Research Article

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Design of a Robust sliding mode controller for bioreactor cultures in overflow metabolism via an interdisciplinary approach

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Abstract: Microorganism culture is highly complex due to the different metabolic pathways, which are very complex. A metabolic phenomenon called overflow is a challenge to overcome in automatic control tasks of microorganism cultures. In this study, a nonlinear algorithm by sliding modes (sliding mode nonlinear control, SMNC) is proposed for the robust regulation of a fed-batch bioreactor in the presence of parametric and system perturbations. A control design is obtained using Lyapunov functions by techniques to propose a control law such that it is robust, only the output signals (biomass and volume) are used, and the reaction rates do not have to be wholly known. Therefore, a novel and simple controller capable of solving the above problems is obtained.

Keywords: sliding mode control, control, bioreactor, fed-batch, overflow

1 Introduction

The demand for bio-products, such as food and medicines, is continuously growing, as is the world's population. At present, a large part of these biotechnological products are obtained using culture microorganisms in bioreactors [1,2]. One way to optimize these products is to increase and assure that biological growth rates are adapted under robust conditions to possible distortions by incorrect mathematical modeling, hypersensitivity, distortions in the inputs, etc. [3–5]. Besides, these growing conditions must be reproducible between batches in an automated way. Modeling, optimization, and control are fundamental tools for achieving these objectives. Dynamics of microorganisms' culture in bioreactors are commonly characterized by significant non-linearity, problems of bilinearity in the inputs of the systems, and strong interactions between the main variables of this: biomass, nutrients, hydrogen potential (pH), temperature, etc. [6,7]. An added difficulty is the high uncertainty and sensitivity of these since the mathematical models' parameters and states are commonly challenging to predict and estimate or even merely unknown.

In bioreactors with high-density culture, the limitations to oxygen transfer mean that the microorganism must use alternative metabolic pathways to obtain the energy necessary for growth and its maintenance due to the low solubility of oxygen the intense competition that exists for it [8]. These alternative routes consist of intermediate roads where high value-products are produced but usually toxic for microorganisms' same cultures [9,10]. The organisms will always choose an aerobic path as long as there is sufficient substrate and oxygen; when the concentration of oxygen decays to a specific limit, microorganisms' culture will change toward an anaerobic metabolic route [11,12].

The cultivation of microorganisms in high-density bioreactors is usually very complex, and therefore modeled

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under certain basic assumptions, in such a way to allow mathematical models to arrive sufficiently rich in information but without becoming unanalytically treatable. On an industrial scale, microorganisms' cultivation is carried out in the liquid phase in discontinuous bioreactors, (batch and fed-batch) inhomogeneous conditions [13]. In these equipment, it is usually controlling some essential variables such as temperature, pressure, and pH. In a high-density culture of microorganisms, it is necessary to monitor or control the biomass, the primary substrate (limiting), dissolved oxygen (DO), and some cultivation products [14]. In the case of in-line nutrient sensors, there are few prototypes of in-line nutrient meters; therefore, state observers are often used for in-line estimation [15].

An added difficulty is the high uncertainty and sensitivity of these since the mathematical models' parameters and states are commonly challenging to predict and estimate or even merely unknown [16]. Therefore, for the automatic control theory, the operation of this type of bioreactors in high-density cultures is a problematic challenge with conventional tools, even with more modern tools such as adaptive, output interval feed back control, etc., face problems due to the large uncertainties commonly encountered. Therefore, this problem is still open [17–22].

Recently, theoretical research has been highlighted as a contribution to solving the problem of culture of high-density microorganisms [23–25]. A remarkable result is an optimal control of the open loop, the technique of the so-called “search of the extreme”; it showed good performance, but the optimal open-loop control is usually not robust to internal disturbances and in the inputs to the system [8]. Adaptive controllers have also been used [18], which are especially useful for controlling systems containing uncertain parameters but fixed mathematical model structures. These are usually robust to variations in the parameters. However, these methods are not robust to perturbations non-parametric [26].

Techniques such as the model predictive control (MPC) have been studied in recent decades in the control of bioprocesses, usually used to maximize the concentration of biomass but which maintain problems in the robustness of internal disturbances very common in bioprocesses [27]. In addition, robust techniques such as sliding mode controllers have maintained better performance than MPC in such biological systems [28].

