C. F. Isernhagen*, D. Schäfer, M. Grass, and T. M. Buzug

Three-dimensional anisotropic regularization for limited angle tomography

Abstract: Limited angle tomography is a challenging task in medical imaging. Due to practical limitations during the image acquisition, the sinogram is recorded incompletely and thus the quality of the reconstruction is deteriorated by streak artifacts. These artifacts are characterized by fast changes of the local intensity gradients and increase the total variation (TV). Generally, an energy functional is optimized which leads to a minimized Total Variation Minimization (TVM). As an outcome, noise and artifacts are reduced while edges are preserved. Anyway, often the orientation of the streak artifacts is not considered at all. Therefore, anisotropic regularization is used to reduce noise and distortions under specific directions.

Keywords: limited angle tomography; anisotropic regularization; denoising

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1 Introduction

Practical limitations during the image acquisition like obese patients often lead to the fact that the raw data is recorded incompletely. Thus, the quality of the reconstructed image is decreases by streaking artifacts which often prohibits usage in clinical routines.

As Rudin et al. showed in [1], the minimization of the total variation can be used to reduce artifacts and noise while edges in the image domain are preserved. Recently Chen et al. [2] extended this approach by introducing anisotropic filtering with directional derivatives to enhance the image quality and sharpen the edges.

2 Methods

Rudin, Osher and Fatemi outlined the benefits of Total variation (TV) or L_1-norm of derivatives for denoising compared with other methods like L_2-norm related minimization algorithms or maximum entropy methods. The total variation of an image f in the image domain Ω is defined as

$$\mathrm{TV}(f) = \int_{\Omega} |\nabla f| \; d\Omega = \|\nabla f\|_1 \tag{1}$$

where $\|\cdot\|_1$ describes the L_1-norm of the gradient of f. The fact that limited angle artifacts appear perpendicular to the direction of the missing projections motivates the idea of anisotropic regularization to reduce these streaks and their impact of disturbances. To minimize such an expression, one has to find the roots of the partial derivatives:

$$\frac{\partial\,\mathrm{TV}(f)}{\partial f} \stackrel{!}{=} 0. \tag{2}$$

Since discontinuities may arise using (2), Persson et al. suggest to use the approximation defined by

$$\nabla f_{x,y,z} = \begin{pmatrix} [f_{x,y,z}]_x \\ [f_{x,y,z}]_y \\ [f_{x,y,z}]_z \end{pmatrix} = \begin{pmatrix} f_{x+1,y,z} - f_{x,y,z} \\ f_{x,y+1,z} - f_{x,y,z} \\ f_{x,y,z+1} - f_{x,y,z} \end{pmatrix}$$

where $[\cdot]_x$, $[\cdot]_y$ and $[\cdot]_z$ denote the x, y or z-component of the given vector. Consequently, (2) can be written as

$$\begin{aligned} \frac{\partial\,\mathrm{TV}(f)}{\partial f} \approx & \left(\frac{[f_{x,y,z}]_x}{\|\nabla f_{x,y,z}\|_\varepsilon} - \frac{[f_{x-1,y,z}]_x}{\|\nabla f_{x-1,y,z}\|_\varepsilon} \right) \\ & + \left(\frac{[f_{x,y,z}]_y}{\|\nabla f_{x,y,z}\|_\varepsilon} - \frac{[f_{x,y-1,z}]_y}{\|\nabla f_{x,y-1,z}\|_\varepsilon} \right) \\ & + \left(\frac{[f_{x,y,z}]_z}{\|\nabla f_{x,y,z}\|_\varepsilon} - \frac{[f_{x,y,z-1}]_z}{\|\nabla f_{x,y,z-1}\|_\varepsilon} \right). \end{aligned}$$

Thereby, $\|\cdot\|_\varepsilon$ defines the modified norm $\|(r_x, r_y, r_z)\|_\varepsilon = \sqrt{r_x^2 + r_y^2 + r_z^2 + \varepsilon^2}$ to stabilize the derivatives.

2.1 Anisotropic regularization

The investigated and extended algorithm in this paper is the algorithm for iterative anisotropic total variation minimization (aTVM) of Chen et al. [2]. It was designed to remove artifacts that strongly depend on the direction of projection (by using directional derivatives). The fact that limited angle artifacts appear perpendicular to the direction of the missing projections encourage the use of anisotropic

***Corresponding Author: C. F. Isernhagen:** Institute of Medical Engineering, University of Lübeck, Lübeck, Germany, E-mail: isernhagen@imt.uni-luebeck.de

D. Schäfer, M. Grass: Philips Research, Hamburg, Germany, E-mail: dirk.schaefer@philips.com, michael.grass@philips.com

T. M. Buzug: Institute of Medical Engineering, University of Lübeck, Lübeck, Germany, E-mail: buzug@imt.uni-luebeck.de

regularization to reduce these streaks. The anisotropic TV minimization can be described as an optimization problem constrained by the fact that the projection of the recovered image $\chi(f)$ should match the projections p.

$$\min_f \sum_{i=1}^{N_\alpha} \omega_i \left\| \vec{\nabla}_{\alpha_i} f \right\|_1 \quad s.t. \quad \chi(f) = p. \tag{3}$$

Here i denotes the ith direction under the angle α_i used for the directional derivative while N_α denotes the total number of directions used. The directional derivatives are used to minimize the TV of the image f, while preserving fine details. A weighted and normalized edge image is calculated by

$$v = \sum_{i=1}^{N_\alpha} \omega_i \frac{\partial \left\| \vec{\nabla}_{\alpha_i} f \right\|_1}{\partial f}, \qquad v = \frac{v}{\|v\|_2} \tag{4}$$

and minimized by a gradient descent with the update parameter

$$s(n) = s_{\text{start}} \cdot \left(\frac{s_{\text{final}}}{s_{\text{start}}} \right)^{\frac{n}{N_{\text{TV}}}}. \tag{5}$$

The starting and final regularization parameter s_{start} and s_{final}, respectively, cause a cooling down effect of the strength of regularization to prevent oversmoothing. The weights ω_i are defined by

$$\omega_i = \sum_{j=1}^{N_v} \sin(\alpha_i - r_j(\theta)) \tag{6}$$

where N_v denotes the number of projections and $r_j(\theta)$ the angle between the ray j and the rotated axis. In polar coordinates the weight for a given angle is defined by the distance towards the origin.

