341	0.9274	0.9205		
0.3	0.9608	0.9668	0.9699	0.9707	0.9696	0.9696	0.9672	0.9637	0.9593	0.9542	0.9486	0.9426
0.4	0.9610	0.9732	0.9817	0.9873	0.9905	0.9905	0.9917	0.9915	0.9900	0.9874	0.9841	0.9800
0.5	0.9552	0.9737	0.9880	0.9988	1.0067	1.0067	1.0123	1.0158	1.0178	1.0183	1.0178	1.0162
Profit Margin 100%												
0.3	1.1074	1.1117	1.1129	1.1118								
0.4	1.1104	1.1206	1.1270	1.1304	1.1314	1.1314	1.1305					
0.5	1.1082	1.1247	1.1367	1.1451	1.1505	1.1505	1.1536	1.1547	1.1542			
0.6		1.1237	1.1415	1.1552	1.1654	1.1654	1.1729	1.1779	1.1810	1.1824	1.1824	

Table 2. Innovation Patterns in the Step by Step Model without Mutually Exclusive Innovation												
Delta		0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1
Profit Margin = 10%												
0.01	1.8066	1.7939	1.7816	1.7696	1.7581	1.7468	1.7359	1.7253	1.7149	1.7049	1.6951	
Profit Margin = 25%												
0.01	1.6496	1.6289	1.6092	1.5906	1.5729	1.5561	1.5400	1.5247	1.5100	1.4959	1.4824	
0.1	1.4972	1.4832	1.4699	1.4571	1.4448	1.4331	1.4218	1.4109	1.4004	1.3903	1.3804	
0.2								1.3115	1.3043	1.2973	1.2905	
Profit Margin = 50%												
0.01	1.4824	1.4568	1.4330	1.4107	1.3897	1.3699	1.3511	1.3332	1.3160	1.2995	1.2836	
0.1	1.3804	1.3618	1.3442	1.3276	1.3118	1.2968	1.2824	1.2686	1.2552	1.2423	1.2298	
0.2	1.2905	1.2774	1.2650	1.2531	1.2417	1.2308	1.2203	1.2101	1.2002	1.1905	1.1810	
0.3				1.1913	1.1835	1.1759	1.1686	1.1614	1.1543	1.1474	1.1406	
0.4								1.1193	1.1148	1.1102	1.1057	
Profit Margin = 75%												
0.01	1.3699	1.3421	1.3160	1.2915	1.2682	1.2458	1.2242	1.2032	1.1825	1.1621	1.1419	
0.1	1.2968	1.2754	1.2552	1.2360	1.2175	1.1997	1.1822	1.1651	1.1482	1.1313	1.1144	
0.2	1.2308	1.2151	1.2002	1.1857	1.1717	1.1580	1.1445	1.1312	1.1178	1.1044	1.0909	
0.3	1.1759	1.1649	1.1543	1.1440	1.1338	1.1237	1.1136	1.1035	1.0934	1.0830	1.0725	
0.4		1.1216	1.1148	1.1080	1.1011	1.0943	1.0873	1.0801	1.0728	1.0653	1.0575	
0.5					1.0720	1.0680	1.0638	1.0594	1.0547	1.0497	1.0444	
0.6								1.0402	1.0378	1.0352	1.0321	
0.7											1.0201	
Profit Margin = 100%												
0.01	1.2836	1.2532	1.2242	1.1962	1.1689	1.1419	1.1148	1.0874	1.0595	1.0309	1.0014	
0.1	1.2298	1.2056	1.1822	1.1594	1.1369	1.1144	1.0916	1.0685	1.0447	1.0201	0.9946	
0.2	1.1810	1.1626	1.1445	1.1267	1.1089	1.0909	1.0725	1.0539	1.0339	1.0135	0.9921	
0.3	1.1406	1.1270	1.1136	1.1002	1.0865	1.0725	1.0580	1.0430	1.0272	1.0106	0.9931	
0.4	1.1057	1.0966	1.0873	1.0777	1.0678	1.0575	1.0466	1.0351	1.0229	1.0099	0.9960	