Sliding mode control (SMC) control is a robust controller, which can be used to reject all types of disturbances; therefore, this technique is the right choice for the case of cultures of high-density microorganisms, since it allows to avoid complex unmodeled uncertainties and external disturbances in a more feasible way than

classical techniques [29]. However, there are few works related to the use of sliding controllers for the implementation in bioreactors in overflow metabolism precursors, and this research topic is still open [30]. Since robust sliding techniques can resolve the discontinuous uncertainties inherent to this type of system, SMC control can significantly improve to efficiency in high-density microorganism cultures in the overflow metabolism presence, i.e., it is known that when a microorganisms changes its metabolic rate discontinuously, this problem is solved by using sliding techniques [30]. We propose a control law that takes no more than the biomass signal and the volume without knowing the real reaction rate, thus becoming a novel and realistic solution for real-time fermentations and processes. It is an extension of the paper published in ref. [6] since the previous study involved complete knowledge of all states and reaction rates to construct and calculate the controller. In the present work, we study the design of an SMC controller based on a reduced model that avoids the singularity, for its design only occupies the output signals (biomass and volume). Stability analyses in Lyapunov and a realistic numerical study based on simulations in the Matlab 2019a language are proposed. A broad explanation of the base modeling is presented in Section 2. Section 3 presents the control problem. Section 4 presents the main stage of this work since it presents the robust controller design. Section 5 presents the numerical results. Finally, Section 6 presents the conclusions of this work.

2 Mathematical model

To arrive at mathematical models and the design of automatic control techniques, it is necessary to make assumptions about the system to obtain robust controllers. Previous research has already been done on the high-density culture model, as shown in ref. [18]. Therefore, the following model is presented:

$$\begin{aligned}
 \dot{x}_1 &= (k_{1,1}r_1 + k_{1,2}r_2 + k_{1,3}r_3)x_1 - \frac{u(t)}{x_6}x_1 \\
 \dot{x}_2 &= -(k_{2,1}r_1 + k_{2,2}r_2)x_1 - \frac{u(t)}{x_6}(x_2 - \alpha_1) \\
 \dot{x}_3 &= -(k_{3,2}r_2 + k_{3,3}r_3)x_1 - \frac{u(t)}{x_6}x_3 \\
 \dot{x}_4 &= -(k_{4,1}r_1 + k_{4,3}r_3)x_1 - \frac{u(t)}{x_6}x_4 + \beta(x_4) \\
 \dot{x}_5 &= (k_{5,1}r_1 + k_{5,2}r_2 + k_{5,3}r_3)x_1 - \frac{u(t)}{x_6}x_2 + \gamma(x_5) \\
 \dot{x}_6 &= u(t).
 \end{aligned} \tag{1}$$

The above model is based on the following design conditions and characteristics:

- (1) The bioreactor is completely homogeneous (perfect blend).
- (2) The maternal model is unstructured, i.e., it does not include dynamics at the microscopic level.
- (3) It is kept in a bioreactor by fed-batch that maintains a perfect control of temperature (isothermal culture) and pH, so no balance of energy or hydrogen protons are required.
- (4) The microorganism presents overflow metabolism, which is about a nonstop change of metabolism between the aerobic and anaerobic phases.
- (5) The products of anaerobic fermentation are toxic and inhibit the growth of biomass.
- (6) The concentrations of the components are denoted by the following variables: x_1 biomass, x_2 nutrients, x_3 products, x_4 oxygen, x_5 carbon dioxide, and x_6 variable volume of the reactor, where $k_{j,i}$ are constants of stoichiometric and yields and r_i are the metabolic reaction rates. The control $u(t)$ corresponds to the system input volumetric flow, α_1 denotes the substrate concentration in the input, $\beta(x_4) = \lambda_\beta(\beta^* - x_4)$, $\gamma(x_5) = \lambda_\gamma(\gamma^* - x_5)$, where β^* is the CO_2 saturation concentration and γ^* is the oxygen saturation concentration. The λ_β and λ_γ are mass transfer coefficients.
- (7) The most important variable to control is the biomass concentration x_1 ; it is expected to maintain its dynamics, avoiding the anabolic metabolism; therefore, a design of control will be on this variable.
- (8) There is a substrate feedback filter in the bioreactor, so that it is possible to remove excess substrate such that $u(t) = F_s - F_{\text{fil}}$, where F_s is the inflow and F_{fil} is the nutrients outflow.
- (9) The only variables available for online measurement and calculation of a controller are biomass x_1 and volume x_6 .

