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Experimental evaluation of different weighting schemes in magnetic particle imaging reconstruction

Abstract: Magnetic Particle Imaging (MPI) is a new imaging technique with an outstanding sensitivity, a high temporal and spatial resolution. MPI is based on the excitation and detection of magnetic tracer material by using magnetic fields. The spatial resolution strongly depends on the reconstruction parameters and on the selection and weighting of the system function frequency components. Currently, no fundamental strategy to weight the system function for the reconstruction is given. In this contribution, the influence on the spatial resolution of different selection and weighting methods is analyzed. Thereby, a new strategy is proposed to select and weight the components with respect to their mixing order. As a result, it is confirmed that a weighted system function provides better results of image reconstruction than a non-weighted one. In addition to this, it is shown that the usage of the mixing order in combination with established weightings improves the resolution.

Keywords: MPI; Reconstruction; Weighting Schemes

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1 Introduction

Magnetic Particle Imaging (MPI) is a novel imaging technique based on the detection of superparamagnetic iron oxide particles [1]. The generation of a detectable signal is based on the non-linear magnetization of those particles. This signal is the induced voltage generated by the excitation of the particles. This excitation is realized with orthogonal sinusoidal drive fields generated by orthogonal sets of coils [2]. Currently, two reconstruction methods are established to obtain the image of the particle distribution, the time-domain reconstruction, known as the x-space approach [3], and the frequency-domain reconstruction [2, 4].

In frequency space, typically a system matrix is used to calibrate the system. Therefore, a system function (SF) for every position in the field of view (FOV) is acquired. It has been shown in [5] that the image reconstruction can be improved with an additional weighting of individual SF frequency components.

In this contribution, the mixing orders of measured SFs, as described in [4], are analyzed. In order to evaluate those mixing orders with respect to their influence on the reconstruction results, different regularization, selection, and weighting schemes are used.

2 Material and methods

The measurements used in this work are performed with the MPI preclinical demonstrator from the Philips research laboratories, Hamburg, Germany [6]. These measurements are performed with a 16 mT drive field amplitude on all three axes and a gradient strength of \( G_x = G_y = 1.25 \, \text{T/m/\mu}_0 \) and accordingly \( G_z = 2.5 \, \text{T/m/\mu}_0 \). The frequencies to generate a Lissajous measurement cycle with a repetition time \( T_R = \frac{528 \Delta f}{f_x}, f_y = \frac{561 \Delta f}{f_y}, \) and \( f_z = \frac{544 \Delta f}{f_z} \) [4]. Thereby, \( \Delta f \) is given by \( \frac{1}{T_R} = 46.42 \, \text{Hz} \). The FOV is sampled at \( 20 \times 36 \times 36 = 25920 \) positions with a voxel and a delta probe size of 1 mm³. The phantom consists of seven tubes centered in the volume and filled with a tracer similar to Resovist with a concentration of 0.03 mol(Fe)/l. Each tube has a diameter of 2 mm and a distance of 2 mm to the next tube. In the following, a characterization of the SF is given followed by the description of the reconstruction. With respect to the reconstruction, the regularization, the selection, and the weighting of the frequency components are represented in relation to the mixing order.
2.1 System Function

The SF is a complex valued matrix, which includes all information about the system and the particle behavior in every sampled position of the FOV [7].

In [4], the characteristics of a 3D SF have been presented. The system matrix of one receive channel $G$ is defined as

$$G = \begin{pmatrix} \tilde{s}(f_1, p_1) & \tilde{s}(f_1, p_2) & \cdots & \tilde{s}(f_1, p_n) \\ \tilde{s}(f_2, p_1) & \tilde{s}(f_2, p_2) & \cdots & \tilde{s}(f_2, p_n) \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{s}(f_m, p_1) & \tilde{s}(f_m, p_2) & \cdots & \tilde{s}(f_m, p_n) \end{pmatrix},$$

with $\tilde{s}$ as the signal in Fourier space, the rows $M_{freq}$, representing the number of stored frequencies $f_i$, and the columns $N_{samp}$, the number of sampled positions in the FOV $p_j$. In this work, $\tilde{s}$ is given with $M_{freq} = 26929$ frequencies per channel.

Every frequency component can be described by a linear combination of the excitation frequencies for each drive field channel and positive/negative integers $n_x, n_y$ and $n_z$ like

$$f = |n_x f_x + n_y f_y + n_z f_z|$$

[4]. The mixing order $n_{mo}$ of the frequencies is defined as

$$n_{mo} = |n_x| + |n_y| + |n_z|.$$ (3)

An example visualization of the minimal mixing order $n_{mo}$ over all frequencies for one channel is presented in Fig. 1b. Furthermore, the measured signal-to-noise ratio (SNR) for the same channel is visualized in Fig. 1a. It can be seen that there is a direct link between a high SNR and a low mixing order. As proposed in [8], this correlation can be used to choose frequencies with high SNR via the mixing order.

2.2 Image reconstruction

Image reconstruction in MPI decodes the particle distribution from the measured signal in the sampled FOV. The previously described SF is necessary for the system matrix based frequency-domain reconstruction. For this reconstruction, a discrete system of equations can be established, which in the matrix vector notation is given as

$$G c = \tilde{u},$$

with $c$ being the desired particle concentration and $\tilde{u}$ being the Fourier transformed voltage signal. Due to the dimension of the system matrix, this inverse problem is ill-posed, so that a regularization is needed. Furthermore, it has been shown in [5] that a weighting matrix can be used to improve the solution of the problem. The reconstruction can be expressed as a weighted regularized least-squares problem

$$||W(G S c - \tilde{u})||^2 + \lambda ||c||^2 \rightarrow \min$$

(5)

with $W$ as the weighting matrix, $G_S$ as the system matrix with selected components and $\lambda$ as the regularization parameter. In the practical realization, a reconstruction kernel $k = W \cdot G_S$ including information about the selected, weighted frequency components is generated. Further, as described in [5], a non-negativity and non-complexity constraint justified by the particle physics can be applied to refine the reconstructions. The reconstruction, which is used in this work, follows the approach presented in [9].