0.5		1.0694	1.0638	1.0579	1.0514	1.0444	1.0368	1.0285	1.0196	1.0099	0.9994	
0.6				1.0394	1.0361	1.0321	1.0275	1.0223	1.0164	1.0097	1.0022	
0.7						1.0201	1.0182	1.0157	1.0124	1.0085	1.0038	
0.8								1.0082	1.0074	1.0059	1.0037	
0.9											1.0018	

Table 3. Patenting Patterns in the Step by Step Model without Mutually Exclusive Innovation												
Delta		0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1
Profit Margin = 10%												
0.01	0.1844	0.1956	0.2063	0.2166	0.2265	0.2359	0.2450	0.2537	0.2621	0.2702	0.2779	
Profit Margin = 25%												
0.01	0.3124	0.3274	0.3411	0.3536	0.3650	0.3754	0.3850	0.3937	0.4017	0.4090	0.4157	
0.1	0.4491	0.4571	0.4643	0.4709	0.4769	0.4823	0.4872	0.4917	0.4958	0.4994	0.5027	
0.2								0.5738	0.5750	0.5760	0.5768	
Profit Margin = 50%												
0.01	0.4157	0.4273	0.4369	0.4448	0.4511	0.4561	0.4598	0.4624	0.4640	0.4647	0.4646	
0.1	0.5027	0.5082	0.5124	0.5156	0.5179	0.5192	0.5198	0.5196	0.5188	0.5173	0.5153	
0.2	0.5768	0.5779	0.5783	0.5782	0.5774	0.5762	0.5744	0.5723	0.5697	0.5667	0.5634	
0.3				0.6262	0.6237	0.6209	0.6179	0.6146	0.6111	0.6073	0.6033	
0.4								0.6478	0.6440	0.6400	0.6358	
Profit Margin = 75%												
0.01	0.4561	0.4612	0.4640	0.4647	0.4636	0.4608	0.4565	0.4506	0.4435	0.4349	0.4251	
0.1	0.5192	0.5198	0.5188	0.5164	0.5127	0.5078	0.5018	0.4948	0.4866	0.4774	0.4672	
0.2	0.5762	0.5734	0.5697	0.5651	0.5597	0.5534	0.5464	0.5386	0.5300	0.5206	0.5105	
0.3	0.6209	0.6163	0.6111	0.6054	0.5991	0.5923	0.5849	0.5770	0.5684	0.5594	0.5497	
0.4		0.6497	0.6440	0.6379	0.6315	0.6247	0.6175	0.6099	0.6019	0.5934	0.5845	
0.5					0.6574	0.6511	0.6445	0.6376	0.6304	0.6228	0.6148	
0.6								0.6603	0.6541	0.6475	0.6406	
0.7											0.6621	
Profit Margin = 100%												
0.01	0.4646	0.4619	0.4565	0.4484	0.4379	0.4251	0.4099	0.3925	0.3728	0.3508	0.3266	
0.1	0.5153	0.5096	0.5018	0.4922	0.4806	0.4672	0.4520	0.4349	0.4159	0.3949	0.3721	
0.2	0.5634	0.5556	0.5464	0.5358	0.5238	0.5105	0.4957	0.4794	0.4615	0.4421	0.4210	
0.3	0.6033	0.5946	0.5849	0.5742	0.5625	0.5497	0.5357	0.5206	0.5042	0.4865	0.4674	
0.4	0.6358	0.6270	0.6175	0.6073	0.5963	0.5845	0.5717	0.5580	0.5433	0.5275	0.5105	
0.5		0.6532	0.6445	0.6353	0.6254	0.6148	0.6035	0.5914	0.5785	0.5646	0.5498	
0.6				0.6583	0.6498	0.6406	0.6309	0.6205	0.6094	0.5975	0.5848	
0.7						0.6621	0.6541	0.6454	0.6361	0.6262	0.6155	
0.8								0.6661	0.6586	0.6505	0.6418	
0.9											0.6639	