The model describes, therefore, the cell catabolism through the following three main reactions:

The $r_i(t)$ reaction rates of (1) are the most complex part of the previous model, since in these, the nonlinearities and the possible discontinuous phenomena are due to overflow, and these have been modeled in the literature as follows:

$$r_1(t) = \alpha_1 \min(r_s, r_{\text{crit}}) \quad (2)$$

$$r_2(t) = \alpha_2 \max(0, r_s - r_{\text{crit}}) \quad (3)$$

$$r_3(t) = \alpha_3 \max\left(0, \frac{k_{4,1}(r_s - r_{\text{crit}})}{\alpha_3} \frac{x_3}{x_3 + \alpha_4}\right) \quad (4)$$

$$r_s = \alpha_5 \frac{x_2}{x_2 + \alpha_7} \quad (5)$$

$$r_{\text{crit}} = \frac{\alpha_6}{k_{4,1}} \frac{x_4}{(\alpha_8 + x_4)} \frac{\alpha_9}{(\alpha_9 + x_3)}. \quad (6)$$

Above terms use the common form of Monod array where α_5 and α_6 are the maximal constant of specific growth rates, α_7 and α_8 are the saturation constants of the corresponding nutrients and oxygen, and α_9 is the inhibition constant. α_4 and α_3 represent the coefficients characterizing, respectively, the yield between the oxygen and substrate consumptions. As it could see that the growth rates r_i are extremely changeable, they depend on the metabolism and the internal conditions of the system. These can be considered as a parameter that varies within time, therefore, by changing variables (1) to a full state mode, the following is achieved:

$$\begin{aligned} \dot{x} &= \Psi(u(t), x)\rho(t) + g(u(t), x) \\ y &= Cx, \end{aligned} \quad (7)$$

$$\text{where } \Psi(u(t), x) = \begin{bmatrix} k_{x1} & k_{x2} & k_{x3} \\ k_{s1} & k_{s2} & k_{s3} \\ k_{p1} & k_{p1} & k_{p1} \\ k_{o1} & k_{o2} & k_{o3} \\ k_{c1} & k_{c2} & k_{c3} \\ 0 & 0 & 0 \end{bmatrix} x_1, \quad \rho(t) = \begin{bmatrix} r_1(t) \\ r_2(t) \\ r_3(t) \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ and } g(u(t), x) = \begin{bmatrix} -\frac{x_1}{x_6}u(t) \\ -\frac{(x_2 - s_{\text{in}})}{x_6}u(t) \\ -\frac{x_3}{x_6}u(t) \\ -\frac{x_4}{x_6}u(t) + \beta(x_4) \\ -\frac{x_5}{x_6}u(t) + \gamma(x_5) \\ x_6 \end{bmatrix}.$$

3 Approach to the control problem

The biological system represented in (2)–(6) can be easily disturbed, due to changes that may occur due to changes in the internal metabolism of the microorganism. For this reason, there must be a problem of designing a robust control law to ensure the proper functioning of the bioreactor. In particular, we are interested in finding a control

law that will help us maximize the concentration of biomass. As mentioned earlier, a microorganism can have several metabolic pathways to obtain energy (oxidative metabolism and metabolism fermentative) [6].

The mathematical model of $r_i(t)$ reaction rates (2)–(6) is based on the literature [17]. This model describes that during a culture in a bioreactor with high-density cells, these are expected to shift their metabolism due to their limited oxidative capacity. When the substrate is in excess glucose consumption rate $r_S > r_{crit}$, the cells deliver a byproduct x_3 via the fermentative pathway. This system with excess oxygen will be the ideal model, i.e., the reference system $r_{1,r}(t) = r_{crit,r}(t)$. When the oxygen decreases in concentration, the dilution rate should be compensated in a robust way to avoid the phenomenon of overflow with the intention of not allowing fermentation and maximizing the biomass (Figure 1).