2.2.1 Selection of individual frequency components

Many of the components of the SF are highly noisy and therefore should not be used for the reconstruction. A possible way to improve the reconstruction is to introduce a preprocessing step, where only specific components are selected. This aspect suggests to choose the components based on an SNR threshold [5]. This means, only the components with an SNR above the threshold are used for the reconstruction.

The correlation between the SNR and the mixing order, which is visualized in Fig. 1, is presented in [4]. Based on that relation, the selection of the frequency compo-
nents using the mixing order is determined in this work. The idea is to select only those frequency components that can be generated by a given minimal mixing order threshold.

### 2.2.2 Weighting of the system function

In [5], it has been proven that an additional weighting of the system of equations as defined in (5) does not change the mathematical correctness. A weighting has the effect to raise or decrease the rows of the matrix and therefore the corresponding frequencies in every position. It has been proposed in [5] to use a weighting, that adjusts the energy of the system matrix rows. The representation for the k-th row of $G$ is $g_k$, so that the weighting can be written as

$$\tilde{w}_k = ||g_k||_2^{-2}$$

where $W = \text{diag}(\tilde{w}_k^M_{k=1})$ and $||.||_2^{-2}$ denote the squared inverse Euclidean norm.

An additional approach is to weight the system matrix with the squared inverse background norm $iBG2$, which includes information about the sensitivity of the receive coils. The background information can be obtained by empty measurements $g_{\text{empty}}$ during the acquisition of the SF.

A new strategy, presented in this work, is to use the mixing order for the weighting, like $\tilde{w}_k = n_{\text{mo}}$. The idea is to increase the influence of the components with a high mixing order and hence a low SNR, to achieve a homogeneous distribution of the SNR over all components. Additionally, the mixing order weighting is extended to integrate system properties by a multiplication with $iBG2$, like

The idea is to integrate system information and especially information about the receive channels.

### 2.2.3 Regularization

The main idea of regularization is to establish a balance between the discrepancy of $|Gc - \hat{u}|^2$ and the energy of the solution $|c|^2$. Possible approaches to determine the mathematically optimal regularization parameter $\lambda$ are presented in [5]. In this work, the regularization parameter $\lambda$ is defined as

$$\lambda = l \cdot \lambda_0 = l \frac{||x||_F^2}{r},$$

with $l$ being a scaling factor of the measurement dependent $\lambda_0$, $||x||_F^2$ is the squared Frobenius norm of the kernel $\kappa$, and $r$ the number of voxels. In (7) it is shown that the regularization is directly linked to the weighting and selection of the SF. Therefore, the following regularization will be described via the scaling factor $l$, independent of the used weighting and selection.

### 3 Results and discussion

In this work, several weighting and selection aspects are investigated. The achieved reconstruction results are presented in Fig. 2 for both investigated selection methods, i.e. the SNR threshold and the mixing order threshold. Further, the different weighting types are shown: the non-weighted, the energy based, the $iBG2$, the mixing order, and the product of mixing order and $iBG2$. In addition to this, the scaling factor for all reconstructions is given.

The results of the $iBG2$ weighting demonstrate a better resolution in comparison with the non-weighted and energy-based weighting. However, some noise patterns are visible, what could be explained by the very low regularization. The reconstruction results of the weighting with the mixing order are satisfactory. This is very significant,
because the information used for the weighting and selecting are independent of the measured SNR.

In addition to the aforementioned approaches, reconstructions are presented, where the mixing order is combined with the iBG2. It is visible that this approach improves the reconstruction in comparison to the non-weighted and energy based method, so that a resolution of almost 2 mm also in the low gradient direction y is achieved. Furthermore, the noise in the background is at an acceptable level.

All results include some minor artifacts, which can be explained by the fact that the images are not post processed and that the areas where the artifacts occur are not sampled by the Lissajous trajectory.

In the reconstructions shown in Fig. 2, only minor differences between the selection methods are visible, which could be explained by the selection of nearly the same frequency components. This emphasizes the importance of the mixing order, not only for the weighting, but also for the selection of the frequency components carrying the important information for image reconstruction.

4 Conclusion

In this work, different weighting and selection methods for the frequency components of the MPI system function are analyzed. It has been shown that the idea presented in [4], where the system is characterized with the mixing order, can be used for the weighting as well as for the selection of the frequency components. Furthermore, it has been shown that a combination of the theoretical mixing order and system information improves the reconstruction results in comparison to the presented weighting schemes.

Summarized, it could be shown that a weighting and selection of the system function components based on the mixing order of the excitation frequencies improves the image reconstruction.

In the future, the theoretical aspects of the SF should be investigated further. Through this, it might be possible to improve the resolution even further.

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Author’s Statement

Conflict of interest: P. Szwargulski, M. Ahlborg, C. Kaethner and T. M. Buzug state no conflict of interest. J. Rahmer is employed by Philips. Material and Methods: Informed consent: Informed consent is not applicable. Ethical approval: The conducted research is not related to either human or animals use.

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