Table 4. Joint Probabilities of Mutual Innovation													
Delta													
h	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1		
Profit Margin = 10%													
0.01	0.8953	0.8880	0.8809	0.8739	0.8671	0.8604	0.8539	0.8476	0.8414	0.8353	0.8294		
Profit Margin = 25%													
0.01	0.8017	0.7890	0.7770	0.7655	0.7547	0.7445	0.7347	0.7255	0.7167	0.7084	0.7004		
0.1	0.6782	0.6689	0.6601	0.6517	0.6438	0.6363	0.6292	0.6225	0.6161	0.6101	0.6044		
0.2	0.5184 0.5144 0.5107 0.5072												
Profit Margin = 50%													
0.01	0.7004	0.6858	0.6726	0.6607	0.6501	0.6405	0.6320	0.6244	0.6178	0.6119	0.6069		
0.1	0.6044	0.5939	0.5845	0.5761	0.5686	0.5621	0.5563	0.5512	0.5469	0.5432	0.5402		
0.2	0.5072	0.5008	0.4951	0.4902	0.4859	0.4822	0.4791	0.4765	0.4744	0.4728	0.4717		
0.3	0.4141			0.4125	0.4112	0.4103	0.4096	0.4098	0.4097	0.4101	0.4101		
0.4	0.3514 0.3527 0.3541 0.3558												
Profit Margin = 75%													
0.01	0.6405	0.6281	0.6178	0.6094	0.6027	0.5977	0.5942	0.5922	0.5917	0.5926	0.5949		
0.1	0.5621	0.5537	0.5469	0.5416	0.5377	0.5351	0.5337	0.5335	0.5345	0.5366	0.5399		
0.2	0.4822	0.4777	0.4744	0.4722	0.4710	0.4706	0.4712	0.4727	0.4751	0.4783	0.4824		
0.3	0.4112	0.4100	0.4096	0.4099	0.4108	0.4124	0.4146	0.4174	0.4209	0.4249	0.4296		
0.4	0.3508			0.3527	0.3549	0.3576	0.3606	0.3641	0.3680	0.3722	0.3769	0.3821	
0.5					0.3112	0.3153	0.3197	0.3243	0.3292	0.3343	0.3398		
0.6									0.2862	0.2915	0.2969	0.3025	
0.7									0.0000	0.0000	0.0000	0.2699	
Profit Margin = 100%													
0.01	0.6069	0.5992	0.5942	0.5919	0.5921	0.5949	0.6002	0.6081	0.6186	0.6320	0.6483		
0.1	0.5402	0.5358	0.5337	0.5337	0.5358	0.5399	0.5460	0.5543	0.5648	0.5775	0.5927		
0.2	0.4717	0.4706	0.4712	0.4734	0.4771	0.4824	0.4892	0.4977	0.5079	0.5199	0.5339		
0.3	0.4101	0.4118	0.4146	0.4185	0.4235	0.4296	0.4369	0.4454	0.4553	0.4665	0.4792		
0.4	0.3558	0.3596	0.3641	0.3693	0.3753	0.3821	0.3897	0.3981	0.4075	0.4180	0.4296		
0.5	0.3139		0.3197	0.3259	0.3326	0.3398	0.3475	0.3558	0.3648	0.3746	0.3851		
0.6				0.2880	0.2951	0.3025	0.3102	0.3184	0.3270	0.3361	0.3458		
0.7					0.2699 0.2776				0.2855	0.2937	0.3023		
0.8									0.2567	0.2646	0.2726	0.2809	
0.9													0.2544

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4 Appendix 1

Appendix 1 Table 1. Standard Step By Step Model Leveled R&D												
Delta		0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1
Profit Margin = 10%												
0.1	0.3064	0.3218	0.3366	0.3507	0.3643	0.3774	0.3901	0.3901	0.4024	0.4144	0.4260	0.4373