Here, the sub-indices r will denote the reference variable in all the variables defined earlier. It is well known that the input flow is given by an exponential function in our reference model we proposed to use the flow proposed in ref. [23]. Therefore, such that the nonlinear reference system is as follows:

$$\begin{aligned} \dot{x}_{1,r} &= (k_{1,1}r_{1,r})x_{1,r} - \frac{u_r(t)}{x_{6,r}}x_{1,r} \\ \dot{x}_{2,r} &= -(k_{2,1}r_{1,r})x_{1,r} - \frac{u_r(t)}{x_{6,r}}(x_{2,r} - s_{in}) \\ \dot{x}_{3,r} &= -\frac{u_r(t)}{x_{6,r}}x_{3,r} \\ \dot{x}_{4,r} &= -k_{4,1}r_{1,r}x_{1,r} - \frac{u_r(t)}{x_{6,r}}x_{4,r} + \lambda_\beta(\beta^* - x_4) \\ \dot{x}_{5,r} &= k_{5,1}r_{1,r}x_{1,r} - \frac{u_r(t)}{x_{6,r}}x_{5,r} + \lambda_y(y^* - x_5) \\ \dot{x}_{6,r} &= u_r(t) \\ u_r(t) &= bk_{1,1}r_{1,r} \exp(k_{1,1}r_{1,r}t). \end{aligned} \quad (8)$$

The overflow dynamic is quasi-optimal and unstable, because it potentially leads to a point of product production, i.e., fermentation [27], which is analogous to an unstable stability point in the inverted pendulum. As illustrated in Figure 2, which has already been utilized in ref. [31], it is appropriate to seek a quasi-optimal reference that ensures the maximum without the risk of instability and overflow.

4 Robust controller design

Based on previous research [6,17], it is known that it is possible to design a controller based on a robust trajectory

Figure 1: Overflow bioreactor and metabolism diagram (yeasts, bacteria, animal cells, etc.).

control of systems in overflow metabolism presence; for this, it is necessary to make a design by the variable to control and monitor; in this work, we want to keep the biomass dynamics at its maximum point, i.e., the $r_1 = r_{crit}$ also has the problem of bilinearity, which is why a logarithmic basis change will be proposed to deal with this analytical inconvenience, which can have repercussions on the calculation of singularities. The operation close to r_{crit} is a challenge from the control point of view because usually, this term is uncertain and variable. Any failure in operation or any uncontrolled perturbation can drive the system into fermentative metabolism, which affects the growth rate. The fundamental idea is to design a controller

Figure 2: Overflow problem, when the critical path is lower than the oxidative path, the system remains in oxidative mode, but when this threshold is reached, the system starts to ferment.

based on a reduced to the complete model, demonstrating asymptotic convergence in biomass state x_1 as a function of biomass:

$$\dot{x}_{1,r} = k_{1,1}r_{1,r}x_{1,r} - \frac{u_r(t)}{x_{6,r}}x_{1,r}. \quad (9)$$

With the above model, it is easy to see that it has been possible to reduce the problems due to bilinearity; therefore, a reduced model for the plant in the same sense shown above is proposed:

$$\dot{x}_1 = k_{1,1}r_1x_1 - \frac{u(t)}{x_6}x_1 + \delta x_1, \quad (10)$$

where $\delta(t) = \delta$ represents the terms and uncertainties due to model reduction and the interaction with the other state variables, it is proposed that this perturbation $\delta(t)$ is bounded and Lipschitz concerning the state such that: $|\delta(t)| \leq l_1|e|$ and $e = x_{1,r} - x_1$. Now, therefore, if the dynamics of the error is superimposed, and the two dynamics are substituted:

$$\begin{aligned} \dot{e} &= k_{1,1}r_{1,r}x_{1,r} - \frac{u_r(t)}{x_{6,r}}x_{1,r} - k_{1,1}r_1x_1 + \frac{u(t)}{x_6}x_1 - \delta x_1 \\ \dot{e} &= \underbrace{de - \delta x_1}_{\Delta_e} - \frac{u_r(t)}{x_{6,r}}x_{1,r} + \frac{u(t)}{x_6}x_1. \end{aligned} \quad (11)$$

Assumption 1

Due to the incredible complexity of this type of systems, the term $d = k_{1,1}(r_{1,r} - r_1)$ is usually very uncertain, which is known that by the nature of the system, this term is bounded, this term cannot be rebounded, When the system is at the limit of the fermentation, the r_{crit} , there is $0 < \Delta_e \leq l_{max}$ such that it is a constant yield such that Δ_e is a bounded function.