Finally, in an iterative reconstruction the weighted correction term

$$s(n-1) \cdot v$$

in the nth iteration is added to the previous image and the step parameter updated afterwards:

$$f^{(n)} = f^{(n-1)} + s(n-1) \cdot v. \tag{7}$$

3 Results

For our results we used a scanning range of 160°, $s_{\text{start}} = 1$, $s_{\text{final}} = 10^{-4}$, $N_{\text{TV}} = 100$, $N_\alpha = 3$ and $\varepsilon = 10^{-8}$ with equiangular distributed α_i. We used this scheme in an iterative reconstruction approach, the regularization is applied once an iteration is finished. Afterwards, the smoothed image is provided as a prior for the next iteration. Figure 1 shows the resulting images for all different methods under investigation. The corresponding similarity values are given in Table 1. The similarity is expressed in terms of the structural similarity index (SSIM) [4], which considers not only the gray values but also the structural information present in the image with floating windows or patches. Table 1 clearly states that the anisotropic method (SART + aTVM) outperforms the other algorithms. Our algorithm provides comparable results to the standard denoising approach, the ROF-denoising model, but also uses directional derivatives. Figure 2 shows the trend depending on the number of iterations. It can clearly be seen that the ROF-denoising and the anisotropic method give comparable results regarding the SSIM values. Moreover, both trend to even better results and clearly outperform the isotropic method.

Table 1: Similarity results for all methods: SART without regularization, with isotropic regularization (SART + iTVM), with anisotropic regularization (SART + aTVM) and with the ROF-denoising (SART + ROF), all results for 30 iterations

Method	SSIM
SART	0.8550
SART + iTVM	0.9066
SART + aTVM	0.9404
SART + ROF	0.9376

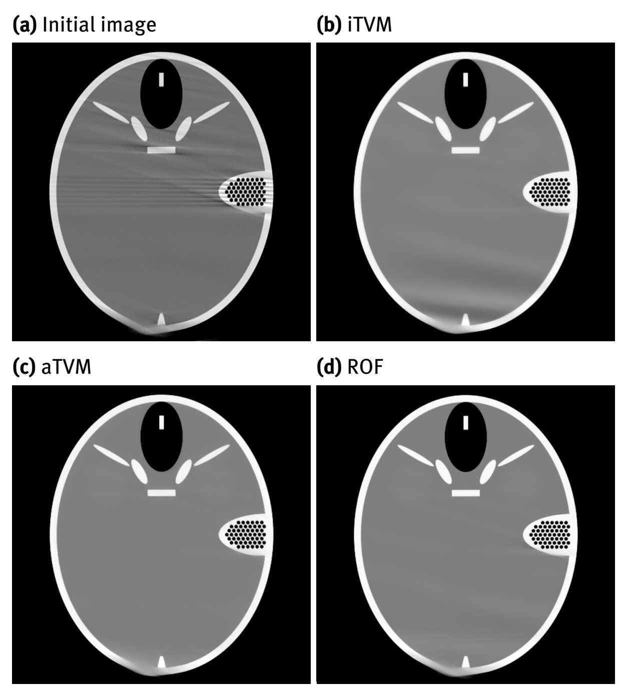

Figure 1: Initial and resulting images, (a): initial image, (b) image with isotropic method (SART + iTVM), (c) image with anisotropic method (SART + aTVM), (d) image with ROF-denoising (SART + ROF)

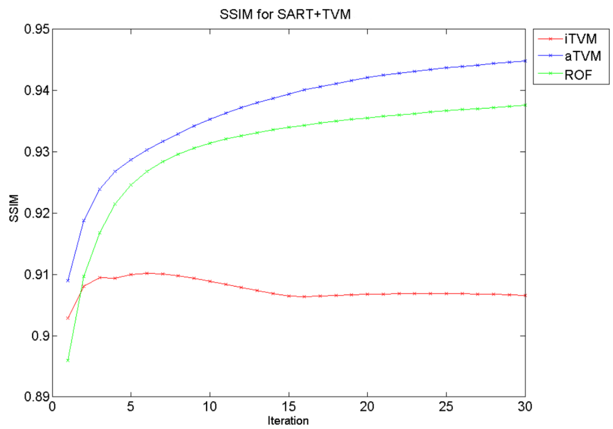

Figure 2: Similarity results for each iteration with all three methods used

4 Conclusion

This paper features a new method to preserve fine details in three dimensional limited angle tomography image reconstruction by using an anisotropic regularization scheme. The results demonstrate clear advantages over the isotropic method and can compete with standard approaches. Moreover, this algorithm handles high noise levels with low photon counts exceptionally well.

Author's Statement
Conflict of interest: Authors state no conflict of interest. Material and Methods: Informed consent: Informed consent has been obtained from all individuals included in this study. Ethical approval: The research related to human use has been complied with all the relevant national regulations, institutional policies and in accordance the tenets of the Helsinki Declaration, and has been approved by the authors' institutional review board or equivalent committee.

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