0.2	0.1742	0.1873	0.2000	0.2123	0.2243	0.2359	0.2472	0.2583	0.2690	0.2796	0.2899	0.2899
Profit Margin = 25%												
0.1	0.4099	0.4339	0.4568	0.4788	0.5000	0.5205	0.5403	0.5596	0.5782	0.5964	0.6141	0.6141
0.2	0.3385	0.3613	0.3831	0.4042	0.4245	0.4442	0.4633	0.4819	0.5000	0.5176	0.5349	0.5349
0.3	0.2831	0.3042	0.3245	0.3442	0.3633	0.3819	0.4000	0.4176	0.4349	0.4517	0.4681	0.4681
Profit Margin = 50%												
0.2	0.5349	0.5681	0.6000	0.6307	0.6602	0.6888	0.7165	0.7434	0.7695	0.7950	0.8198	0.8198
0.3	0.4681	0.5000	0.5307	0.5602	0.5888	0.6165	0.6434	0.6695	0.6950	0.7198	0.7440	0.7440
0.4	0.4124	0.4426	0.4718	0.5000	0.5274	0.5539	0.5798	0.6050	0.6296	0.6536	0.6770	0.6770
Profit Margin = 75%												
0.2	0.6888	0.7301	0.7695	0.8075	0.8440	0.8794	0.9136	0.9467	0.9790			
0.3	0.6165	0.6566	0.6950	0.7320	0.7677	0.8023	0.8358	0.8683	0.9000	0.9309	0.9610	0.9610
0.4	0.5539	0.5925	0.6296	0.6654	0.7000	0.7336	0.7662	0.7979	0.8288	0.8590	0.8884	0.8884
0.5	0.5000	0.5368	0.5724	0.6068	0.6402	0.6726	0.7042	0.7349	0.7649	0.7942	0.8229	0.8229
Profit Margin = 100%												
0.2	0.8198	0.8677	0.9136	0.9576								
0.3	0.7440	0.7909	0.8358	0.8790	0.9207	0.9610						
0.4	0.6770	0.7225	0.7662	0.8083	0.8490	0.8884	0.9267	0.9638				
0.5		0.6619	0.7042	0.7450	0.7845	0.8229	0.8602	0.8964	0.9318	0.9663		
0.6				0.6884	0.7267	0.7638	0.8000	0.8353	0.8697	0.9033	0.9362	0.9362

Appendix 1 Table 2. Standard Step By Step Model Uneveled R&D													
Delta		0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1	
Profit Margin = 10%													
h	0.1	0.2258	0.2192	0.2132	0.2077	0.2026	0.1979	0.1934	0.1893	0.1854	0.1817	0.1783	
	0.2	0.1458	0.1372	0.1292	0.1216	0.1145	0.1078	0.1015	0.0955	0.0899	0.0845	0.0793	
Profit Margin = 25%													
	0.1	0.3135	0.3018	0.2910	0.2810	0.2718	0.2632	0.2552	0.2477	0.2407	0.2340	0.2278	
	0.2	0.2706	0.2576	0.2456	0.2345	0.2242	0.2145	0.2054	0.1969	0.1888	0.1812	0.1740	
	0.3	0.2355	0.2220	0.2094	0.1975	0.1864	0.1759	0.1660	0.1567	0.1478	0.1394	0.1314	
Profit Margin = 50%													
	0.2	0.4167	0.3993	0.3832	0.3684	0.3546	0.3418	0.3298	0.3186	0.3080	0.2981	0.2887	
	0.3	0.3761	0.3576	0.3405	0.3246	0.3098	0.2960	0.2830	0.2707	0.2592	0.2483	0.2380	
	0.4	0.3409	0.3218	0.3041	0.2874	0.2719	0.2572	0.2434	0.2303	0.2180	0.2063	0.1951	
Profit Margin = 75%													
	0.2	0.5305	0.5097	0.4907	0.4731	0.4568	0.4416	0.4274	0.4141	0.4017			
	0.3	0.4871	0.4651	0.4448	0.4260	0.4086	0.3923	0.3770	0.3627	0.3492	0.3365	0.3244	