Therefore, tracking error dynamics is as follows:

$$\dot{e} = \Delta_e - \frac{u_r(t)}{x_{6,r}}x_{1,r} + \frac{u(t)}{x_6}x_1. \quad (12)$$

Now, if we propose the following sliding surface and its corresponding derivative:

$$s = e + k_1 \int e dt \quad (13)$$

$$\dot{s} = \dot{e} + k_1 e. \quad (14)$$

If we substitute the dynamics of the error:

$$\dot{s} = \Delta_e - \frac{u_r(t)}{x_{6,r}}x_{1,r} + \frac{u(t)}{x_6}x_1 + k_1 e. \quad (15)$$

Now, it is known that the sliding surface is sought to be stable; therefore, we have the following:

$$\dot{s} = -k_2 \text{sgn}(s) - k_3 s. \quad (16)$$

Such that:

$$-k_2 \text{sgn}(s) - k_3 s = \Delta_e - \frac{u_r(t)}{x_{6,r}}x_{1,r} + \frac{u(t)}{x_6}x_1 + k_1 e. \quad (17)$$

From here, the following sliding mode controller is proposed. In the control design, it is necessary to assume $\Delta_e = 0$ since it is a term with uncertainty, to be rejected by the control. Therefore, the following robust controller is proposed:

$$u(t) = \frac{x_6}{x_1} \left(-k_1 e - k_2 \text{sgn}(s) - k_3 s + \frac{u_r(t)}{x_{6,r}}x_{1,r} \right). \quad (18)$$

Main theorem SMNC (sliding mode nonlinear control)

In the system (7) where assumption 1 is satisfied, a robust control loop (18) will keep the system asymptotically stable in the presence of perturbations on the main state x_1 with $k_2 = l_{max}$ for $k_{1,2,3} > 0$.

Proof. Let it be the dynamic error, if the control law is substituted to it

$$\dot{e} = -k_2 \text{sgn}(s) - k_3 s - k_1 e + \psi_\delta.$$

Therefore, sliding surface is

$$\begin{aligned} \dot{s} &= \dot{e} + k_1 e \\ \dot{s} &= -k_2 \text{sgn}(s) - k_3 s + \psi_\delta. \end{aligned} \quad (19)$$

We propose a Lyapunov function and its derivatives sliding surface:

$$\begin{aligned} V &= 0.5s^2 \\ \dot{V} &= s\dot{s} \\ \dot{V} &= s(-k_2 \text{sgn}(s) - k_3 s + \Delta_e) \\ \dot{V} &\leq -k_3 s^2 - |s|(k_2 - l_{max}). \end{aligned} \quad (20)$$

Since $k_2 = l_\psi$,

$$\begin{aligned} s\dot{s} &\leq -k_3 s^2 \\ \dot{s} &\leq -k_3 s \\ s(t) &\leq s(0) \exp(-k_3 t), \end{aligned} \quad (21)$$

the sliding surface converges asymptotically to zero; therefore, the control is robust. $\square$

5 Numerical experiments

The simulation considers a yeast culture in a high-density fed-batch bioreactor, whose objective is to avoid to overflow metabolisms. This simulation is based on experimental data reported in ref. [18] and on numerical simulation work reported previously. The control proposed in this work is the robust control shown in the previous section.

As mentioned earlier, this controller has more advantages than the one shown in 2015 [6], since the previous controller required full knowledge of r_{crit} in the control computation, which can be a problem in addition to the singularity problem and the monitoring of all states. We in this work make a robust extension that only deals with the two measurable states (biomass and volume) without the need for complete knowledge of rate growth r_1 . Furthermore, in this simulation study, a perturbation of almost 50% of the nominal value is proposed to study the robustness of the system to parameter changes with the presented controller (Table 2). This type of parametric robustness is widely used in the area because bioreactors typically present this type of real problem [15].

To carry out the numerical simulation, it is proposed to use a numerical method, Dorman-Price of variable step on Matlab Simulink 2019 language. The following table shows the simulation parameters used for both the reference dynamics and the plant dynamics.