	0.4	0.4485	0.4256	0.4044	0.3847	0.3663	0.3490	0.3328	0.3176	0.3032	0.2895	0.2766	
	0.5	0.4142	0.3908	0.3690	0.3486	0.3295	0.3114	0.2945	0.2784	0.2632	0.2488	0.2351	
Profit Margin =100%													
	0.2	0.6270	0.6035	0.5819	0.5620								
	0.3	0.5819	0.5570	0.5341	0.5129	0.4933	0.4750						
	0.4	0.5411	0.5152	0.4912	0.4690	0.4483	0.4289	0.4108	0.3937				
	0.5		0.4777	0.4529	0.4298	0.4083	0.3880	0.3690	0.3510	0.3341	0.3180		
	0.6				0.3950	0.3728	0.3518	0.3321	0.3134	0.2956	0.2788	0.2628	

5 Appendix 2

Appendix 2 Table 1. Step By Step Model without Mutually Exclusive Innovation Leveled R&D													
Delta													
h	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1		
Profit Margin = 10%													
0.01	0.9462	0.9423	0.9385	0.9348	0.9312	0.9276	0.9241	0.9206	0.9173	0.9140	0.9107		
Profit Margin = 25%													
0.01	0.8954	0.8883	0.8815	0.8750	0.8687	0.8628	0.8572	0.8517	0.8466	0.8416	0.8369		
0.1	0.8236	0.8179	0.8124	0.8073	0.8024	0.7977	0.7932		0.7890	0.7849	0.7811	0.7774	
0.2									0.7200	0.7173	0.7146	0.7122	
Profit Margin = 50%													
0.01	0.8369	0.8281	0.8201	0.8129	0.8063	0.8003	0.7950	0.7902	0.7860	0.7823	0.7791		
0.1	0.7774	0.7706	0.7645	0.7590	0.7541	0.7497	0.7458	0.7425	0.7395	0.7370	0.7350		
0.2	0.7122	0.7077	0.7037	0.7001	0.6971	0.6944	0.6922	0.6903	0.6888	0.6876	0.6868		
0.3				0.6435	0.6422	0.6413	0.6406		0.6402	0.6400	0.6401	0.6404	
0.4									0.5928	0.5939	0.5951	0.5965	
Profit Margin = 75%													
0.01	0.8003	0.7925	0.7860	0.7806	0.7763	0.7731	0.7709	0.7696	0.7692	0.7698	0.7713		
0.1	0.7497	0.7441	0.7395	0.7359	0.7333	0.7315	0.7306	0.7304	0.7311	0.7326	0.7348		
0.2	0.6944	0.6912	0.6888	0.6872	0.6863	0.6860	0.6865	0.6876	0.6893	0.6916	0.6945		
0.3	0.6413	0.6403	0.6400	0.6402	0.6409	0.6422	0.6439	0.6461	0.6487	0.6519	0.6555		
0.4		0.5923	0.5939	0.5958	0.5980	0.6005	0.6034	0.6066	0.6101	0.6139	0.6181		
0.5					0.5578	0.5615	0.5654	0.5695	0.5738	0.5782	0.5829		
0.6								0.5350	0.5399	0.5449	0.5500		
0.7											0.5195		
Profit Margin = 100%													
0.01	0.7791	0.7741	0.7709	0.7694	0.7695	0.7713	0.7747	0.7798	0.7865	0.7950	0.8051		
0.1	0.7350	0.7320	0.7306	0.7306	0.7320	0.7348	0.7389	0.7445	0.7515	0.7599	0.7699		
0.2	0.6868	0.6860	0.6865	0.6881	0.6908	0.6945	0.6995	0.7055	0.7127	0.7210	0.7307		
0.3	0.6404	0.6417	0.6439	0.6469	0.6508	0.6555	0.6610	0.6674	0.6748	0.6830	0.6923		
0.4	0.5965	0.5997	0.6034	0.6077	0.6126	0.6181	0.6242	0.6310	0.6384	0.6465	0.6554		