The reference dynamics are generally presented as oxidative dynamics in the absolute absence of the fermentative effect, i.e., when $r_1(t)$ is oxidative, assuming that the rate of oxygen consumption is never larger than the rate of carbon metabolism, as established in earlier work. With parameters in Tables 1 and 2 and $u_r(t) = k_{1,1}r_{1,r} \exp(k_{1,1}r_{1,r}t)$, where

Table 1: Kinetic, mass transfer coefficients, and input parameters

Parameters	Reference	Plant	Units
S_{in}	350	350	g/L
γ^*	0.035	0.035	g x_4 /L
β^*	1.286	1.286	g x_5 /L
α_1	1	1	g of x_1 /g of x_2
α_2	1	1	g of x_1 /g of x_2
α_3	1	2	g of x_2 /g of x_3
α_4	0.1	0.1	g of x_4 /g of x_2
α_5	3.5	3.5	h^{-1}
α_6	0.256	0.256	h^{-1}
α_7	0.1	0.1	g of x_2 /L
α_8	0.0001	0.0001	g of x_4 /L
α_9	10	10	g of x_3 /L
λ_β	1,000	1,000	h^{-1}
λ_γ	1,000	1,000	h^{-1}
$x_1(0)$	0.4	0.4	g of x_1 /L
$x_2(0)$	0.5	0.5	g of x_2 /L
$x_3(0)$	0	0	g of x_3 /L
$x_4(0)$	0.035	0.035	g of x_4 /L
$x_5(0)$	1.286	1.286	g of x_5 /L
$x_6(0)$	6.81	6.81	L

Table 2: Yield coefficients

Parameters	Reference	Plant	Units
$k_{1,1}$	0.49	$0.49 * 0.5$	g of x_1 /g of x_2
$k_{1,2}$	0.05	$0.05 * 0.5$	g of x_1 /g of x_2
$k_{1,3}$	0.72	$0.72 * 0.5$	g of x_1 /g of x_3
$k_{2,1}$	1	$1 * 0.5$	g of x_2 /g of x_1
$k_{2,2}$	1	$1 * 0.5$	g of x_2 /g of x_1
$k_{3,2}$	1	0.48	—
$k_{3,3}$	1	0.48	g of x_3 /g of x_2
$k_{4,1}$	0.3968	0.3968	g of x_4 /g of x_2
$k_{4,2}$	0	0	g of x_4 /g of x_2
$k_{4,3}$	1.104	1.104	g of x_4 /g of x_3
$k_{5,1}$	0.5897	0.5897	g of x_5 /g of x_3
$k_{5,2}$	0.4621	0.4621	g of x_5 /g of x_3
$k_{5,3}$	0.6249	0.6249	g of x_5 /g of x_3
$x_1(0)$			

$$\dot{x}_{1,r} = k_{1,1}r_{1,r}x_{1,r} - \frac{u_r(t)}{x_{6,r}}x_{1,r} \quad (22)$$

with

$$r_{1,r} = \alpha_5 \frac{x_{2,r}}{x_{2,r} + \alpha_7}. \quad (23)$$

Tables 1 and 2 show the parameters obtained from the experimentation [17]. In this work, we compare the SMNC with a PI Controller and the open-loop, and design to open loop is shown [23]. It is proposed to use the SMNC controller on the complete system and thus verify the effect of the control law, and how the system maintains stability in the presence of parametric uncertainties (see Tables 1 and 2); in turn, to inject realism to the numerical simulation, it is proposed to add Gaussian noise to the two output signals x_1 and x_2 , to match the accurate signals of these sensors.

$$\dot{x} = \Psi(u(t), x)\rho(t) + g(u(t), x)$$

$$y = Cx$$

$$u_{\text{SMNC}}(t) = \frac{x_6}{x_1} \left(-k_1 e - k_2 \text{sgn}(s) - k_3 s + \frac{u_r(t)}{x_{6,r}} x_{1,r} \right) \quad (24)$$

$$u_{\text{PI}}(t) = -k_p e - k_i \int e dt$$

$$u_{\text{open}}(t) = 74.9\mu \exp(\mu t).$$

To compare the resulting dynamics due to the input of the three different control signals, we propose to compare them concerning the calculation of the integral-squared error (ISE). In addition, the dynamics of the reaction rates r_1 shows how the SMNC controller maintains the dynamics in the oxidative pathway, i.e., the minimum product

Figure 3: Biomass dynamics as a function of the 3 different types of simulated controllers.

pressure (x_3) is proposed. Based on the parameters and studies in other works such as [6,18], 22 h of numerical simulation are proposed.