0.5		0.5603	0.5654	0.5709	0.5767	0.5829	0.5895	0.5965	0.6040	0.6120	0.6206		
0.6				0.5366	0.5432	0.5500	0.5570	0.5643	0.5718	0.5797	0.5880		
0.7							0.5195	0.5268	0.5343	0.5420	0.5498		
0.8									0.5067	0.5144	0.5221		
0.9												0.5044	

Appendix 2 Table 2. Step By Step Model without Mutually Exclusive Innovation Unlevered R&D												
h	Delta											
	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1	
Profit Margin = 10%												
0.01	0.9698	0.9673	0.9647	0.9622	0.9596	0.9570	0.9544	0.9518	0.9492	0.9466	0.9440	
Profit Margin = 25%												
0.01	0.9308	0.9241	0.9173	0.9105	0.9036	0.8967	0.8897	0.8827	0.8756	0.8684	0.8612	
0.1	0.8879	0.8811	0.8744	0.8675	0.8606	0.8536	0.8466	0.8395	0.8324	0.8252	0.8179	
0.2								0.7946	0.7874	0.7801	0.7728	
Profit Margin = 50%												
0.01	0.8612	0.8466	0.8317	0.8166	0.8013	0.7857	0.7698	0.7537	0.7374	0.7209	0.7041	
0.1	0.8179	0.8033	0.7883	0.7732	0.7578	0.7422	0.7263	0.7103	0.6940	0.6775	0.6608	
0.2	0.7728	0.7581	0.7431	0.7279	0.7125	0.6969	0.6811	0.6651	0.6489	0.6325	0.6159	
0.3				0.6857	0.6703	0.6547	0.6390	0.6231	0.6070	0.5908	0.5744	
0.4								0.5842	0.5683	0.5523	0.5361	
Profit Margin = 75%												
0.01	0.7857	0.7618	0.7374	0.7125	0.6871	0.6612	0.6348	0.6079	0.5805	0.5528	0.5246	
0.1	0.7422	0.7183	0.6940	0.6692	0.6439	0.6182	0.5920	0.5653	0.5383	0.5109	0.4831	
0.2	0.6969	0.6731	0.6489	0.6242	0.5991	0.5737	0.5478	0.5215	0.4949	0.4679	0.4406	
0.3	0.6547	0.6311	0.6070	0.5826	0.5578	0.5326	0.5072	0.4813	0.4552	0.4287	0.4019	
0.4		0.5922	0.5683	0.5442	0.5198	0.4950	0.4700	0.4447	0.4191	0.3932	0.3670	
0.5					0.4850	0.4607	0.4362	0.4114	0.3864	0.3612	0.3357	
0.6								0.3814	0.3571	0.3325	0.3077	
0.7											0.2829	
Profit Margin = 100%												
0.01	0.7041	0.6699	0.6348	0.5988	0.5621	0.5246	0.4865	0.4479	0.4089	0.3697	0.3306	
0.1	0.6608	0.6268	0.5920	0.5564	0.5201	0.4831	0.4455	0.4075	0.3691	0.3304	0.2918	
0.2	0.6159	0.5822	0.5478	0.5127	0.4769	0.4406	0.4037	0.3664	0.3286	0.2907	0.2526	
0.3	0.5744	0.5411	0.5072	0.4726	0.4376	0.4019	0.3658	0.3293	0.2923	0.2551	0.2176	
0.4	0.5361	0.5033	0.4700	0.4362	0.4018	0.3670	0.3318	0.2961	0.2600	0.2236	0.1870	
0.5		0.4688	0.4362	0.4031	0.3696	0.3357	0.3014	0.2666	0.2315	0.1961	0.1604	
0.6				0.3733	0.3407	0.3077	0.2744	0.2407	0.2066	0.1723	0.1376	
0.7						0.2829	0.2506	0.2179	0.1850	0.1517	0.1182	
0.8								0.1980	0.1662	0.1341	0.1017	
0.9											0.0878	