It is easy to see in Figure 3 that how the SMNC controller is far superior to its counterparts; even despite the substantial distortions due to the parametric changes, the system maintains the desired trajectory even despite noise generation. In its second place, the PI controller maintains adequate but not-so-robust dynamics; PI controllers are usually the first line ones as they have consistently demonstrated a good performance [quotes my paper]. In the case of the open-loop, it is clear that this is not an option in the presence of noise [15] (Table 3).

In Figure 4, it can appreciate the dynamics of the control; even though the SMNC control maintains a superior performance, it has an important dynamic on the actuators, which is a challenge to solve in the programming stage, an alternative is the use of low-pass filters and even the extension of fractional calculation in its implementation, but this remains open for further study. As the ISE dynamics in Figure 5 shows, the SMNC controller is far superior to the PI control and the open-loop, the latter of which had to be scaled to 10% of actual to compare with the closed-loop techniques. The SMNC is almost twice as

Figure 4: Control dynamics on bioreactor.

good as the PI control case, which is why this type of controller is a good choice for solving robust performance in the face of disturbances and overflow in bioreactors.

Table 4 shows some indices to compare the dynamics of the controllers based on the indices used in the literature: the integral of the time-weighted absolute error (ITAE), the integral of the absolute error (IAE), and the integral of the square error (ISE) [28]

It is clear that the SMNC control outperforms the other controls in two indexes. However, it should be noted that the PI-type controller has been shown in the literature [6,31] to be an excellent control for bioprocesses because it has proven to be robust in both input disturbance and parametric presence. Nonetheless, the

Figure 5: ISE dynamics of the numerical experiment.

Table 3: Parameters control

Parameters control	Value
k_1	0.5
k_2	0.05
k_3	0.72
k_p	0.3
k_i	0.03

Table 4: Error rate comparison

Error index	SMNC	PI	Open-Loop
ISE	9.2×10^{-4}	2.99×10^{-3}	0.11
ITAE	0.9086	0.8786	18.5
IAE	0.091	0.1531	1.5

Figure 6: Dynamics of the abrupt constant disturbance on input channel in the system.

SMNC controller outperforms the other controls by 30% in ISE and 59% in IAE (Table 4).

5.1 Abrupt disturbances rejection numerical simulation

This section illustrates the simulation results of the overflow system in the presence of constant abrupt perturbations that appear and fade overtime during the fermentation simulation. The same condition and settings described earlier will be used to ensure that a sudden and constant disturbance develops on the input channel, is shown in Figure 6.

As can be seen in Figure 7 and Table 5, the SMNC controller in this simulation stage (input distortion rate) is far superior to the other two types of control laws. In

Figure 7: Biomass dynamics in the presence of input disturbances.

Table 5: Error rate comparison

Error index	SMNC	PI	Open-Loop
ISE	2.2×10^{-3}	0.08	0.42
ITAE	0.94	4.29	21.87
IAE	0.11	0.70	2.51

comparison with ISE, the SMNC is 380% higher than PI, ITAE 456%, and IAE almost 700%.

6 Conclusion

This paper proposed a stable SMNC controller that is built based on a reduced model on the primary and measurable variable to solve the biomass trajectory tracking problem in a high-density culture in the presence of parametric distortions. The resulting controller has a simple structure when compared to other controllers suggested in the literature since it only occupies the knowledge of the output, biomass, and volume, which stimulates its use in an accurate operation, and also shows promising results in the presence of white noise. Using part of the measurable model, we investigated the asymptotic convergence of the biomass dynamics and asymptotic stability in the presence of variable distortions, the above, due to the use of sliding mode functions. We tested the controller output on the complete bioreactor model by numerical simulations, and it was found that the biomass dynamics followed a predetermined trajectory despite system perturbations. Compared with the PI control and open-loop, it was demonstrated that the SMNC maintains a considerable advantage since it improves the ISE by more than 30%. With input distortion rate, it is far superior with ISE and the SMNC is 380% higher than PI. This type of controller is suitable for implementation, but without saying that their performance can be further improved, currently, techniques such as quasi-sliding mode or fractional computation are being studied, but this is the subject of future work.

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